# Specific Frequency of Globular Clusters in Different Galaxy Types

Ahmed H. Abdullah, Pavel Kroupa

*Abstract*—Globular clusters (GC) are important objects for tracing the early evolution of a galaxy. We study the correlation between the cluster population and the global properties of the host galaxy. We found that the correlation between cluster population $(N_{GC})$ and the baryonic mass $(M_b)$ of the host galaxy are best described as $10^{-5.6038}M_b$. In order to understand the origin of the U -shape relation between the GC specific frequency $(S_N)$ and $M_b$ (caused by the high value of $S_N$ for dwarfs galaxies and giant ellipticals and a minimum $S_N$ for intermediate mass galaxies$\approx 10^{10}M\odot$), we derive a theoretical model for the specific frequency $(S_{Nth})$. The theoretical model for $S_{Nth}$ is based on the slope of the power-law embedded cluster mass function $(\beta)$ and different time scale $(\Delta t)$ of the forming galaxy. Our results show a good agreement between the observation and the model at a certain $\beta$ and $\Delta t$. The model seems able to reproduce higher value of $S_{Nth}$ of $\beta = 1.5$ at the midst formation time scale.

*Keywords*—Galaxies, dwarf, globular cluster, specific frequency, formation time scale.

## I. INTRODUCTION

**G**LOBULAR clusters (GCs) are spherical concentrations of stars $(10^4\_10^7$ stars), which are made up of population II objects (i.e., old stars) and are regarded as one of first stellar systems to form in the early Universe. The luminosity and compact size (half-light radii of a few pc) of GCs lead to the brightest objects that can be recognized around galaxies out to galactocentric radii $\approx 200$ kpc [1]. Globular clusters are found within different morphological types of galaxies, from irregulars to spiral and elliptical galaxies. Probably most of the GCs formed at the same time as their host galaxy, so that the global properties of the GCs can be considered as a key object to study the formation and evolution of galaxies. For the purpose of developing an understanding of the formation efficiency of globular clusters as a function of galaxy luminosity (or mass), their total number normalized to specific frequency $S_N$. Specific frequency which is the number of globular clusters $N_{GC}$ per unit $V$-band luminosity, normalized at $Mv = -15$ [2]

$$S_N = N_{GC}10^{0.4(M_v+15)} \tag{1}$$

This simple parameter $S_N$ was introduced by Harris and van den Bergh [2] as a measure of the richness of a GC system normalized to the host galaxy luminosity. This measure shows

Ahmed H. Abdullah is with the Department of Astronomy, College of Science, University of Baghdad, Baghdad, 10071, Iraq (e-mail: vipahmed.hasan@gmail.com).

Pavel Kroupa is with the Argelander Institut für Astronomie der Universität Bonn, Auf dem Hügel 71, D-53121 Bonn, Germany, Helmholtz-Institut fuer Strahlen- und Kernphysik, Universität Bonn, Nussallee 14-16, D-53115 Bonn, Germany.

that the $S_N$ varies between galaxies of different morphological type. The spiral galaxies have $S_N$ between 0.5 to 2 [3]-[5]. For more luminous ellipticals galaxies the $S_N$ ranges from $\approx 2$ to 10 and tends to increase with luminosity. The $S_N$ increases from few to several dozen for early-type dwarfs galaxies [6]-[8], and for late type dwarfs galaxies [9], [10].

The $S_N$ is affected by environments, for dEs galaxy in denser environments $S_N$ is higher while for Es galaxy in rich cluster is smaller [11]-[14]. The cD galaxy (central dominant elliptical galaxy) having $S_N$ value larger than 10 [15], [16]. Forbes et al. [17] proposed a tidal stripping model of GCs from smaller galaxies to explain the increasing value of $S_N$ in cD galaxies. In this work, we construct a model to calculate the theoretical specific frequency $(S_{Nth})$ for early type galaxies depending on the slope of the embedded cluster mass function $(\beta)$ and different formation time scale of the galaxy $(\Delta t)$. The effect of these two parameters $\beta$ and $\Delta t$ on $S_{Nth}$ will be investigated. The outline of this paper is as follows: the description of theoretical specific frequency $S_{Nth}$ is presented in Section II. Comparison between the observed data and $S_{Nth}$ is presented in Section III. Finally Section IV contains our discussion and conclusion.

## II. THEORETICAL SPECIFIC FREQUENCY $(S_{Nth})$

According to (1) we derive an analytical model for the theoretical specific frequency $S_{Nth}$, which is the number of globular clusters $N_{GC}$ per unit luminosity or (Mv), the galaxy magnitude (Mv) can by converted into a mass $(M_{gal})$ by using mass-to-light-ratio $\psi$,

$$S_{Nth} = \frac{N_{GC}}{M_{gal}} \times \psi 10^6 \tag{2}$$

Beginning with a power-law globular cluster mass function (CMF),

$$\xi_{ecl}(Mecl) = K_{ecl}M_{ecl}^{-\beta} \tag{3}$$

where $\xi_{ecl}$ is the mass function of embedded cluster and $K_{ecl}$ is the normalization constant, here we constrain the power law slope of the (CMF) $\beta$ between (1.5 _ 2.5) [18]-[20]. Using the empirical relation which derived by [20], which represents the relation between the maximum cluster mass $(M_{ecl,max})$ and the star-formation rate (SFR) of the host galaxy based on a linear regression fit to the observational data from [21] for absolute magnitude (Mv) of the brightest young cluster and star formation rate.

$$M_{ecl,max} = K_{ML}SFR^{0.75} \times 10^{6.77} \tag{4}$$

where $K_{ML} = 0.0144$ is the typical mass-to-light-ratio for young globular cluster population [20], [22]. The total mass of a population of star cluster $M_{gal}$ is,

$$M_{gal} = \int_{M_{ecl,min}}^{M_{ecl,max}} \xi_{ecl}(Mecl) M_{ecl} dM_{ecl} \qquad (5)$$

where $M_{ecl,min}$ is the minimal mass of a star cluster which can be expressed as 5M⊙ which is the lower mass observed in the Taurus-Auriga aggregate [23], [20].

Using (3) and (5) at $\beta \neq 2$, in order to determine the normalization constant $K_{ecl}$

$$K_{ecl} = \frac{M_{gal} \times (2 - \beta)}{(M_{ecl,max})^{(2-\beta)} - (5M\odot)^{(2-\beta)}} \qquad (6)$$

The number of GC ($N_{GC}$), can also be expressed with the CMF at a minimum cluster mass which we take to be $10^4 M\odot$, as Baumgardt and Makino [24] suggested this as the minimum mass remaining bound as a cluster after 13 Gyr.

$$N_{GC} = \int_{10^4 M\odot}^{M_{ecl,max}} \xi_{ecl}(Mecl) M_{ecl} dM_{ecl} \qquad (7)$$

putting (3) into (7) then,

$$N_{GC} = \frac{K_{ecl}}{1 - \beta} [M_{ecl,max}^{(1-\beta)} - (10^4 M\odot)^{(1-\beta)}] \qquad (8)$$

Substituting the value for $K_{ecl}$ from (6), we have

$$N_{GC} = M_{gal} \frac{2 - \beta}{1 - \beta} \frac{M_{ecl,max}^{(1-\beta)} - (10^4 M\odot)^{(1-\beta)}}{M_{ecl,max}^{(2-\beta)} - (5M\odot)^{(2-\beta)}} \qquad (9)$$

From (4) we substitute the $M_{ecl,max}$, when SFR is depends on time and on mass. Having obtained $N_{GC}$ and $M_{gal}$ the $S_{Nth}$ follows from

$$S_{Nth} = \frac{2 - \beta}{1 - \beta} \times$$
$$\frac{[(SFR)^{0.75} \times K_{ML} \times 10^{6.77}]^{(1-\beta)} - (10^4 M\odot)^{(1-\beta)}}{[(SFR)^{0.75} \times K_{ML} \times 10^{6.77}]^{(2-\beta)} - (5M\odot)^{(2-\beta)}}$$
$$\times \psi 10^6 M\odot$$
$$\qquad (10)$$

where $SFR = M_b/\Delta t$ here ($M_b$) is the baryonic mass and $\Delta t$ is the formation time scale.

The E and dE galaxies formed under different physical boundary conditions [25], [26], which need different formation time-scales. In Fig. 1 we show the theoretical specific frequency from equation (10) as a function of baryonic mass ($M_b$) with CMF power law indices $\beta = 2.3$. We have tested eight values of $\Delta t_S$ ranging from $10^3$ to $10^{10}$ yr, also assume $\psi = 1$ (mass -to-light ratio for the galaxy), we will see later whether this assumption was reasonable. In these calculations we ignore any value of $S_{Nth}$ that are zero or negative. Here $\beta = 2.3$ which agrees with the work of [20] and later we continuum of slop in the range between $(1.5 - 2.5)$ to include the entire slopes. It is clearly indicates the trend of $S_{Nth}$, at lower mass the higher scale time deviations are stronger, which is also notice $S_{Nth}$, remained constant =2.4 for high
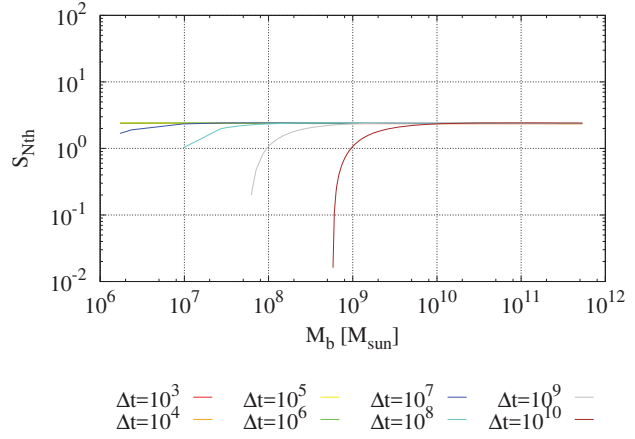


Fig. 1 Theoretical specific frequency $S_{Nth}$ versus wide range of galaxy mass $M_b$. Color selected from left to right represent increasingly different formation time scale of the galaxy ($\Delta t$) ranging from flatter to curves $(10^3, 10^4, 10^5, 10^6, 10^7, 10^8, 10^9$ and $10^{10})$ yr, assume $\psi = 1$ (mass-to-light ratio for the galaxy ) and CMF with $\beta = 2.3$
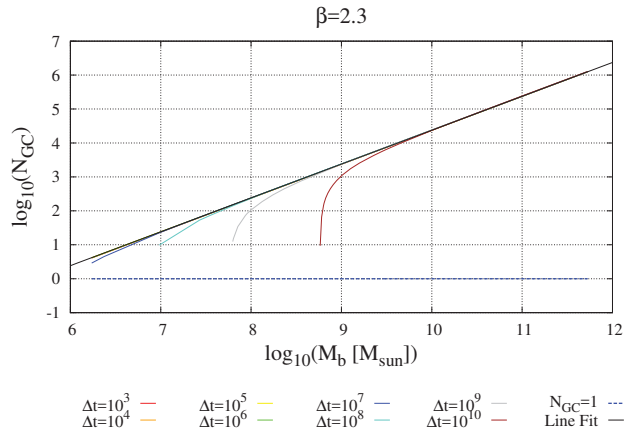


Fig. 2 The number of GCs versus the baryonic mass of galaxy for different time of galaxy formation $(10^3$ to $10^{10})$ yr (uppermost to lowermost curves), at $\beta = 2.3$ and $\psi = 1$. The blue line corresponds to the one GC ($N_{GC}=1$). The black line is the best-fit line (11)

mass of galaxy $M_b > 10^{10} M\odot$, on this basis and according to equation (2) the $N_{GC}$ is approximately proportional to $M_{gal}$.

The correlation between the $N_{GC}$ and $M_b$ which is demonstrated in Fig. 2 shows as expected previously a linear (log scale) in behavior at high mass of galaxy $M_b > 10^{10} M\odot$ which is achieved independent of $\Delta t$ and also shifted by one power of ten for small $M_b$. We derive a best-fit relation (black line):

$$log_{10}(N_{GC}) = log_{10}(M_b) + b \qquad (11)$$

where $b = -5.6038$ the $log_{10}(N_{GC})$ - intercept

$$N_{GC} = 10^{-5.6038} M_b \qquad (12)$$

Equation (2) can be expressed with (12) as

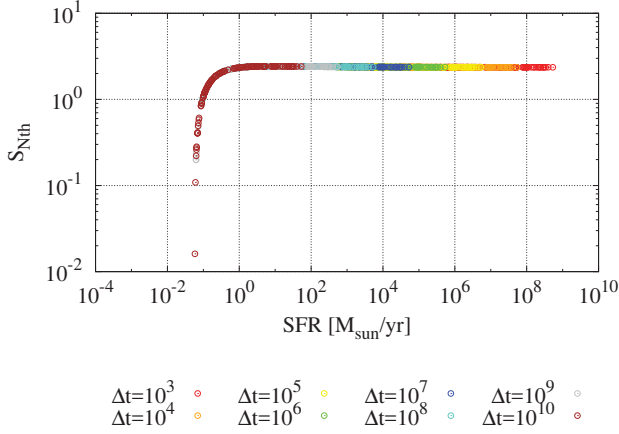$$S_{Nth} = \frac{10^{-5.6038} M_b}{M_b} 10^6 \qquad (13)$$

Fig. 3 shows how long it take this galaxy to form at a given mass, it also can be seen that the specific frequency $S_{Nth}$ behaves largely independently of the SFR i.e. The different formation time does not influence any of the results.

We compared the formation time $\Delta t_{MW}$ for the Milky Way (MW) which has a known specific frequency with the literature values $\Delta t_{MW}$ = (0.5 - 1) Gyr [27]. At a given specific frequency $S_{Nth}$ for MW (2.3) and with mass $M_{MW}$ = $10^8 M\odot$, which is estimated from the mass of the old population II spheroid, which together with the GC, at a certain value of $\beta$= 2.3, mass-to-light-ratio = 1 and $N_{GC}$ =150. Having obtained this values, and after calculating equation 10 numerically, the $\Delta t_{MW}$ equal 7.058e+08 yr (0.705 Gyr). This value lies exactly in the range $\Delta t_{MW}$ = 0.5 - 1 Gyr [27], and shows that the model with $\psi$ = 1 gives the correct physical leading results. In this work we assume 6 values of the power low slope of the (CMF) $\beta$: 1.5, 1.7, 1.9, 2.1, 2.3 and 2.5. To better estimate the difference between various models, we have plotted in Fig. 4 the $S_{Nth}$ versus baryonic mass for a singular case to the power law slope of the (CMF) in the range $\beta$ between (1.5 - 2.5), for each plot we assume eight possible values of different formation time ranging from $10^3$ to $10^{10}$ (red to brown). It can be clearly seen that the lines become separate from each other at a low mass, while for high mass > ($10^{10}$ ) they become flatter at $\beta$ (2.1, 2.3 and 2.5 ) and disperse at $\beta$ (1.5, 1.7 and 1.9). Thus we see how the results are sensitively dependent on the power law index $\beta$. Fig. 5 shows the same behavior of $\beta$ for a range between (1.5 - 2.5) and different formation times which ranging from $10^3$ to $10^{10}$ same as Fig. 4 but for $N_{GC}$ and mass of galaxies.



Fig. 3 Theoretical Specific frequency, $S_{Nth}$, versus the star formation rate (SFR) of host galaxy, the different colors of circles mark galaxies with different formation time ($\Delta t$) range from $10^3$ to $10^{10}$ yr, $SFR = M_b/\Delta t$ remained constant for $SFR > 1 M\odot /yr$

$S_{Nth}$ = 2.48 is similar to the previous value in Fig. 1 when $S_{Nth}$, remained constant =2.4 for high mass of galaxy.

If we compare between equations 9 and 12, we find the origin of the proportionality, we can see that the fraction term must be constant. The term with $M_{ecl,max}$ (SFR) in the numerator of the formula will be reduced by about an order of magnitude when $M_{gal}$ is increased by a factor of ten.The situation is similar for the denominator, but the term $M_{max}$ (SFR) does not fall so strongly with Mgal and here always smaller than $(5M\odot)^{2-\beta}$
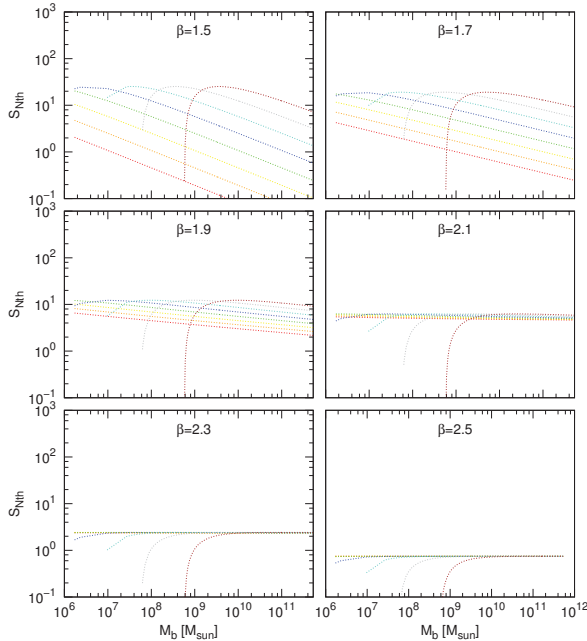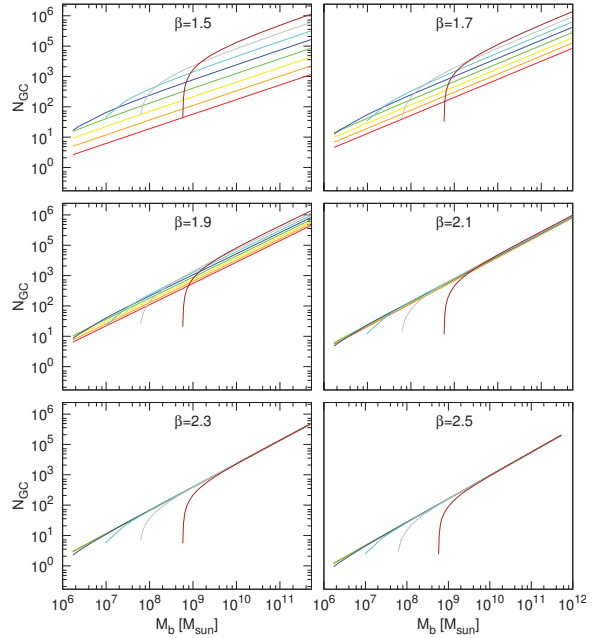


Fig. 4 Theoretical specific frequency $S_{Nth}$ versus wide range of galaxy mass $M_b$ for different value of $\beta$. The coloured dot lines indicate different formation time scale (as in Fig. 1)



Fig. 5 Number of globular cluster $N_{GC}$ versus mass of galaxy for different formation time scale (as in Fig. 2) and for different value of $\beta$
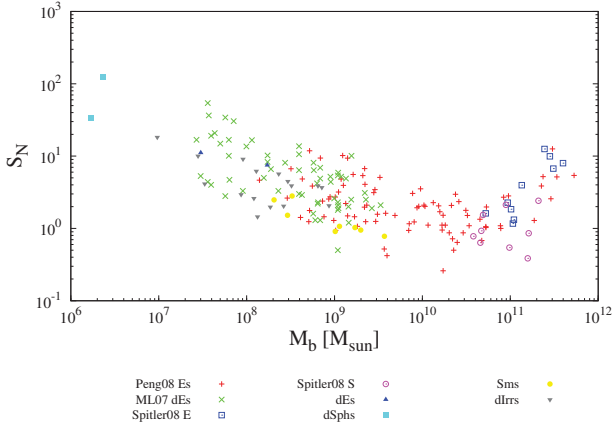
Fig. 6 The specific frequency of globular cluster versus baryonic mass for a range of galaxy morphologies. The various symbol type and colours which explained in the figure legend, represent different sources of data: red plus signs (ellipticals) from [7], green crosses (dwarf ellipticals) from [6], blue squares (elliptical) [32], pink circles (spiral) [32]. While for nearby dwarf galaxies blue solid triangles (dEs), cyan solid square (dSphs), yellow solid circles(Sms) and grey solid triangles (dIrrs) from [33], [34]



Fig. 7 Comparison between the observed Data and model with different $\beta$ and different formation time

## III. COMPARISON BETWEEN THE OBSERVED DATA AND THEORETICAL SPECIFIC FREQUENCY($S_{Nth}$)

The specific frequencies of GCs ($S_N$) are important tools for the aim of understanding the evolution of the galaxies [28], [13]. In Fig. 6 we show the observed samples of galaxies (the main source of this data is [8]), we demonstrate the general tendency of the specific frequency of GCs ($S_N$) versus range of galaxy mass ($M_V = -11$ to$-23$ mag ), which takes a 'U' -shape as usual. At the low -mass and high- mass end of the scale, the $S_N$ value is higher compared with the galaxy at intermediate mass which becames close to one. The value of $S_N$ is high in dwarf spheroidal (dSph) galaxies in contrast to spiral galaxies which have smaller $S_N$. We also notice that the $S_N$ of giant elliptical is higher than for giant spirals, which is due to the formation of globular cluster during collision of spiral galaxies which forms ellipticals [29]-[31]. $S_N$ for dSphs (low mass) is higher than dwarf irregular (dIrs), which might suggest that dSphs progenitors of dIrss and dEs [35], [8]. The general trend is towards increasing $S_N$ above and below Mv$\approx$ -20 mag ($\approx 4.7 \times 10^{10} M\odot$), regardless the galaxy type [8]. The median for the whole sample is ($S_N$=2.56) and 58% of the sample are located below $S_N$=3.

## IV. DISCUSSION AND CONCLUSIONS

We investigated the correlation between($S_N$) and baryonic mass, which shows a 'U'-shape, the ($S_N$) value is higher for dwarfs and supergiants (the low and high- mass end of the scale) compared to the galaxy at intermediate mass which take value nearly to one. The population ($N_{GC}$) correlates linear (log scale) with ($N_{GC}$) at high mass of galaxy $M_b >$ $10^{10} M\odot$. This agrees with previous studies that suggest that the specific frequency ($S_N$) is a function of galaxy mass, which holed irrespective of galaxy morphology.

The Comparison between the data in Fig. 6 and the theoretical specific frequency, we performed the model at 6
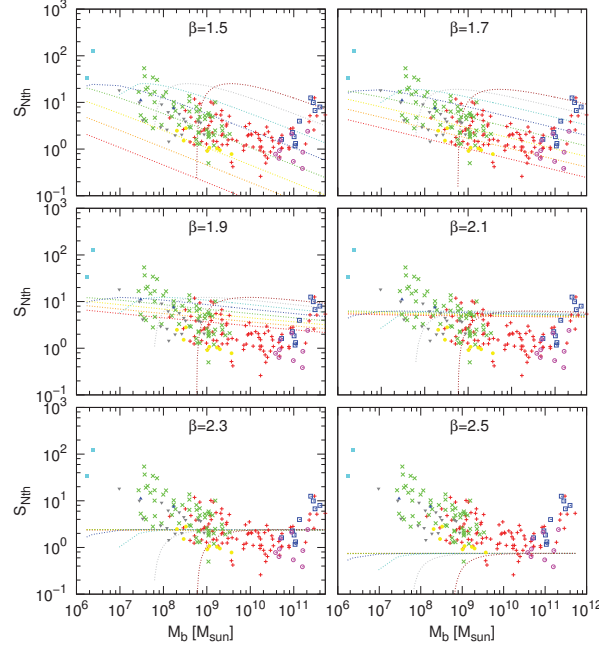
different a simple power law index $\beta$ ranging from ( 1.5 to 2.5) and for eight different formation time scale ($\Delta t$), that can be understood as being due to the observation of entire galaxy population instead of individual galaxies. In addition the mass of galaxy varies with the formation time scale. The low mass with high $S_N$ is identical to short formation time scale which can be described with small $\beta$ while high mass with low ($S_N$) is better described by high $\beta$. In Fig. 7 we show our results for $S_{Nht}$ as a function of the $M_b$ for different values of $\beta$ and different formation time scale and compare our models with observational data.
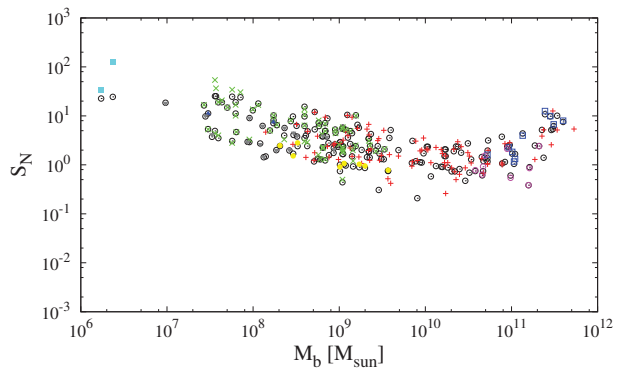


Fig. 8 Specific frequency ($S_N$) versus baryonic mass ($M_b$). Black circles represents the model with best value of different formation time and $\beta$ which explain the observation data. The various symbol types as in the Fig. 6

The model with $\beta$ = 2.1, 2.3 and 2.5 can reproduce the data for the specific frequency smaller than 6.2, 2.4, 0.74

respectively and the models with $\beta$ = 1.5 , 1.7 and 1.9 agree with the $S_{Nth}$ for a wide range of $\Delta$t. The model with $\beta$ =2.5 fail to fit the most data galaxy especially at $S_N > 0.74$, and also all model for $\Delta$ t = $10^{10}$ are biased towards high mass, therefore fail also to fit the data at low mass.

Furthermore, all the models except $\beta$= 2.5 predict the specific frequency significantly for Sms and S galaxies while the models at 1.5, 1.7 and 1.9 predict the $S_N$ for the vast majority of $S_N$ for the Es, ML07 dEs, E, dEs, dIrrs, galaxies.

However, we point out that the model with $\beta$= 1.5 produces a larger range of $S_{Nth}$ which fits the data much better, and for the models $\beta$ =1.5 and $\beta$ =1.7 the mildest $\Delta$t can reproduce the high $S_{Nht}$, We also see a possible hint that the embedded cluster mass function may become top-heavy (smaller $\beta$) in major galaxy-wide star burst, which was also suggested by [36]. In Fig. 8 shows the observational data and a best value of $\beta$ and $\Delta t$ which is obtained by measuring the smallest vertical distance between the data and model $distance =|$ $S_N - S_{Nth}$ |. The histogram distributions in Fig. 9 shows the number of best value of $\beta$ at a period time which can explain the specific frequency of observed data and illustrates that the $\beta$= 1.5 best value to explains the observed data . The maximum theoretical specific frequency characterized by a specific $\beta$ at a given formation time scale is shown in Fig. 10. Nevertheless, some dwarfs stand out as having high $S_N$ which also can be described by $\beta$ smaller than that we use in our calculations. With smaller $\beta$ we get correspondingly higher specific frequencies, and to get extension must include galaxies which are formed in a starburst, such as elliptical and dwarf elliptical galaxies.

The Milky Way appear to be in the formation time scale $t_{MW}$ equal to 0.705 Gyr according to the $S_{Nth}$ relation.
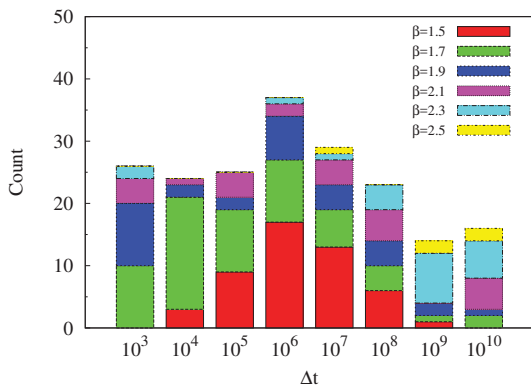


Fig. 9 The histogram for the best $\beta$ at a different time scale which can explain the specific frequency of observed data. The coloured bars are explained in the figure legend
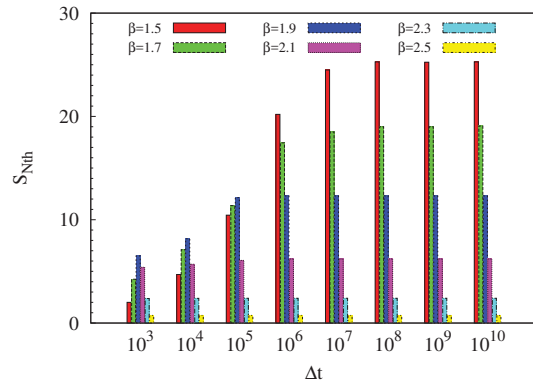
## ACKNOWLEDGMENT

Fig. 10 The maximum theoretical specific frequency versus formation time scale at a different value of $\beta$. The coloured bars are explained in the figure legend

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