

Aliasing Free and Additive Error in Spectra for Alpha Stable Signals

R. Sabre

Abstract—This work focuses on the symmetric alpha stable process with continuous time frequently used in modeling the signal with indefinitely growing variance, often observed with an unknown additive error. The objective of this paper is to estimate this error from discrete observations of the signal. For that, we propose a method based on the smoothing of the observations via Jackson polynomial kernel and taking into account the width of the interval where the spectral density is non-zero. This technique allows avoiding the “Aliasing phenomenon” encountered when the estimation is made from the discrete observations of a process with continuous time. We have studied the convergence rate of the estimator and have shown that the convergence rate improves in the case where the spectral density is zero at the origin. Thus, we set up an estimator of the additive error that can be subtracted for approaching the original signal without error.

Keywords—Spectral density, stable processes, aliasing, p-adic.

I. INTRODUCTION

RECENT years have shown an increasing interest in the study of stationary stable processes and in general stationary processes with infinite variance. The harmonizable process is an important example of a symmetric α -stable stationary process, and its properties have been considered by numerous authors like [1]-[10] to name a few.

Stable symmetric processes find applications in various fields including: signal and image processing, hydrology, economics, electronic and electric, communications and radar applications ... see [11]-[23].

In this work, we consider, as in [4], a complex stationary symmetric α -stable (S α S) process: $X = \{X(t), -\infty < t < +\infty\}$, with $\alpha \in (0, 2)$; more specifically, X is a complex-valued stochastic process for which the finite dimensional characteristic function is:

$$E \exp \left\{ i \operatorname{Re} \sum_{j=1}^n z_j X(t_j) \right\} = \exp \left\{ -C_\alpha \int_{-\infty}^{+\infty} \left| \sum_{j=1}^n z_j e^{it_j u} \right|^\alpha \phi(u) du \right\}$$

with $z_j = x_j - iy_j$ and $C_\alpha = (\alpha\pi)^{-1} \int_0^\pi |\cos(\theta)|^\alpha d\theta$, where ϕ is a nonnegative integrable function called the spectral density of X . This spectral density plays a role analogous to that played by the usual power spectral density function of a second order stationary process. It is clear that the spectral

density ϕ fully describes the distribution of the process X . Alternatively X has the integral representation

$$X(t) = \int_{-\infty}^{\infty} e^{itu} d\xi(u) \quad (1)$$

where ξ is a complex-valued S α S process with independent isotropic increments. The stochastic integral in (1) is defined by means of convergence in probability, for details see [1], [5].

The spectral density function was already estimated by [4], when the process is continuous-time, by [24] when the process is discrete-time and by [25] when the process is p-adic-time.

This paper deals with the fairly common situation in practice, namely that the observations of the process are tainted by an unknown and constant error. The process $X_t = a + Z_t$ is observed instead of the process Z alone. The constant a is estimated in [26] when the process is discrete-time.

Our goal is to establish nonparametric estimate of a , from sample of the process $X(t_n)$ at instants t_n , where the sampling instants t_n are equally distant, i.e., $t_n = n\tau$, $\tau > 0$. It is known that aliasing occurs. For more details about aliasing phenomenon, see [27], [28]. To avoid this difficulty, we suppose that the spectral density ϕ is vanishing for $|\lambda| > \Omega$ where Ω is a nonnegative real number. The value of τ is chosen such that $\tau\Omega < \pi$.

In this paper we particularly study some cases where the spectral density is zero at the origin: $\phi(\lambda) = \sin^{2k\alpha}(\lambda/2) g(\lambda)$ and $\phi(\lambda) = |\lambda|^\beta g(\lambda)$. We show that the rate of convergence of the estimator of the constant “ a ” is improved.

This paper is organized as follows: The second section gives some definitions and properties of Jackson polynomial kernel and an estimator of the constant a is defined. We show that this estimate converges in probability to a . Then we show that the estimate converges to a in L_p ($p < \alpha$). Since the second moment of this process does not exist, the convergence in L_p substitutes the quadratic. In third section, the spectral density of Z is assumed vanishing at origin precisely: $\phi(\lambda) = |\lambda|^\beta g(\lambda)$. We improve the rate of convergence depending to the value of β . The fourth section is reserved to numerical studies.

R. Sabre is with the Laboratory Biogeoscience Laboratory Biogeosciences UMR CNRS, University of Burgundy, Agrosup, Department DSIP, Dijon, France (e-mail: r.sabre@agrosupdijon.fr).

II. ESTIMATION OF THE CONSTANT ERROR

As in [20], [24], we give the definition of the Jackson polynomial kernel: Let Z_1, \dots, Z_N observations of the process

$Z: (Z_{(n)})_{0 \leq n \leq N-1}$, where N satisfies:

$$N-1=2k(n-1) \quad \text{with} \quad n \in N \quad k \in N \cup \{1/2\}$$

if $k=1/2$ then $n=2n_1-1, n_1 \in N$.

The Jackson's polynomial kernel is defined by:

$$|H_N(\lambda)|^\alpha = |A_N H^{(N)}(\lambda)|^\alpha$$

where

$$H^{(N)}(\lambda) = \frac{1}{q_{k,n}} \left(\frac{\sin(\frac{n\lambda}{2})}{\sin(\frac{\lambda}{2})} \right)^{2k} \quad \text{with}$$

$$q_{k,n} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left(\frac{\sin(\frac{n\lambda}{2})}{\sin(\frac{\lambda}{2})} \right)^{2k} d\lambda.$$

In addition, we have $A_N = (B_{\alpha,N})^{\frac{1}{\alpha}}$ with

$$B_{\alpha,N} = \int_{-\pi}^{\pi} |H^{(N)}(\lambda)|^\alpha d\lambda.$$

We give the following lemmas which are used in the reminder of this paper. Their proofs are given in [24].

Lemma1. There is a non negative function h_k such as:

$$H^{(N)}(\lambda) = \sum_{m=-k(n-1)}^{k(n-1)} h_k \left(\frac{m}{n} \right) \cos(m\lambda)$$

Lemma2. Let

$$B'_{\alpha,N} = \int_{-\pi}^{\pi} \left| \frac{\sin \frac{n\lambda}{2}}{\sin \frac{\lambda}{2}} \right|^{2k\alpha} d\lambda \quad \text{and}$$

$$J_{N,\alpha} = \int_{-\pi}^{\pi} |u|^\gamma |H_N(\lambda)|^\alpha d\lambda,$$

where $\gamma \in]0, 2]$. Then

$$B'_{\alpha,N} \begin{cases} \geq 2\pi \left(\frac{2}{\pi} \right)^{2k\alpha} n^{2k\alpha-1} & \text{if } 0 < \alpha < 2 \\ \leq \frac{4\pi k\alpha}{2k\alpha-1} n^{2k\alpha-1} & \text{if } \frac{1}{2k} < \alpha < 2 \end{cases} \quad \text{and}$$

$$J_{N,\alpha} \leq \begin{cases} \frac{\pi^{\gamma+2k\alpha}}{2^{2k\alpha} (\gamma-2k\alpha+1) n^{2k\alpha-1}} & \text{if } \frac{1}{2k} < \alpha < \frac{\gamma+1}{2k} \\ \frac{2k\alpha \pi^{\gamma+2k\alpha}}{2^{2k\alpha} (\gamma+1)(2k\alpha-\gamma-1) n^\gamma} & \text{if } \frac{\gamma+1}{2k} < \alpha < 2 \end{cases}$$

The estimator of the a is constructed as follows:

$$\hat{a} = \frac{\tau^\alpha A_N}{H_N(0)} \sum_{n'=-k(n-1)}^{k(n-1)} h_k \left(\frac{n'}{n} \right) X(\tau n' + \tau k(n-1)). \quad (2)$$

Theorem1. Let p a real number such that $0 < p < \alpha$. Then

$$|\hat{a} - a|^p = O\left(\frac{1}{n^{p\alpha}}\right)$$

Proof. From (1) and (2), we have

$$\hat{a} = \frac{\tau^\alpha A_N}{H_N(0)} \sum_{n'=-k(n-1)}^{k(n-1)} h_k \left(\frac{n'}{n} \right) \int_{-\infty}^{\infty} \exp[i(\tau n' + \tau k(n-1))\lambda] d\xi(\lambda) + a.$$

As in [1], the characteristic function of $(\hat{a} - a)$ can be written as:

$$E \exp[i\Re r \bar{r}(\hat{a} - a)] = \exp[-C_\alpha |r|^\alpha \int_{-\infty}^{\infty} \tau \left| \frac{A_N}{H_N(0)} \sum_{n'=-k(n-1)}^{k(n-1)} h_k \left(\frac{n'}{n} \right) e^{in'\tau\lambda} \right|^\alpha d\xi(\lambda)].$$

where $r = r_1 + ir_2$. It is easy to show that:

$$E \exp[i\Re r \bar{r}(\hat{a} - a)] = \exp(-C_\alpha |r|^\alpha \psi_N),$$

where

$$\psi_N = \int_{-\infty}^{\infty} \frac{|H_N(\lambda)|^\alpha}{|H_N(0)|^\alpha} \phi\left(\frac{\lambda}{\tau}\right) d\lambda = \sum_{j \in \mathbb{Z}} \int_{(2j-1)\pi}^{(2j+1)\pi} \frac{|H_N(\lambda)|^\alpha}{|H_N(0)|^\alpha} \phi\left(\frac{\lambda}{\tau}\right) d\lambda.$$

Putting $\lambda = y - 2\pi j$ and using the fact that H_N is 2π -periodic, we obtain

$$\psi_N = \sum_{j \in \mathbb{Z}} \int_{-\pi}^{\pi} \frac{|H_N(\lambda)|^\alpha}{|H_N(0)|^\alpha} \phi_j(y) dy,$$

where $\phi_j(y) = \phi\left(\frac{y}{\tau} - \frac{2\pi}{\tau} j\right)$. Let j be an integer such that

$-\Omega < \frac{y-2\pi j}{\tau} < \Omega$. Using the fact that $\tau\Omega < \pi$ and $|y| < \pi$,

we get $|j| < \frac{\tau\Omega}{2\pi} + \frac{1}{2} < 1$ and then $j = 0$. Consequently:

$$\psi_N = \int_{-\pi}^{\pi} \frac{|H_N(\lambda)|^\alpha}{|H_N(0)|^\alpha} \phi\left(\frac{y}{\tau}\right) dy.$$

The function ϕ being bounded on $[-\pi, \pi]$ and $|H_N(\cdot)|^\alpha$ being a kernel, it can be shown that $\int_{-\pi}^{\pi} |H_N(\lambda)|^\alpha \phi(\lambda) d\lambda$ is converging to $\phi(0)$. On the other hand, from lemma 2, we have:

$$\frac{1}{|H_N(0)|^\alpha} = \frac{B'_{\alpha,N}}{n^{2k\alpha}} = O(1/n). \quad (3)$$

Therefore ψ_N converges to 0. Consequently, the characteristic function of $\hat{a} - a$ converges to 1 when N approaches infinity. Hence we have the convergence in probability of \hat{a} to a .

We study now the convergence of \hat{a} to a in L_p where $0 < p < \alpha$, which replaces the convergence in mean square, because the second order moment of X is infinity. Let

$$D_p = \Re e \int_{-\infty}^{\infty} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1 - e^{ir \cos \theta}}{|r|^{1+p}} dr d\theta.$$

Let $t = r'e^{i\theta'}$ and $x = \rho e^{i\tau_0}$. Assuming now $r = \varepsilon r'$, $\tau = \tau' - \tau_0$

$$D_p = \Re e \int_{-\infty}^{\infty} \int_{-\frac{\pi}{4}+\tau_0}^{\frac{\pi}{4}+\tau_0} \frac{1 - e^{i\varepsilon r' \cos(\tau' - \tau_0)}}{|\varepsilon|^{1+p} |r'|^{1+p}} \varepsilon dr' d\theta'$$

$$D_p |x|^p = \Re e \int_0^\infty \int_{-\frac{\pi}{4}+\tau_0}^{\frac{\pi}{4}+\tau_0} \frac{1 - e^{i\rho r' \cos(\tau' - \tau_0)}}{|t|^{1+p}} d|t| d\theta' \\ - \Re e \int_0^\infty \int_{-\frac{\pi}{4}+\tau_0}^{\frac{\pi}{4}+\tau_0} \frac{1 - e^{i\rho r' \cos(\tau' - \tau_0)}}{|t|^{1+p}} d|t| d\theta'.$$

Replacing in this formula x by $\hat{a} - a$, from the previous result we have

$$D_p E |x|^p = \int_{-\infty}^{\infty} \int_{-\frac{\pi}{4}+\tau_0}^{\frac{\pi}{4}+\tau_0} \frac{1 - e^{-C_\alpha |t|^\alpha \psi_N}}{|t|^{1+p}} d|t| d\theta' \\ = \frac{\pi}{2} \int_{-\infty}^{\infty} \frac{1 - e^{-C_\alpha |t|^\alpha \psi_N}}{|t|^{1+p}} dt.$$

Let $u = t[\psi_N]^\frac{1}{\alpha}$ and using (3), we obtain

$$\frac{2}{\pi} C_{p,\alpha} E |\hat{a} - a|^p = (\psi_N)^\frac{p}{\alpha} = O\left(\frac{1}{n^{p/\alpha}}\right)$$

where $C_{p,\alpha} = R_p F_{p,\alpha}^{-1} (C_\alpha)^\frac{p}{\alpha}$ with

$$R_p = \int \frac{1 - \cos(u)}{|u|^{1+p}} du \text{ and } F_{p,\alpha} = \int \frac{1 - e^{-|u|^\alpha}}{|u|^{1+p\alpha}} du.$$

III. THE RATE OF CONVERGENCE FOR PARTICULAR CASES

Suppose that the spectral density is zero at the origin. This assumption gives our estimator a better convergence rate.

Theorem2. Assume that the spectral density is satisfying: $\phi(\lambda) = |\lambda|^\beta g(\lambda)$ where $\beta \in]0, 2k\alpha - 1[$, $\lambda \in [-\pi, \pi]$ and $g(\lambda)$ is a bounded function on $[-\pi, \pi]$ and vanishing for $|\lambda| > \Omega$, continuous in neighborhood of 0 and $g(0) \neq 0$. Then

$$2^{4kp} L \tau^{2k\alpha - \beta} \leq \lim_{N \rightarrow \infty} n^\frac{p(\beta+1)}{\alpha} E |\hat{a} - a|^p \leq \tau^\beta \pi^{4kp} L,$$

where L is the following constant:

$$L = \frac{\pi}{2C_{p,\alpha}} \left[g(0) \int_{-\infty}^{\infty} \frac{|\sin u|^{2k\alpha}}{|u|^{2k\alpha - \beta}} du \right]^\frac{p}{\alpha}.$$

Proof. The function ψ_N can be written as:

$$\psi_N = n^{-2k\alpha} \int_{-\pi}^{\pi} \left| \frac{\sin n\lambda}{\sin \frac{\lambda}{2}} \right|^{2k\alpha} \left| \frac{\lambda}{\tau} \right|^\beta g\left(\frac{\lambda}{\tau}\right) d\lambda$$

Using the inequality: $\left| \sin \frac{x}{2} \right| \geq \frac{x}{\pi}$ $0 \leq x \leq \pi$, we

maximize ψ_N as:

$$\psi_N \leq \pi^{4k\alpha} n^{-2k\alpha} \tau^{-\beta} \int_{-\pi}^{\pi} \frac{\left| \sin \frac{n\lambda}{2} \right|^{2k\alpha}}{|\lambda|^{2k\alpha - \beta}} g\left(\frac{\lambda}{\tau}\right) d\lambda.$$

Putting $n\lambda = u$, we have

$$\psi_N \leq \pi^{4k\alpha} \tau^{-\beta} n^{-1-\beta} \int_{-\infty}^{\infty} \frac{\left| \sin \frac{u}{2} \right|^{2k\alpha}}{|u|^{2k\alpha - \beta}} g\left(\frac{u}{n\tau}\right) du$$

The Lebesgue's dominated convergence theorem gives the following convergence:

$$\lim_{N \rightarrow \infty} \int_{-\infty}^{\infty} \frac{\left| \sin \frac{u}{2} \right|^{2k\alpha}}{|u|^{2k\alpha-\beta}} g\left(\frac{u}{n\tau}\right) du = g(0) \int_{-\infty}^{\infty} \frac{\left| \sin \frac{u}{2} \right|^{2k\alpha}}{|u|^{2k\alpha-\beta}} du.$$

Lemma 2 gives:

$$\lim_{N \rightarrow \infty} n^{\frac{p(\beta+1)}{\alpha}} (\psi_N)_{\alpha}^{\frac{p}{\alpha}} \leq \pi^{4kp} \left(g(0) \int_{-\infty}^{+\infty} \frac{\left| \sin \frac{u}{2} \right|^{2k\alpha}}{|u|^{2k\alpha-\beta}} du \right)^{\frac{p}{\alpha}}. \quad (4)$$

Thus ψ_N converges to zero. Using the following inequality

$|\sin x| \leq |x| \quad \forall x \in [-\pi, \pi]$, we obtain:

$$\psi_N \geq 2^{4k\alpha} n^{-2k\alpha} \tau^{2k\alpha-\beta} \int_{-\pi}^{\pi} \frac{\left| \sin \frac{n\lambda}{2} \right|^{2k\alpha}}{|\lambda|^{2k\alpha-\beta}} g\left(\frac{\lambda}{\tau}\right) d\lambda.$$

Therefore

$$\begin{aligned} \psi_N &\geq 2^{4k\alpha} n^{-\beta-1} \tau^{2k\alpha-\beta} \int_{-\pi}^{\pi} \frac{\left| \sin \frac{u}{2} \right|^{2k\alpha}}{|u|^{2k\alpha-\beta}} g\left(\frac{u}{n\tau}\right) du \\ \lim_{N \rightarrow \infty} n^{\frac{p(\beta+1)}{\alpha}} (\psi_N)_{\alpha}^{\frac{p}{\alpha}} &\geq 2^{4kp} \tau^{2k\alpha-\beta} \left(g(0) \int_{-\infty}^{+\infty} \frac{\left| \sin \frac{u}{2} \right|^{2k\alpha}}{|u|^{2k\alpha-\beta}} du \right)^{\frac{p}{\alpha}}. \end{aligned}$$

The first equality of (4) reaches the result of this theorem.

Theorem3. Assuming that the spectral density satisfies:

$$\phi(\lambda) = \sin^{2k\alpha} \left(\frac{\lambda}{2} \right) g(\lambda)$$

where the function g is integrable on $[-\pi, \pi]$, $g(0) \neq 0$ and vanishing for $|\lambda| > \Omega$. Then

$$\begin{aligned} cte \pi 2C_{p,\alpha} \tau^{p\alpha} \left(\int_{-\pi}^{\pi} g(\lambda) d\lambda \right)^{\frac{p}{\alpha}} &\leq \lim_{N \rightarrow \infty} n^{2pk} E |\hat{a} - a|^p \\ &\leq \frac{\pi}{2C_{p,\alpha}} \tau^{\frac{p}{\alpha}} \left(\int_{-\pi}^{\pi} g(\lambda) d\lambda \right)^{\frac{p}{\alpha}} \end{aligned}$$

Proof. From the definition of ψ_N , we have

$$\psi_N = n^{-2k\alpha} \int_{-\pi}^{\pi} \left| \sin \frac{n\lambda}{2\tau} \right|^{2k\alpha} g\left(\frac{\lambda}{\tau}\right) d\lambda.$$

As $1 \leq k\alpha$ and the sinus function is smaller than 1, the next expression is connected to

$$\psi_N \leq n^{-2k\alpha} \int_{-\pi}^{\pi} \left| \sin \frac{n\lambda}{2\tau} \right|^{2k\alpha} g\left(\frac{\lambda}{\tau}\right) d\lambda$$

$$\psi_N \leq n^{-2k\alpha} \left[\int_{-\pi}^{\pi} g\left(\frac{\lambda}{\tau}\right) d\lambda \right].$$

So, from lemma 2, we have:

$$\lim_{N \rightarrow \infty} \psi_N n^{2k\alpha} \leq \tau \int_{-\pi}^{\pi} g(\lambda) d\lambda.$$

Using the fact that the sinus function is between -1 and 1 and $k\alpha < [k\alpha] + 1$ where $[k\alpha]$ represents the integer part of $k\alpha$, we obtain

$$\psi_N \geq n^{-2k\alpha} \int_{-\pi}^{\pi} \left[\left(\sin \frac{n\lambda}{2\tau} \right)^2 \right]^{[k\alpha]+1} g\left(\frac{\lambda}{\tau}\right) d\lambda$$

$$\psi_N \geq n^{-2k\alpha} \frac{n^{-1}}{2B'_{\alpha,N}} \int_{-\pi}^{\pi} \left[\frac{1 - \cos\left(\frac{n\lambda}{\tau}\right)}{2} \right]^{[k\alpha]+1} g\left(\frac{\lambda}{\tau}\right) d\lambda.$$

The binomial formula gives:

$$2^{[k\alpha]+1} \psi_N \geq n^{-2k\alpha} \tau \int_{-\pi}^{\pi} g(\lambda) d\lambda$$

IV. SIMULATION

The proposed estimator can be applied to concrete situations. For example, the arrival time of a signal sent by Wi-Fi using new wireless transmission technologies can be modeled by alpha stable process. Indeed, [29] proposed an arrival time model based on Poisson distributions. Reference [15] provided a model based on stable alpha distributions. The sum of arrival times modeled by independent and isotropic Poisson distributions can be represented by a stable harmonizable process like that given in (1), see [30]. When these signals are collected in an aquatic environment with a constant disturbance, the process will be added with a constant error ($X_n = Z_n + a$).

Throughout this section, we give the simulation of the studied process:

$$Z_t = \int_{-\infty}^{\infty} e^{it\lambda} d\xi(\lambda),$$

where $1 < \alpha < 2$ and ξ is a complex symmetric $S\alpha S$ measure on R with independent and isotropic increments and with control measure m such that $mdx = \phi(x)dx$. In order to achieve this, we use the series representation defined in [26]. Therefore, the process Z given in (8) can be expressed as:

$$Z_t = C_\alpha \left(\int \phi(x) dx \right)^{1/\alpha} \sum_{j=1}^{\infty} \varepsilon_j \Gamma_j^{-1/\alpha} e^{inV_j} e^{i\theta_j}$$

where ε_j is a sequence of i.i.d. random variables such as $P[\varepsilon_j = 0] = P[\varepsilon_j = 1] = 1/2$, Γ_k is a sequence of arrival times of Poisson process, V_j is a sequence of i.i.d. random variables independent of ε_k and of Γ_k having the same distribution of control measure m , which has probability density ϕ , θ_j are independent random variables, having the uniform distribution on $[-\pi, \pi]$, independent of ε_j , Γ_j and V_j .

To generate N values ($N = 101, 501, 1001$) of the process Z_{t_n} , we use the following steps:

- generate 2000 values of ε_j
- generate 2000 values of Γ_j
- generate 2000 values of V_j
- generate 2000 values of θ_j

Then we calculate for all $0 \leq n \leq N$:

$$Z_{t_n} = C_\alpha \left(\int \phi(x) dx \right)^{1/\alpha} \sum_{j=1}^{2000} \varepsilon_j \Gamma_j^{-1/\alpha} e^{inV_j} e^{i\theta_j} \text{ where the spectral}$$

density is chosen as $\phi(x) = |x|^\beta e^{-|x|}$ for $x \in [-\pi, \pi]$ and $\phi(x) = 0$ otherwise and $\alpha = 1.7$ and $k = 4$. The β is taken as 2, between 0 and $2k\alpha - 1$. Afterwards, we generate $X_{t_n} = a + Z_{t_n}$ where a is chosen equal to 40.

We calculate the estimator \hat{a} given in (1) for different sizes of sample $N = 101, 501, 1001$. The result is given in Table I.

TABLE I
THE VALUE OF THE ERROR ESTIMATOR FOR DIFFERENT SIZES OF SAMPLES

$2k\alpha - 1 = 12.6$	$\beta = 2$
N=101	$\hat{a} = 31,5$
N=501	$\hat{a} = 36,3$
N=1001	$\hat{a} = 41,05$

Comparing \hat{a} to 40 (value of the constant a), we find that the estimator \hat{a} increasingly approaches the constant error a when the sample size is large.

V.CONCLUSIONS

We give an estimator of the constant additive error in spectral representation of $S\alpha S$ process. This work could be applied to several cases when processes have an infinite variance and the observation of these processes is perturbed by a constant noise. For example:

- the decomposition of audio signals with background noise by separating the different musical instruments.
- the denoising of a degraded historical record. The signal is considered infinitely variable.

The perspective of this work is to optimize the smoothing parameters to have a better speed of convergence. For this purpose, the cross-validation method will be the most appropriate tool.

REFERENCES

- [1] S. Cambanis (1983) "Complex symmetric stable variables and processes" In P.K.SEN, ed, Contributions to Statistics: Essays in Honour of Norman L. Johnson North-Holland. New York.(P. K. Sen. ed.), pp. 63-79
- [2] S. Cambanis, and M. Maejima (1989). "Two classes of self-similar stable processes with stationary increments". Stochastic Process. Appl. Vol. 32, pp. 305-329
- [3] M.B. Marcus and K. Shen (1989). "Bounds for the expected number of level crossings of certain harmonizable infinitely divisible processes". Stochastic Process. Appl., Vol. 76, no. 1 pp 1-32.
- [4] E. Masry, S. Cambanis (1984). "Spectral density estimation for stationary stable processes". Stochastic processes and their applications, Vol. 18, pp. 1-31 North-Holland.
- [5] G. Samorodnitsky and M. Taqqu (1994). "Stable non Gaussian processes ». Chapman and Hall, New York.
- [6] K. Panki and S. Renming (2014). "Stable process with singular drift". Stochastic Process. Appl., Vol. 124, no. 7, pp. 2479-2516
- [7] C. Zhen-Qing and W. Longmin (2016). "Uniqueness of stable processes with drift." Proc. Amer. Math. Soc., Vol. 144, pp. 2661-2675
- [8] K. Panki, K. Takumagai and W. Jiang (2017). "Laws of the iterated logarithm for symmetric jump processes". Bernoulli, Vol. 23, n° 4 pp. 2330-2379.
- [9] M. Schilder (1970). "Some Structure Theorems for the Symmetric Stable Laws". Ann. Math. Statist., Vol. 41, no. 2, pp. 412-421.
- [10] R. Sabre (2012b). "Missing Observations and Evolutionary Spectrum for Random". International Journal of Future Generation Communication and Networking, Vol. 5, n° 4, pp. 55-64.
- [11] E. Sousa (1992). "Performance of a spread spectrum packet radio network link in a Poisson of interferences". IEEE Trans. Inform. Theory, Vol. 38, pp. 1743-1754
- [12] M. Shao and C.L. Nikias (1993). "Signal processing with fractional lower order moments: Stable processes and their applications", Proc. IEEE, Vol.81, pp. 986-1010
- [13] C.L. Nikias and M. Shao (1995). "Signal Processing with Alpha-Stable Distributions and Applications". Wiley, New York
- [14] S. Kogon and D. Manolakis (1996). "Signal modeling with self-similar alpha- stable processes: The fractional levy motion model". IEEE Trans. Signal Processing, Vol 44, pp. 1006-1010
- [15] N. Azzaoui, L. Clavier, R. Sabre, (2002). "Path delay model based on stable distribution for the 60GHz indoor channel" IEEE GLOBECOM, IEEE, pp. 441-467
- [16] J.P. Montillet and Yu. Kegen (2015). "Modeling Geodetic Processes with Levy alpha-Stable Distribution and FARIMA", Mathematical Geosciences. Vol. 47, no. 6, pp. 627-646.
- [17] M. Pereyra and H. Batalia (2012). "Modeling Ultrasound Echoes in Skin Tissues Using Symmetric alpha-Stable Processes". IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control, Vol. 59, n°. 1, pp. 60-72.
- [18] X. Zhong and A.B. Premkumar (2012). "Particle filtering for acoustic source tracking in impulsive noise with alpha-stable process". IEEE Sensors Journal, Vol. 13, no. 2, pp. 589 - 600.
- [19] Wu. Ligang and W. Zidong (2015). "Filtering and Control for Classes of Two-Dimensional Systems". The series Studies in Systems of, Decision

- and Control, Vol.18, pp. 1-29.
- [20] N. Demesh (1988). "Application of the polynomial kernels to the estimation of the spectra of discrete stable stationary processes". (Russian) Akad.Nauk.ukrain. S.S.R. Inst.Mat. Preprint 64, pp. 12-36
 - [21] F. Brice, F. Pene, and M. Wendler, (2017) "Stable limit theorem for U-statistic processes indexed by a random walk", Electron. Commun. Prob., Vol. 22, no. 9, pp.12-20.
 - [22] R. Sabre (2019). "Alpha Stable Filter and Distance for Multifocus Image Fusion". IJSPS, Vol. 7, no. 2, pp. 66-72.
 - [23] JN. Chen, J.C. Coquille, J.P. Douzals, R. Sabre (1997). "Frequency composition of traction and tillage forces on a mole plough". Soil and Tillage Research, Vol. 44, pp. 67-76.
 - [24] R. Sabre (1995). "Spectral density estimate for stationary symmetric stable random field", Applicationes Mathematicae, Vol. 23, n°. 2, pp. 107-133
 - [25] R. Sabre (2012a). "Spectral density estimate for alpha-stable p-adic processes". Revisita Statistica, Vol. 72, n°. 4, pp. 432-448.
 - [26] R. Sabre (2017). "Estimation of additive error in mixed spectra for stable process". Revisita Statistica, Vol. LXXVII, n°. 2, pp. 75-90.
 - [27] E. Masry, (1978). "Alias-free sampling: An alternative conceptualization and its applications", IEEE Trans. Information theory, Vol. 24, pp.317-324.
 - [28] R. Sabre (1994). « Estimation de la densité de la mesure spectrale mixte pour un processus symétrique stable strictement stationnaire », C. R. Acad. Sci. Paris, Vol. 319, série I, pp. -1310.
 - [29] L. Clavier, M. Rachdi, Y. Delignon, V. Letuc, C. Garnier, P.A. Roland (2001). "Wide band 60GHz indoor channel: characterization and statistical modelling". IEEE 54th VTC fall, Atlantic City, NJ USA, 7-11October, 2001.
 - [30] A. Janicki and A. WERON (1993). "Simulation and Chaotic Behavior of Alpha-stable Stochastic Processes". Series: Chapman and Hall/CRC Pure and Applied Mathematics, New York.

R.Sabre received the PhD degree in statistics from the University of Rouen, France, in 1993 and Habilitation to direct research (HdR) from the University of Burgundy, Dijon, France, in 2003. He joined Agrosup Dijon, France, in 1995, where he is an Associate Professor. From 1998 through 2010, he served as a member of Institut de Mathématiques de Bourgogne, France. He was a member of the Scientific Council AgroSup Dijon from 2009 to 2013. From 2012 to 2019, he has been a member of Laboratoire Electronique, Informatique, and Image (LE2I), France. Since 2019 he has been a member of Laboratory Biogeosciences UMR CNRS University Burgundy. He is author/co-author of numerous papers in scientific and technical journals and conference proceedings. His research interests lie in areas of statistical process and spectral analysis for signal and image processing.