A Genetic Algorithm Approach Considering Zero Injection Bus Constraint Modeling for Optimal Phasor Measurement Unit Placement

G. Chandana Sushma, T. R. Jyothsna

Abstract—This paper presents optimal Phasor Measurement Unit (PMU) Placement in network using a genetic algorithm approach as it is infeasible and require high installation cost to place PMUs at every bus in network. This paper proposes optimal PMU allocation considering observability and redundancy utilizing Genetic Algorithm (GA) approach. The nonlinear constraints of buses are modeled to give accurate results. Constraints associated with Zero Injection (ZI) buses and radial buses are modeled to optimize number of locations for PMU placement. GA is modeled with ZI bus constraints to minimize number of locations without losing complete observability. Redundancy of every bus in network is computed to show optimum redundancy of complete system network. The performance of method is measured by Bus Observability Index (BOI) and Complete System Observability Performance Index (CSOPI). MATLAB simulations are carried out on IEEE -14, -30 and -57 bus-systems and compared with other methods in literature survey to show the effectiveness of the proposed approach.

Keywords—Constraints, genetic algorithm, observability, phasor measurement units, redundancy, synchrophasors, zero injection bus.

I. Introduction

ONE of modern developments in State Estimation (SE) process is the availability of "Synchrophasor" measurements and their incorporation into the SE process for accurate measurements [1]. Synchrophasor measurements allow direct measurement of phase angles associated with current and voltage measurements [1], [2]. PMUs allocated in the network are synchronized to common time reference provided by server in Global Position Systems (GPS).

In power system, complete observability of network is achieved when every bus of network can be estimated with available measurements. Synchrophasor measurements play a dynamic role in power system control and operation. It is active in providing reliable measurements for SE solution. PMU measurements are necessary in such a way that system network should remain observable during abnormal conditions also. However, this process relies on PMU locations to make network completely observable. PMU installation on every bus of network is not possible as its expenditure on installation is high.

In 1990s, Optimal PMU placement (OPP) in power system network is first introduced by [1]-[3]. Integer linear programming methods with different techniques have been proposed for OPP problems in [4]-[7]. A modified algorithm based on integer linear programming is proposed to recognize the effectiveness of channel capacity on PMU placement [8]. In [9], author proposed Binary Integer Linear Programming method for optimal PMU measurements, which are used in dynamic state estimation to obtain accurate measurements in presence of load changes. A Modified Imperialist Competitive Algorithm (MICA) is proposed for OPP in case of normal and abnormal conditions [10]. A Fuzzy based Binary Linear Programming (FBLP) is presented for OPP in case of normal and abnormal conditions to achieve complete observability [11]. An Artificial Bee Colony (ABC) is presented for PMU allocation in network in such way that it can be utilized both in presence of conventional flow measurements and ZI measurements [12]. A meta-heuristic algorithm titled as Modified Binary Cuckoo Optimization Algorithm (MBCOA) is proposed for OPP to obtain full observability [13]. Teaching-Learning-Based-Optimization (TBLO) Algorithm is presented for OPP with complete observability [14]. In [15], a Binary Particle Swarm Optimization (BPSO) is proposed to increase measurement redundancy of network with complete observability. Graph theory [16] is presented for OPP problem to achieve complete observability in which the author used spanning tree formulation to check the observability. In [18], the author proposed a modified Integer Linear Programming (ILP) method to recognize the channel capacity of PMU and optimal placement to attain complete observability. In [19], the author proposed Mixed ILP which deliberates number of channels connected, effect of ZI buses and contingencies. In [20] the author presented Mixed ILP and Non Linear programming for OPP problem. In [21] the author suggested an improved depth first algorithm for OPP problem considering bus weight. In [22] the author proposed Minimum Connectivity Based Reduction (MCBR) technique for placement of PMUs to achieve complete observability. In [23] the author presented a new topology method considering ZI buses and links between buses and leaf nodes to achieve system observability. In [24] the author proposed multi-objective Biogeography Based Optimization (BBO) for OPP problem to make system network completely observable.

ZI buses in network are considered in this paper for optimization. ZI constraints and radial bus measurements are considered as constraints to determine optimal PMU allocation in network. An optimization based on ZI constraints is developed and analyzed. The problem of OPP is considered with modeling of ZI constraints utilizing GA Approach.

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The rest of the paper is organized as follows: Section II deals with problem formulation for OPP with ZI modeling considering GA approach and formulation of performance index; Section III discusses simulation results and analysis of the problem utilizing different IEEE networks and Section IV concludes the problem.

II. PROBLEM FORMULATION

The constrained OPP using GA approach to obtain complete observabilty with optimal redundancy is formulated as

$$J(x) = Min \sum_{k=1}^{N} C_k x_k^2 \qquad (1)$$

Subjected to

$$b(x) = 0, \qquad 0 \le x \le 1 \qquad (2)$$

where x_k is a binary decision variable of bus-k for allocation of PMU in network C_k is PMU cost vector which is considered as a constant value for each bus, N is number of buses, $b(x)$ is a vector of non-linear bus observability constraints. For example, consider the single line diagram of 7-bus network as shown in Fig. 1.
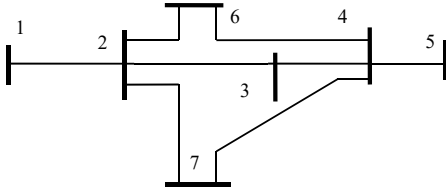

Fig. 1 Single line diagram of 7-bus network

The optimization problem for 7-bus network can be presented as

$$J(x) = Min \sum_{k=1}^{7} C_k x_k^2 \qquad (3)$$

Subjected to nonlinear observability constraints

$$b(x) = \begin{cases} b_1 = (1-x_1)(1-x_2) = 0 \\ b_2 = (1-x_1)(1-x_2)(1-x_3)(1-x_6)(1-x_7) = 0 \\ b_3 = (1-x_2)(1-x_3)(1-x_4) = 0 \\ b_4 = (1-x_3)(1-x_4)(1-x_6)(1-x_7) = 0 \\ b_5 = (1-x_4)(1-x_5) = 0 \\ b_6 = (1-x_2)(1-x_4)(1-x_6) = 0 \\ b_7 = (1-x_2)(1-x_4)(1-x_7) = 0 \end{cases} \qquad (4)$$

With GA approach OPP obtained for seven bus system are 2 and 4 locations. Complete observability is obtained with location of PMUs at 2 and 4 buses in network. Table I shows the fitness value or function value results without ZI modeling.

Bus-no	Function value	Bus-no	Function value
x_1	0.0001	x_5	0.0005
x_2	1.0000	x_6	0.0005
x_3	0.0000	x_7	0.0056
x_4	1.0000		

A. ZI Modeling

The buses in which current flow is negligible are considered as ZI buses. The power flown is very small that it is approximately equal to zero. The power flown is very small that it is approximately equal to zero. These buses are considered in optimization for OPP. The constraints associated with these buses can be modeled to reduce the number of locations for installation of PMUs.

Modeling of ZI constraints in nonlinear frame work is shown by considering an example. Here we suggest a technique to model ZI constraints with in a non-linear frame of work. Consider ZI bus at node 4 as shown in Fig. 2.
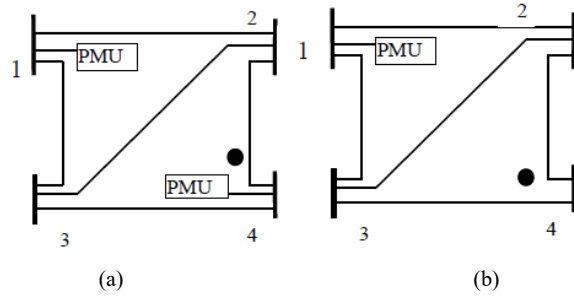

Fig. 2 Single line diagram of 4-bus network (a) with PMU and (b) without PMU at bus-4

When voltage phasors from buses 1 to $(N-1)$ are known then current $I_{k,1}$ is computed as

$$I_{k,1} = Y_{k,1}[e_k - e_1] \qquad (5)$$

where $Y_{k,1}$ is line admittance between bus 1 and k.

Bus N can be observed by computing bus voltage as:

$$e_N = V_1 - Z_{1,N} \sum_{k=2}^{N-1} I_{k,1} \qquad (6)$$

where $Z_{1,N}$ is line impedance between buses 1 and k.

As every ZI node leads to one additional constraint. The minimum number of PMUs required for observability is reduced by total number of ZI buses in network. For example, from Fig. 2, as Bus-4 is ZI bus.

$$I_{24} = I_{12} + I_{32}$$

As line currents are known, voltage at bus-4 can be formulated as

$$V_4 = V_2 - (I_{12} + I_{32})Z_{24}$$

Therefore by applying Kirchhoff law, voltage at node 4 can be computed. Hence PMU at node 4 is not required.

Consider 7-bus network in which bus-2 is considered as ZI bus. Bus-1 is radial bus which is connected to ZI bus. The power in bus-1 and 2 will be approximately equal to zero. In the proposed ZI modeling the constraint of bus -2 i.e., is substituted in (4) such that the constraints are reduced to as follows: The optimization problem for 7-bus network with ZI modeling can be formulated as

$$J(x) = Min \sum_{k=1}^{7} C_k x_k^2 \qquad (7)$$

Subjected to nonlinear observability constraints

$$b(x) = \begin{cases} b_4 = (1-x_3)(1-x_4)(1-x_7) = 0 \\ b_5 = (1-x_4)(1-x_5) = 0 \end{cases} \qquad (8)$$

With the application of ZI modeling, the placement of PMU locations can be reduced. The MATLAB simulation results obtained for the proposed ZI modeling show bus-4 as best function value to locate PMU. Table II shows fitness value or function value with ZI constraint modeling.

TABLE II
FUNCTION VALUE WITH ZI MODELING

Bus-no	Function Value	Bus-no	Function Value
x_1	0.0007	x_5	0.0011
x_2	0.0002	x_6	0.0000
x_3	0.0002	x_7	0.0017
x_4	1.0000		

B. GA Approach for OPP

GA was developed by John Holland at University of Michigan with his colleagues [17]. The procedure for GA constrained for OPP is shown in Fig. 3.

GA is stochastic method of optimization which tries to follow the process of natural evolution of species and genetics. They act on population of individuals, known as parents, best adapted to their atmosphere survive and could replicate. They are then exposed to mechanisms of recombination analogous to those of genetics. Interactions of genes in between parents outcomes information of new individuals, known as children [25]. The GA mainly vary from the other approaches in examination of the finest: They act on a set of alignments (populations) and not a single point. They utilize only values of task to be optimized, not its derivative or other supplementary information. They utilize transition rules. Fig. 3 demonstrates assembly of GA. The processes of different steps of genetic GA are as follows:

- Selection: The selection operator's role is to find which individuals of current population are allowed to replace parents. Selection is constructed on worth of individuals, estimated considering fitness value. The selection process is considered by ranking [25].
- Evaluation: This step is to estimate worth of newly produced individuals. Here, only the function is to be optimized. Assumptions are not considered in the function here, excluding that it could be base for selection process [25].
- Reproduction: The nominated parents are utilized to produce descendants. The two main processes are crossover, that combines genes of 2 parents, and mutation that involves light disturbance of genome. These processes are functional randomly using two limits, probability of crossing and probability of mutation [25]. These probabilities are significant limits, which are considered for quality of final results.

There are numerous types of crossovers and mutations. Among mutation process, there are binary, real, uniform, non-uniform mutation types. Before and after crossover examples are shown in Table III and IV.

TABLE III
BEFORE CROSSOVER

Parent 1	1	1	0	1	1	0	0	1	0
Parent 2	1	1	0	1	1	1	1	0	0

TABLE IV
AFTER CROSSOVER

Child 1	1	1	0	1	1	1	1	0	0
Child 2	1	1	0	1	1	0	0	1	0

Using mutation in GA, the function is able to optimize the PMU locations in terms of (1, 0) with their presence representing '1' and absence representing '0'. Tables V and VI show before and after mutations to show the difference in optimization locations.

TABLE V
BEFORE MUTATION

Parent 1	1	1	0	1	1	0	0	1	0

TABLE VI
AFTER MUTATION

Child 1	1	1	0	0	1	0	0	1	1

Stopping criteria: Ending process at correct time is vital from a real point of view. If there is no data about the directed value of preferred optimum, it is important to know when to end procedure [25]. In this paper, binary based GA approach is followed where allocation of PMU relates to binary decision set with size equal to number of nodes in bus network.

TABLE VII
PRESENCE AND ABSENCE OF PMUS

1	0	1	0	0	1	1	1

In Table VII, '1' represents PMU presence, '0' represents

absence of PMU.
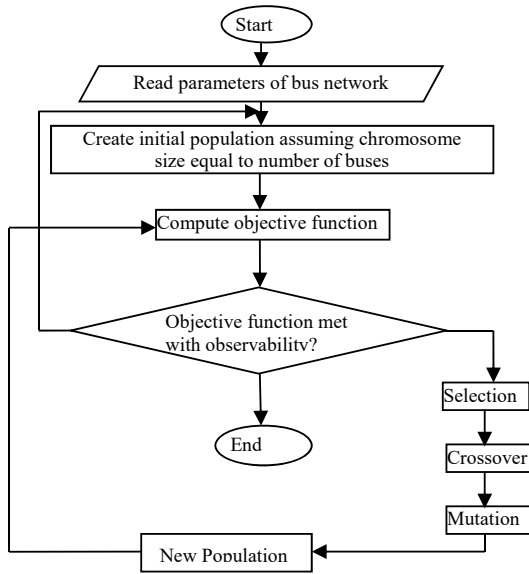

Fig. 3 Constrained GA Approach for OPP

GA steps for constrained PMU Placement are:

Step1. Read the parameters of bus network

Step2. Consider chromosome size is equal to number of N buses

Step3. Create the initial population

Step4. For each individual chromosome, check observability and calculate objective function

Step5. Apply selection operation

Step6. Apply crossover operator

Step7. Apply mutation operator

Step8. Go to step 4 until specified number of generations are completed.

C. CSOPI

To evaluate performance of observability achieved by the optimization method, it is necessary to formulate the best method to check. Here in this paper, we present an index method that checks complete observability of network. Observability of bus is measured by the BOI at every bus and it is defined as

$$\beta_j \leq \Re_j + 1 \tag{9}$$

BOI is limited to number of incident branches ($\Re_j$) plus one. Similarly, the observability of whole system network is estimated with CSOPI which can be derived as the sum of the indices at every bus of the network as:

$$CSOPI = \sum_{j=1}^{N} \beta_j \tag{10}$$

Maximum redundancy of bus can be formulated as

$$Max \sum_{k=1}^{n} by_k \tag{11}$$

Subjected to following constraints

$$\sum_{k=1}^{n} y_k = \mu_0 \tag{12}$$

where μ_0 is minimum number of PMUs obtained for complete network observability, b is bus connectivity matrix, y_k is binary decision variable.

$$y_k = \begin{cases} 1 & \text{if PMU is allocated at bus } k \\ 0 & \text{otherwise} \end{cases}$$

$$b_{i,j} = \begin{cases} 1 & \text{if } i = j \text{ or connected to each other} \\ 0 & \text{otherwise} \end{cases}$$

PMUs are allocated at redundant buses that can increase bus redundancy of network.

III. RESULTS AND ANALYSIS

The ZI modeled PMU placement for different test case network systems is formulated using constrained GA approach and programmed using MATLAB.
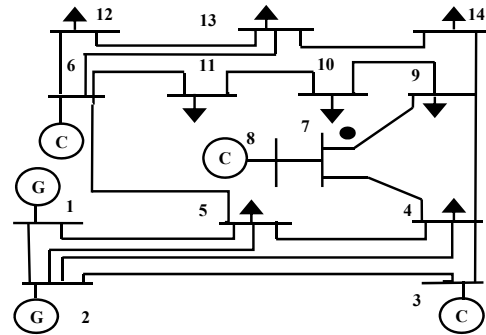

Fig. 4 Single line diagram of 14-bus network

Test cases 14-, 30- and 57 bus network systems are run on Intel(R) core(TM), i3 processor at 2.20 GHz, 4 GB of RAM. The information data of IEEE bus networks such as data of branches, ZI buses and radial buses are shown in Tables VIII-X.

TABLE VIII
14-BUS NETWORK DATA

Test case System	No. of Branches	ZI buses		Radial buses	
		Total Buses	Bus no	Total Buses	Bus no.
14-Bus System	20	1	7	1	8

The single line diagrams of 14-, 30- and 57-bus network are shown in Figs. 4-6. With application of constrained GA

approach, the optimal PMU locations are shown in Table XI. With application of ZI constraint modeling in GA approach, the optimal PMU locations obtained are shown in Table XII.

TABLE IX
30-BUS NETWORK DATA

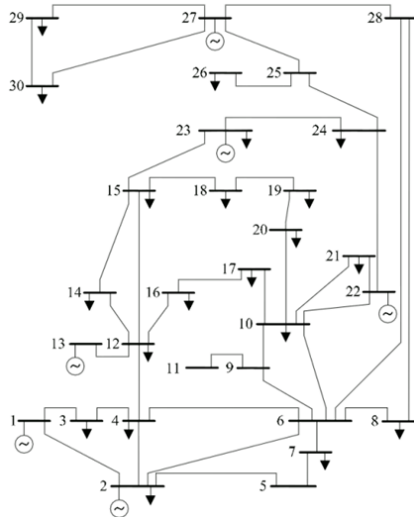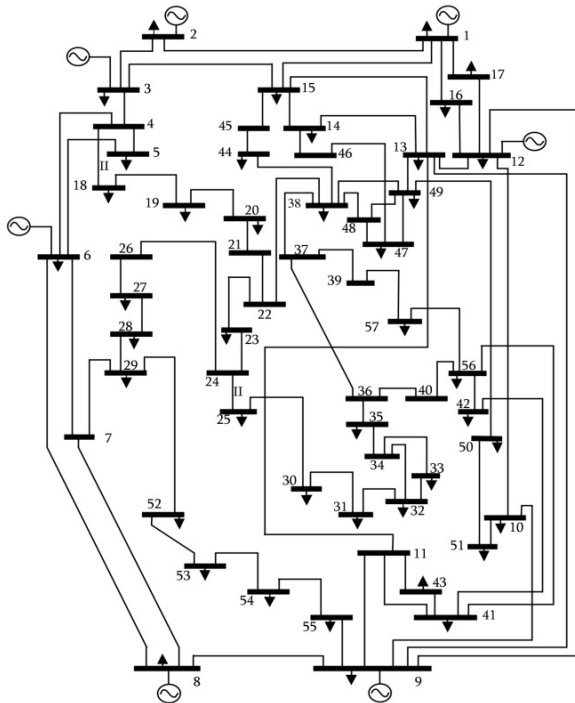| Test case System	No. of Branches	ZI buses		Radial buses	
		Total Buses	Bus no	Total Buses	Bus no.
30-Bus System	41	15	4,7,11,21,22, 24,26,34,36, 37,39,40,45, 46,48	3	11,13, 26

Fig. 5 Single line diagram of 30-bus network

Fig. 6 Single line diagram of 57-bus network

Comparing Tables X and XI and it is observed that placement of PMU with ZI constraint modeling decreases the PMU number thereby reducing the cost of installation of PMUs in network.

TABLE X
57-BUS NETWORK DATA

Test case System	No. of Branches	ZI buses	
		Total Buses	Bus no
57-Bus System	81	6	6,9,22,25,27,28

TABLE XI
PMU LOCATIONS WITHOUT ZI MODELING

IEEE test systems	No of PMUs	PMU locations
14 bus	4	2,6,7,9
30 bus	10	1,6,7,10,11,12,19,23, 25, 30
57 bus	17	1, 6, 12,15,19,22, 25,27,29, 32,36,38,39,41,46, 51, 54

TABLE XII
PMU LOCATIONS WITH ZI MODELING

IEEE test systems	No of PMUs	PMU locations
14 bus	3	2,6,9
30 bus	7	3,7,10,12,19,23,29
57 bus	13	1,6,9,14,20,25,27,32,37,38,50,53,56

The performance of network observability is determined with the proposed CSOPI index. Table XIII shows the performance Index of IEEE test case networks.

TABLE XIII
CSOPI

IEEE Test systems	CSOPI	
	with ZI modeling	without ZI modeling
14 bus	15	19
30 bus	32	42
57bus	57	70

From Table XIII it is observed that with optimal measurements obtained from ZI modeling, redundancy is less compared to without ZI modeling.

TABLE XIV
BRI OF 14-BUS NETWORK

Bus No	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	B11	B12	B13	B14
BRI	1	1	1	3	2	1	2	1	2	1	1	1	1	1

Tables XIV-XVI show the bus redundancy index of 14, 30, and 57 bus systems.

From Tables XIV-XVI, it is observed that the redundancy at weak buses is high compared to other buses due to PMU placement.

The proposed constrained GA approach is compared with other methods in the literature to show its effectiveness. Table XVII shows the comparison of proposed constrained method with other methods.

TABLE XV
BRI OF 30-BUS NETWORK

Bus No	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	B11	B12	B13	B14	B15	B16	B17	B18	B19	B20	B21	B22	B23	B24	B25	B26	B27	B28	B29	B30
BRI	1	2	1	2	1	3	2	1	3	2	1	1	1	1	2	1	1	1	1	2	1	1	1	2	1	1	2	1	1	1

TABLE XVI
BRI OF 57-BUS NETWORK

Bus No	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	B11	B12	B13	B14	B15	B16	B17	B18	B19	B20	B21	B22	B23	B24	B25	B26	B27	B28	B29	B30
BRI	2	1	1	1	1	2	1	1	1	2	1	1	2	2	2	2	1	1	1	2	1	1	1	1	1	1	1	1	1	1

Bus No	B31	B32	B33	B34	B35	B36	B37	B38	B39	B40	B41	B42	B43	B44	B45	B46	B47	B48	B49	B50	B51	B52	B53	B54	B55	B56	B57
BRI	1	1	1	1	1	1	3	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

TABLE XVII
COMPARISON OF PROPOSED CONSTRAINED METHOD WITH OTHER METHODS FOR COMPLETE OBSERVABILITY

Methods	14-bus system	30-bus System	57-bus System
Generalized ILP[4]	4	10	17
MILP[7]	3	-	14
BILP[9]	3	7	-
MICA[10]	4	10	17
FBLP[11]	4	10	17
BABC[12]	4	10	-
MBCOA[13]	4	10	17
TBLO[14]	4	10	-
BPSO[15]	4	10	17
Graph Theory[16]	4	10	
Proposed GA with ZI constraint modeling	3	7	13

IV. CONCLUSION

This paper presented constrained GA approach for OPP to attain complete observability of the networks. ZI constraint modeling is utilized for PMU placement in the network. ZI constraints are modeled in the nonlinear frame with a GA approach to minimize the bus locations. ZI constraint modeling decreases and optimizes the locations for placement of PMUs. Bus Redundancy Index for IEEE bus networks is computed to show redundancy of the buses. Complete observability of network is achieved with ZI constraint modeling in GA approach. CSOPI is presented to check observability of the complete network. IEEE -14, -30 and -57 bus networks are tested with MATLAB programming. This method is compared with other methods in the literature to show its effectiveness.

REFERENCES

[1] A. Abur and A.G. Exposito, Power system state estimation: theory and implementation. CRC press, 2004.

[2] AG. Phadke and J.S. Thorp, Synchronized phasor measurements and their applications, vol. 1. New York, NY: Springer, 2008.

[3] B. Xu, and A. Abur, "Observability analysis and measurement placement for systems with PMUs," *Proc. Power Systems Conference and Exposition*, 2004. IEEE PES, IEEE, 2004.

[4] B. Gou, "Generalized integer linear programming formulation for optimal PMU placement," *IEEE Transactions on Power Systems*, vol. 23, no. 3, 2008, pp.1099-1104.

[5] S. Chakrabarti, E. Kyriakides and D.G. Eliades,"Placement of synchronized measurements for power system observability," *IEEE Transactions on Power Delivery*, vol. 24, no. 1, 2009, pp. 12-19.

[6] B. Gou, "Optimal placement of PMUs by integer linear programming," *IEEE Transactions on power systems*, vol. 23, no. 3, 2008, pp. 1525-1526.

[7] D. Dua, S. Dambhare, R.K.Gajbhiye and S.A.Soman, "Optimal multistage scheduling of PMU placement: An ILP approach," *IEEE Transactions on Power Delivery*, vol. 23, no. 4, 2008, pp. 1812-1820.

[8] F. Rashidi, E. Abiri, T. Niknam and M.R. Salehi, "Optimal placement of PMUs with limited number of channels for complete topological observability of power systems under various contingencies," *International Journal of Electrical Power & Energy Systems*, vol. 67, 2015, pp. 125-137.

[9] M. Ravindra and R. Srinivasa Rao, "Dynamic state estimation solution with optimal allocation of PMUs in presence of load changes," *Proc. Intelligent Control Power and Instrumentation (ICICPI), International Conference on. IEEE*, 2016.

[10] S.A.Taher, H.nMahmoodi, and H. Aghaamouei, "Optimal PMU location in power systems using MICA," *Alexandria Engineering Journal*, vol. 55, no. 1, 2016, pp. 399-406.

[11] R.A. El-Sehiemy, S.H. Aleem, A.Y. Abdelaziz and M.E. Balci, "A new fuzzy framework for the optimal placement of phasor measurement units under normal and abnormal conditions," *Resource-Efficient Technologies*,vol. 3, no. 4, 2017, pp. 542-549.

[12] S. Li and Z. Meng, "Optimal PMU placement based on improved binary artificial bee colony algorithm," *Proc. Transportation Electrification Asia-Pacific (ITEC Asia-Pacific)*, 2017 IEEE Conference and Exp. IEEE, 2017.

[13] M. Dalali, and H.K. Karegar, "Optimal PMU placement for full observability of the power network with maximum redundancy using modified binary cuckoo optimisation algorithm," *IET Generation, Transmission & Distribution*, vol. 10, no. 11, 2016, pp. 2817-2824.

[14] A. Raj and C. Venkaiah, "Optimal PMU placement by teaching-learning based optimization algorithm," *proc. Systems Conference (NSC), 2015 39th National. IEEE* 2015

[15] A. Ahmadi, Y. Alinejad-Beromi, and M. Moradi, "Optimal PMU placement for power system observability using binary particle swarm optimization and considering measurement redundancy," *Expert Systems with Applications*, vol. 38, no. 6, 2011, pp. 7263-7269.

[16] P.K. Ghosh, and A. Tahabilder, "Optimal PMU placement for complete system observability and fault observability using graph theory," *proc. Electrical Engineering Congress (iEECON),* 2017 International. IEEE, 2017.

[17] J.H. Holland, "Genetic algorithms," *Scientific american*, vol. 267, no. 1 1992, pp. 66-73.

[18] F. Rashidi, A. Ebrahim, T. Niknam and M.R.Salehi, "Optimal placement of PMUs with limited number of channels for complete topological observability of power systems under various contingencies," *International Journal of Electrical Power and Energy Systems*, vol. 67, 2015, pp.125-137.

[19] M.V. Khokhlov, A. Obushevs, I.Oleinikova and A. Mutule,"Optimal PMU placement for topological observability of power system: *Robust measurement design in the space of phasor variables*," PES Innovative Smart Grid Technologies Conference Europe (ISGT-Europe), 2016 IEEE. IEEE, 2016.

[20] A. Almunif, and L. Fan, "Mixed Integer Linear Programming and Nonlinear Programming for Optimal PMU Placement," *Power Symposium (NAPS), North America, IEEE conference* 2017.

[21] X. Gao, "An optimal PMU placement method considering bus weight and voltage stability," *Environment and Electrical Engineering (EEEIC), 12th International Conference on. IEEE*, 2013.

[22] A.P. Singh, B. Nagu, N.V. Phanedra babu, and R.V. Jain, "Minimum connectivity based technique for PMU placement in power systems," 2017 *8th International Conference on Computing,*

Communication and Networking Technologies (ICCCNT). IEEE, 2017.

[23] W. Hongyu, X. Cheng, and X. Zong, "Optimal PMU Placement for the System Observability Based on System Topology Model," Trustworthy Systems and their Applications (TSA), 2016 *Third International Conference on. IEEE*, 2016.

[24] K. Jamuna and K. S. Swarup, "Multi-objective biogeography based optimization for optimal PMU placement", *Applied Soft Computing*, vol. 12, no.5, 2012, pp.1503-1510.

[25] Allagui, B., H. Ben Aribia, and Hsan Hadj Abdallah. "Optimal placement of Phasor Measurement Units by genetic algorithm." *Renewable Energies and Vehicular Technology (REVET), 2012 First International Conference on*. IEEE, 2012.