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Elastic and Plastic Collision Comparison Using Finite Element Method

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***Abstract*—**The prevision of post-impact conditions and the behavior of the bodies during the impact have been object of several collision models. The formulation from Hertz's theory is generally used dated from the 19th century. These models consider the repulsive force as proportional to the deformation of the bodies under contact and may consider it proportional to the rate of deformation. The objective of the present work is to analyze the behavior of the bodies during impact using the Finite Element Method (FEM) with elastic and plastic material models. The main parameters to evaluate are, the contact force, the time of contact and the deformation of the bodies. An advantage of using the FEM approach is the possibility to apply a plastic deformation to the model according to the material definition: there will be used Johnson–Cook plasticity model whose parameters are obtained through empirical tests of real materials. This model allows analyzing the permanent deformation caused by impact, phenomenon observed in real world depending on the forces applied to the body. These results are compared between them and with the model-based Hertz theory.

***Keywords*—**Collision, finite element method, Hertz's Theory, impact models.

I. Introduction

THE collision problem dates from the 19th century when Hertz first presented a mathematical model to analyze the behavior of two spheres impacting each other [1].

Although the Hertz model is somewhat old, it has been the basis for many recent models [2], [3]. Basically, all these models take into account the reaction force caused by deformation and this force is estimated by the stiffness and damping coefficients according to the physical and geometrical properties of the spheres. Reference [4] showed that a linear damping model does not truthfully represent the physical nature of the energy transfer process. So, they proposed a model, also based on Hertz's theory, with a non-line damping force defined in terms of the penetration and its corresponding rate.

The development of numerical solutions allowed considering other factors that the collision models neglected due to their inherent limitations. The main tool used to obtain numerical solutions is the FEM, popularized by several commercial software. Even the sound propagation caused by the impact of the spheres can be studied, as in [5].

Several works use FEM to analyze the contact of the spheres. Reference [6] presents numerical experiments in the form of frictional elastoplastic finite element analysis (FEA) of spherical particles in contact.

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A similar work analyzes the energy dissipation using FEM and simulates the normal impact of spherical particles with a substract between then, which may be elastic or elastic-plastic, as depicted in [7].

Reference [8] affirms that best results regarding the parameters of the collision processes (forces, stress, strain, duration of contact, etc.) are obtained using the FEM. It is presented the results of FEM modeling of the collision process after a free fall of a sphere on an elastic-plastic plate of specific dimensions under the additional application of an external force. All these works present a common characteristic: the region of contact has its mesh refined to yield best results, which is also the strategy used in this paper.

II. Collision between Two Spheres

A. Mathematical Approach

As mentioned, the first attempt to model mathematically the collision between two spheres was developed by Hertz [1] who estimated the contact forces during collisions restricted to perfectly elastic bodies. He considered that during the contact between two spheres a and b, for example, the deformation of the spheres with radius R_a and R_b, respectively, is defined by the overlap of spheres.

The overlap is defined as $R_a + R_b - \left|\vec{r}_a - \vec{r}_b\right|$, where $\vec{r}_a$ and $\vec{r}_b$ are the positions of the centers of the spheres a and b. Thus, the equation $R_a + R_b - \left|\vec{r}_a - \vec{r}_b\right|$ is considered as the amount of overlap of sphere a over sphere b, as shown in Fig. 1.

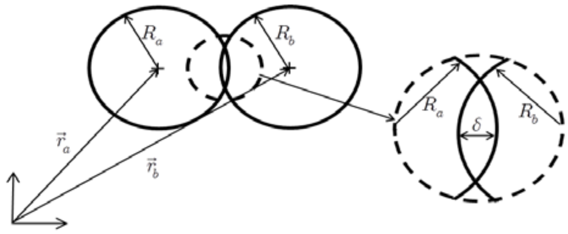

Fig. 1 Overlapping spheres with detail

This deformation generates a repulsive force between the spheres. The repulsive force, F_n, in Hertz's model is given by (1).

$$F_n = k_{HZ}\delta^{1.5} \tag{1}$$

where $\delta = \max(0, R_a + R_b - |\vec{r}_a - \vec{r}_b|)$ is the deformation between the spheres and this value is zero when there is no contact between the spheres and k_{HZ} is the stiffness between the spheres during the collision. As proposed numerically by [9] and experimentally by [10], the stiffness, k_{HZ}, for identical spheres can be calculated by the Theory of Elasticity as shown in (2).

$$k_{HZ} = \frac{5}{8}\left(\frac{E}{(1-\upsilon^2)}\right)\left(\frac{R}{2}\right)^{0.5} \quad (2)$$

where E is the Young modulus, ν is the Poisson coefficient, and R is the radius of the spheres.

There are many other analytical models, as discussed in [11] and in general all of them are based on Hertz's Theory and they take into account the same overlap, δ, presented in Fig. 1 besides the rate of deformation, $\dot{\delta}$.

B. FEM Simulation

In order to analyze the phenomenon of impact during collision, it was developed some FEM simulations of the spheres impact. For all simulations, it is considered that sphere a has an initial velocity and sphere b is at rest. The difference between the FEM simulations is in the materials definition to consider if it is a perfect elastic impact or a plastic one and the initial velocity of sphere a. For the first consideration, the elastic property is defined by the following material parameters for both spheres: modulus of elasticity, E = $1.93 \cdot 10^{11}$ N/m^2, Poisson's coefficient, υ = 0.35, yield stress, $Y = 3.1 \cdot 10^8$ N/m^2, density, ρ = 8030 kg/m^3 and radius, R = 0.0127 m. For the plastic model, the material feature is implemented using Johnson-Cook model, defined by the parameters listed in Table I [12].

TABLE I
JOHNSON-COOK PARAMETERS

Parameter	Value
A (MPa)	253.32
B (MPa)	685.1
n	0.3128
m	2.044
T_{melt} (K)	1698
T_{tr} (K)	296

In Table I, the parameters are defined as follows: *A* is the yield stress, *B* is the pre-exponential factor, *n* is the work-hardening exponent and *m* is the thermal-softening exponent, T_{melt} is the melting point of the material and T_{tr} is the temperature of the material.

As the only difference was the material specification, for both models the model of the representation of one sphere developed by FEM is shown in Fig. 2. The simulation was developed using the Dynamic explicit mode and it was applied an axisymmetric quadratic triangular element defined by CAX6M, element available in ABAQUS / CAE software. The right portion of the sphere a in Fig. 2 had its mesh refined due to this is the contact region to sphere b, that for its part, had the mesh of its left portion also refined. The sphere that will be impacted (sphere b) is not shown in Fig. 2 but the impact line, where the impact will occur, can be thought as a mirror line. The total number of elements of both spheres is 3441 and the total simulation time is about 27 minutes on a computer with an i5-3317U processor with 1.70 GHz and 6 GB RAM.

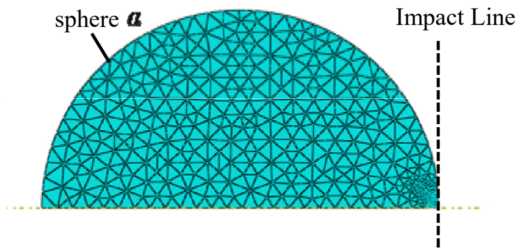

Fig. 2 Mesh applied to one axisymmetric profile

III. Results

A. Elastic Collision

In this section, it will be presented the main results of the simulations. For all FEM simulations, it was used as initial velocity of sphere a the following values: 2, 4, 8, 12, 16 and 20 m/s.

The first result obtained is shown in Fig. 3, where one can see the contact force profile for the initial velocities considered.

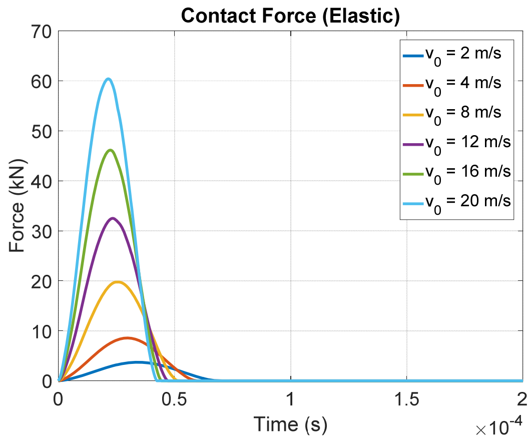

Fig. 3 Contact force for elastic model

Obviously as the initial velocity increases, the maximum contact force also increases. Table II summarizes the maximum force obtained for each initial velocity.

It is also observed (and intuitive) in Fig. 3 that the time of contact decreases as the initial velocity increases. Fig. 4 shows a detailed view of Fig. 3, highlighting the end of contact between the spheres, i.e., when the contact force becomes zero.

TABLE II
MAXIMUM CONTACT FORCE

Initial Velocity (m/s)	Force (kN)
2	3.71
4	8.55
8	19.75
12	32.49
16	46.16
20	60.41

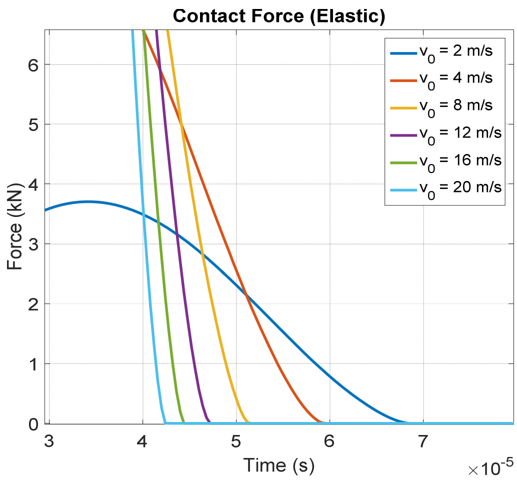

Fig. 4 Detail of the end of contact force for elastic model

Reference [13] shows an approximated formula to estimate the collision time between two elastic spheres using Hertz's model, as one can observe in (3):

$$\tau = 3.21\left(\frac{\mu^2}{e^2 r v}\right)^{\frac{1}{5}} \tag{3}$$

where μ is the reduced mass given by $m / 2$, r is the reduced radius calculated by $R / 2$, v is the velocity before the collision and e is the reduced elastic constant defined by (4):

$$e = \frac{4}{3}\left(\frac{2\left(1-v^2\right)}{E}\right)^{-1} \tag{4}$$

Table III shows the comparison between Hertz estimation collision time, as defined by (3) and MEF estimation collision time. One can verify that the maximum error found occurs for initial velocity of 16 m/s (1.4% of error), showing a good agreement in the results.

In Fig. 5, one can see the contact force versus the penetration (or deformation) of the spheres. Each maximum force is highlighted for the respective initial velocity, as the values depicted in Table II. These points on the chart represent the maximum penetration obtained.

TABLE III
COLLISION TIME

Initial Velocity (m/s)	Hertz Estimation (ms)	MEF Estimation (ms)
2	$6.8254 \cdot 10^{-02}$	$6.84 \cdot 10^{-02}$
4	$5.9419 \cdot 10^{-02}$	$5.96 \cdot 10^{-02}$
8	$5.1727 \cdot 10^{-02}$	$5.16 \cdot 10^{-02}$
12	$4.7698 \cdot 10^{-02}$	$4.72 \cdot 10^{-02}$
16	$4.5031 \cdot 10^{-02}$	$4.44 \cdot 10^{-02}$
20	$4.3066 \cdot 10^{-02}$	$4.28 \cdot 10^{-02}$

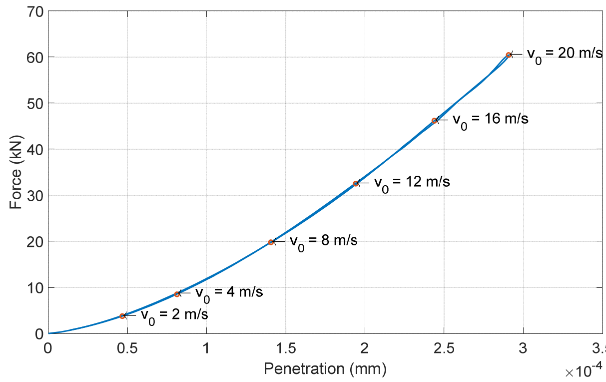

Fig. 5 Contact force versus penetration (elastic simulation)

It is possible to calculate analytically based on Hertz's theory the maximum deformation, δ_M, according to [13], as on can see in (5):

$$\delta_M = \left(\frac{5\mu}{4er^{\frac{1}{2}}}\right)^{\frac{2}{5}} v^{\frac{4}{5}} \tag{5}$$

where μ, r, e and v were already explained. Thus, a comparison between maximum deformation estimated by Hertz, as defined by (5) and maximum deformation obtained by MEF simulation acquired in Fig. 5 is made and the results are listed in Table IV. The maximum error achieved (0.92%) takes place when the initial velocity is 20 m/s, demonstrating a good similarity.

TABLE IV
MAXIMUM PENETRATION

Initial Velocity (m/s)	Hertz Estimation (mm)	MEF Estimation (mm)
2	$4.6496 \cdot 10^{-02}$	$4.6725 \cdot 10^{-02}$
4	$8.0955 \cdot 10^{-02}$	$8.1126 \cdot 10^{-02}$
8	$1.4095 \cdot 10^{-01}$	$1.4079 \cdot 10^{-01}$
12	$1.9496 \cdot 10^{-01}$	$1.9403 \cdot 10^{-01}$
16	$2.4541 \cdot 10^{-01}$	$2.4373 \cdot 10^{-01}$
20	$2.9337 \cdot 10^{-01}$	$2.9066 \cdot 10^{-01}$

B. Plastic Collision

Using Johnson-Cook model whose parameters are specified in Table I, a similar FEM simulation was developed with the same initial velocities applied in the elastic FEM simulation.

The contact force profiles can be observed in Fig. 6.

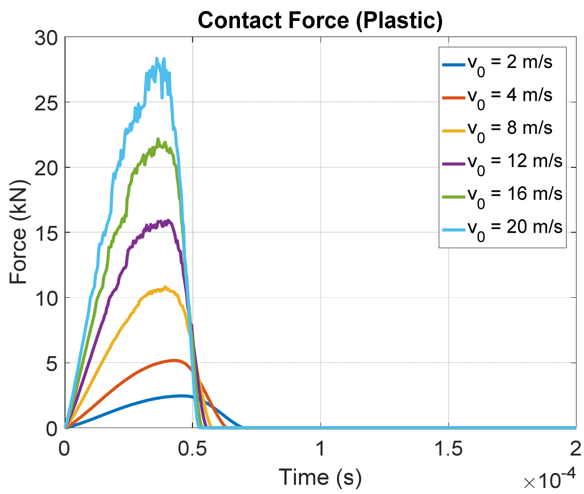

Fig. 6 Contact force for plastic model

As expected, the maximum peak forces in plastic simulation are lower than in elastic one, as one can see in Table V.

TABLE V
MAXIMUM ELASTIC AND PLASTIC CONTACT FORCE

Initial Velocity (m/s)	Elastic Force (kN)	Plastic Force (kN)	Difference (%)
2	3.71	2.45	33.87
4	8.55	5.16	39.61
8	19.75	10.82	45.19
12	32.49	15.93	50.97
16	46.16	22.18	51.96
20	60.41	28.35	53.06

As a manner to facilitate the comprehension of the amount of decreasing in maximum peak force of plastic simulation compared to elastic simulation, it was plotted in Fig. 7 where one can see the difference between the peak forces for both plastic and elastic simulations versus initial velocity.

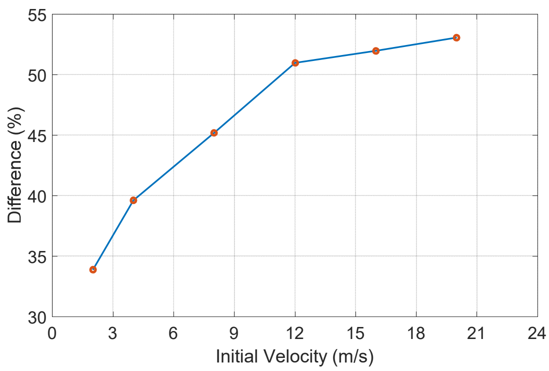

Fig. 7 Difference between the peak forces versus initial velocity

The last parameter analyzed is the contact force versus the penetration when the plastic deformation is considered. In Fig. 8, we can observe a permanent deformation increasing as initial velocity also increases once the amount of energy is bigger for higher initial velocities.

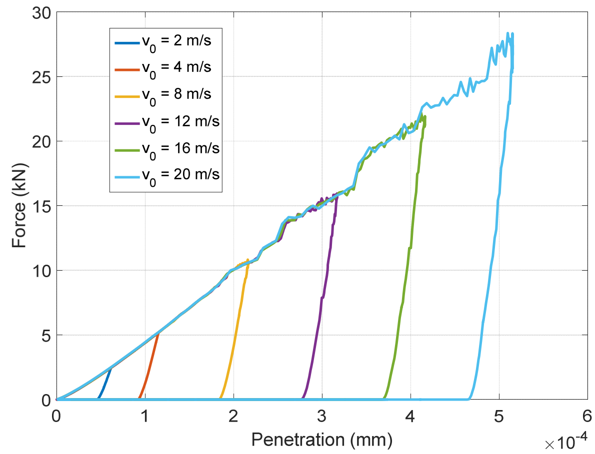

Fig. 8 Contact force versus penetration (plastic simulation)

In order to evaluate both elastic and plastic simulations of the contact force versus penetration, Fig. 9 joins the plots of Figs. 5 and 8, respectively the elastic and plastic simulations of impact.

It is clear the difference between the simulations evidenced by the different maximum forces, as well as the permanent deformations present in the plastic model, as larger as the initial velocity is increased.

Another difference in both models is the elastic simulation is perfect elastic and the plastic simulation is "perfect plastic", i.e., even for very small velocity, there will be a plastic deformation.

Finally, one last comparison that can be made is the maximum deformation reached. The maximum deformation in the plastic simulation is always higher than the maximum deformation reached by elastic simulation, considering the same initial velocity.

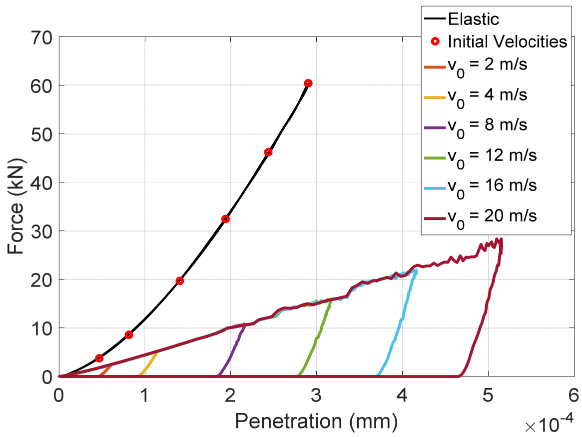

Fig. 9 Comparison contact force versus penetration (elastic simulation)

IV. CONCLUSION

Despite of the models of impact between solid bodies does not represent a recent discovery, but it still arouses interest in scientific community due to so many applications in academy and industry.

Hertz model is definitely the first to attempt to understand this unique phenomenon so called impact. After its pioneering, many other authors looked for evolutions for his model and took into account energy dissipation, not considered in perfect elastic models as Hertz model.

The FEM is used for solving so many problems and certainly Hertz never dreamed about such tool to solve his contact problem (once he passed away still in the XIX century).

The contact force measured by FEM in elastic model simulation and principally the time of contact between the spheres prove the agreement between Hertz model and numerical simulation. The other parameter compared, namely maximum penetration also shows an excellent agreement between the models.

Comparing the FEM simulation using Johnson–Cook plasticity model with FEM simulation using elastic model, we can observe the corresponding macroscopic behavior looks like the expected. However, the slope in force versus penetration plot is quite different, indicating an unusual behavior in elastic and plastic FEM models. It seems like the plastic model has no elasticity and the relation between stress and strain is totally different for both models.

We believe an elastoplastic model can be used to avoid these differences found between the models, applying a simulation even more realistic and in the future, we intend to investigate the elastoplastic models to use them in the simulations.

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