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Damping and Stability Evaluation for the Dynamical Hunting Motion of the Bullet Train Wheel Axle Equipped with Cylindrical Wheel Treads

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***Abstract*—**Classical matrix calculus and Routh-Hurwitz stability conditions, applied to the snake-like motion of the conical wheel axle, lead to the conclusion that the hunting mode is inherently unstable, and its natural frequency is a complex number. In order to analytically solve such a complicated vibration model, either the inertia terms were neglected, in the model designated as geometrical, or restrictions on the creep coefficients and yawing diameter were imposed, in the so-called dynamical model. Here, an alternative solution is proposed to solve the hunting mode, based on the observation that the bullet train wheel axle is equipped with cylindrical wheels. One argues that for such wheel treads, the geometrical hunting is irrelevant, since its natural frequency becomes nil, but the dynamical hunting is significant since its natural frequency reduces to a real number. Moreover, one illustrates that the geometrical simplification of the wheel causes the stabilization of the hunting mode, since the characteristic quartic equation, derived for conical wheels, reduces to a quadratic equation of positive coefficients, for cylindrical wheels. Quite simple analytical expressions for the damping ratio and natural frequency are obtained, without applying restrictions into the model of contact. Graphs of the time-depending hunting lateral perturbation, including the maximal and inflexion points, are presented both for the critically-damped and the over-damped wheel axles.

***Keywords*—**Bullet train, dynamical hunting, cylindrical wheels, damping, stability, creep, vibration analysis.

I. Introduction

TRADITIONALLY, vibration analysis associated to the snake-like movement of the railway vehicle, is performed under the assumption that the wheel tread is slightly conical [1], [2], and moreover, the whole wheel axle can be circumscribed by a double-cone [3], [4]. Simplest analytical expression for the natural frequency of the wheel axle hunting mode was found based on a so-called *geometrical model* [1], [2], in which the inertial effect, i.e. the mass of the wheel axle, was neglected. Since geometrical model cannot predict the dissipation induced by the contact of the wheels with the rails [5], [6], in absence of damping, once started, the hunting vibration of the wheel axle cannot be naturally halted [7], [8]. Moreover, as emphasized in this work, the geometrical model cannot describe the hunting motion of a wheel axle equipped with cylindrical wheel treads, which are commonly employed by the bullet trains [9], [10].

Improvement of the geometrical hunting vibration model, by including the inertial effect, was achieved under different types of assumptions regarding the creep ratio $\bar{f}$ (ratio of the lateral creep coefficient to the longitudinal creep coefficient), and also regarding the dimensionless contact width $\bar{b}$ (ratio of the track span to the yawing diameter of gyration) of the wheel axle [4], [11]. Thus, hunting analysis is customarily simplified by considering that the lateral creep coefficient almost equals the longitudinal creep coefficient ($\bar{f} \cong 1$) [1]-[6], [11]-[15]. Additionally, unitary dimensionless contact width ($\bar{b} \cong 1$) was widely used, and this assumption is acceptable for various types of railway carriages [16]-[18]. Recently, the severer restriction $\bar{b} \cong \bar{f} \cong 1$, usually used in the study of the hunting motion, was replaced by a milder restriction ($(\bar{b}^2 - \bar{f})^2 \cong 0$) [4]. Based on the derived expressions for the damping coefficients and damped natural frequency, one clarified the influence of the train speed, wheel conicity, dimensionless mass of the wheel axle, creep ratio, and, ratio of the track span to the yawing diameter, on these dynamic parameters [4], [11]. Unfortunately, even these advanced models fail to properly depict the hunting movement for the simpler constructive case of a wheel axle furnished with cylindrical wheel treads.

Accordingly, in this work, the vibration model is improved to allow for the accurate evaluation of the damping and stability associated to the dynamical hunting motion for the bullet train wheel axle, of negligible wheel conicity.

II. Hunting Model for the Railway Wheel Axle Equipped with Conical Wheel Treads

Due to the conical geometry of the wheels, values of contact radius with the rails are different at the left and right wheels, and this produces a wedge effect that opposes the lateral perturbation δ and the angular perturbation θ [4]. Hunting vibration of the railway wheel axle can be described by the following set of two differential equations [1]-[4], [11]:

$$\begin{cases} m\ddot{\delta} + \dfrac{f_2}{V}\dot{\delta} - f_2\theta = 0 \\ mR_z^2\ddot{\theta} + \dfrac{f_1 b^2}{V}\dot{\theta} + \dfrac{f_1 b\lambda}{r}\delta = 0 \end{cases} \tag{1}$$

where m denotes the mass of the wheel axle, mR_z^2 is the yawing moment of inertia, corresponding to a rotation of the wheel axle around the vertical z axis, R_z is the yaw gyration

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radius, f_1 is the longitudinal creep coefficient, and f_2 is the lateral creep coefficient. Creep coefficients are taken for the whole wheel axle, i.e. they include the creep effects at both the left and right contact points of the wheels with the rails.

Neglecting in (1), the inertial terms of translation ($m\ddot{\delta}$) and rotation ($mR_z^2\ddot{\theta}$), i.e. neglecting the mass of the wheel axle ($m \cong 0$), and applying Laplace transformation to the simplified set of equations, the characteristic equation of the *geometrical hunting mode* can be obtained as [1]-[4], [11]:

$$s^2 + \frac{\lambda V^2}{rb} = 0 \tag{2}$$

where r is the contact radius of the unperturbed wheel axle, b is the semi-span of the parallel rails, λ is the slope of the conical wheel tread, and V is the velocity of the wheel axle along the rails. Under such severe simplification of the model, from (2) a real value for the natural circular frequency of the geometrical hunting motion is obtained as [1]-[4], [11]:

$$\omega_g = V\sqrt{\frac{\lambda}{rb}} \tag{3}$$

Since (2) does not contain a term in s, it appears that in the absence of damping, once excited, the hunting vibration of the wheel axle cannot be naturally attenuated [4]. Moreover, since for negligible wheel conicity ($\lambda = 0$), the natural circular frequency becomes nil ($\omega_g = 0$), and it seems that the geometrical model cannot describe the hunting for the wheel axle furnished with cylindrical wheel treads.

Concerning the *dynamical hunting mode*, if the inertia terms are not neglected in (1), after performing the Laplace transform, a quartic characteristic equation, with a missing term in s, is obtained as [4], [11]:

$$A_4 s^4 + A_3 s^3 + A_2 s^2 + A_1 s + A_0 = 0 \tag{4}$$

Thus, polynomial coefficients of (4) are given by [4]:

$$\begin{cases} A_4 = 1 \quad ; \quad A_3 = \dfrac{f_1 b^2 + f_2 R_z^2}{mR_z^2 V} = \omega_c(\bar{b}^2 + \bar{f}) \quad ; \quad A_1 = 0 \\ A_2 = \dfrac{f_1 f_2 b^2}{m^2 R_z^2 V^2} = \omega_c^2 \bar{b}^2 \bar{f} \quad ; \quad A_0 = \dfrac{f_1 f_2 b\lambda}{m^2 R_z^2 r} = \omega_c^2 \omega_g^2 \bar{b}^2 \bar{f} \end{cases} \tag{5}$$

where $\bar{f} = f_2 / f_1$ is the creep ratio, i.e. the ratio of the lateral creep coefficient f_2 to the longitudinal creep coefficient f_1, $\omega_c = f_1/(mV)$ is the creep circular frequency, and $\bar{b} = b/R_z$ is the dimensionless contact width, i.e. the ratio of the track span $2b$ to the yawing diameter $2R_z$ of gyration.

Routh-Hurwitz stability conditions for quartic characteristic equation can be generally written as [19], [20]:

$$\begin{cases} A_4 > 0 \quad ; \quad A_3 > 0 \quad ; \quad A_0 > 0 \\ A_2 A_3 - A_1 A_4 > 0 \\ A_1 A_2 A_3 - A_1^2 A_4 - A_0 A_3^2 > 0 \end{cases} \tag{6}$$

Evaluating the conditions (6) for the quartic equation (4), one observes that the first four conditions are satisfied as follows:

$$\begin{cases} A_4 = 1 > 0 \quad ; \quad A_3 = \omega_c(\bar{b}^2 + \bar{f}) > 0 \\ A_0 = \omega_c^2 \omega_g^2 \bar{b}^2 \bar{f} > 0 \\ A_2 A_3 - A_1 A_4 = A_2 A_3 = \omega_c^3 \bar{b}^2 \bar{f}(\bar{b}^2 + \bar{f}) > 0 \end{cases} \tag{7}$$

However, since the last condition of (6) is not satisfied:

$$A_1 A_2 A_3 - A_1^2 A_4 - A_0 A_3^2 = -\omega_c^4 \omega_g^2 \bar{b}^2 \bar{f}(\bar{b}^2 + \bar{f})^2 < 0 \tag{8}$$

the wheel axle hunting occurs as an inherently unstable motion.

Moreover, by rewriting (1) of the dynamical hunting mode in matrix form [8], [21]:

$$[M] \cdot \begin{bmatrix} \ddot{\delta} \\ \ddot{\theta} \end{bmatrix} + [C] \cdot \begin{bmatrix} \dot{\delta} \\ \dot{\theta} \end{bmatrix} + [K] \cdot \begin{bmatrix} \delta \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{9}$$

where the mass matrix $[M]$ is given by:

$$[M] = \begin{bmatrix} m & 0 \\ 0 & mR_z^2 \end{bmatrix} \tag{10}$$

and the damping matrix $[C]$, and the stiffness matrix $[K]$ are given by:

$$[C] = \frac{1}{V}\begin{bmatrix} f_2 & 0 \\ 0 & f_1 b^2 \end{bmatrix} \quad ; \quad [K] = \begin{bmatrix} 0 & -f_2 \\ f_1 b\lambda / r & 0 \end{bmatrix} \tag{11}$$

In these circumstances, the characteristic equation attached to the natural circular frequency ω_n of the system can be written as [8], [21]:

$$\det([K] - \omega_n^2 [M]) = 0 \;\Rightarrow\; \begin{vmatrix} -m\omega_n^2 & -f_2 \\ f_1 b\lambda / r & -mR_z^2 \omega_n^2 \end{vmatrix} = 0 \tag{12}$$

leading to the following quartic equation in ω_n :

$$\omega_n^4 + \omega_c^2 \omega_g^2 \bar{b}^2 \bar{f} = 0 \;\Leftrightarrow\; \omega_n^4 + A_0 = 0 \tag{13}$$

Thus, although a real value (3) was found for the natural circular frequency of the geometrical hunting motion, from (13), an imaginary value, i.e. a complex number is obtained for the natural circular frequency of the dynamical hunting movement:

$$\omega_n^2 = \pm\omega_c\omega_g\bar{b}\sqrt{\bar{f}}\,i \tag{14}$$

Physical significance of (14) can be explained as follows. Linear system of equations (1) can be generally decoupled into two state space modes, with four components, corresponding to the generalized displacements (δ, θ) and velocities $(\dot{\delta}, \dot{\theta})$ [22]. The state space modes are defined by means of the complex eigenvectors of the system containing magnitudes and phase angles. The appearance of spatially varying phase angles implies travelling wave behavior of the mode shape as the oscillation proceeds through a cycle. This is a major and important difference from the synchronous standing oscillation found for the classical damped systems. The complex modal coordinates, which determine the modal amplitude and the modal phase, can be defined by means of the eigenvectors and the initial conditions. This feature is induced by the normality properties of the complex eigenvectors [22]. Moreover, from (14), one observes that for negligible wheel conicity (λ=0), the geometrical hunting frequency becomes nil ($\omega_g = 0$), and hence, the natural circular frequency becomes zero ($\omega_n = 0$). Consequently, it seems that also the dynamical models, previously proposed [4], [11], cannot describe, without proper adjustment, the hunting movement of the wheel axle supplied with cylindrical wheel treads.

III. Hunting Model for the Railway Wheel Axle Equipped with Cylindrical Wheel Treads

In the case of bullet trains carriages, such as the Japanese bullet train, *Shinkansen*, the geometrical hunting mode of the wheel axle appears as irrelevant since for cylindrical wheel treads (λ=0) the geometrical circular frequency becomes nil ($\omega_g = V\sqrt{\lambda/(rb)} = 0$). However, although the last coefficient of (4) becomes nil ($A_0 = \omega_c^2\omega_g^2\bar{b}^2\bar{f} = 0$), the dynamical hunting mode occurs as still significant, since the characteristic quartic equation (4) reduces to the following quadratic equation:

$$s^2(s^2 + A_3 s + A_2) = 0 \quad \Rightarrow \quad s^2 + A_3 s + A_2 = 0 \tag{15}$$

In the case of cylindrical treads, all three coefficients of the quadratic equation (15) are positive (see (5)), i.e., they have the same sign. Therefore, based on the Routh-Hurwitz stability conditions [19]-[20], the dynamical hunting mode appears as stabilized.

Solutions of (15) are real and can be calculated as:

$$s_{1,2} = \frac{-\omega_c(\bar{b}^2 + \bar{f}) \mp \omega_c\left|\bar{b}^2 - \bar{f}\right|}{2} \quad ; \quad s_1 \le s_2 \tag{16}$$

Then, the following three possibilities can be taken into account:

1) In the case $\bar{b}^2 > \bar{f}$, solutions (16) reduce to:

$$s_1 = -\omega_c\bar{b}^2 \quad ; \quad s_2 = -\omega_c\bar{f} \tag{17}$$

2) In the case $\bar{b}^2 = \bar{f}$, solutions (16) reduce to:

$$s_1 = s_2 = -\omega_c\bar{b}^2 = -\omega_c\bar{f} \tag{18}$$

3) In the case $\bar{b}^2 < \bar{f}$, solutions (16) reduce to:

$$s_1 = -\omega_c\bar{f} \quad ; \quad s_2 = -\omega_c\bar{b}^2 \tag{19}$$

In order to identify the damping spontaneously occurring at the contact between the cylindrical wheels and the rails, during the hunting motion of the railway wheel axle, one pays attention to similarities between (17)-(19), and solutions (20):

$$s_{3,4} = -\zeta\omega_n \mp \omega_n\sqrt{\zeta^2 - 1} \tag{20}$$

which correspond to (21), i.e. to the characteristic equation of a classical damped one-degree of freedom vibration system, consisted of a spring connected in parallel to a dashpot [7], [8]:

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \tag{21}$$

Here, ω_n is the natural circular frequency, and ζ is damping ratio. Real solutions of the characteristic equation (21) can be obtained only in the case when the mechanical system is over-damped, i.e. $\zeta > 1$, or critically-damped, i.e. $\zeta = 1$ [7]. Note that, solutions (20) in correlation with (17) for $\bar{b}^2 > \bar{f}$, and (19) for $\bar{b}^2 < \bar{f}$, allow for identification of the damping ratio ζ and natural frequency ω_n as follows:

$$\zeta = \frac{\bar{b}^2 + \bar{f}}{2\bar{b}\sqrt{\bar{f}}} > 1 \quad ; \quad \omega_n = \omega_c\bar{b}\sqrt{\bar{f}} = \frac{\bar{b}\sqrt{f_1 f_2}}{mV} \tag{22}$$

On the other hand, solutions (20) in correlation with (18) for $\bar{b}^2 = \bar{f}$, permit the identification of the damping ratio ζ and natural frequency ω_n as:

$$\zeta = 1 \quad ; \quad \omega_n = \omega_c\bar{b}^2 = \omega_c\bar{f} = \frac{f_2}{mV} \tag{23}$$

From (22), the partial derivatives of the damping ratio can be calculated as:

$$\begin{cases} \dfrac{\partial\zeta}{\partial\bar{b}} = \dfrac{\bar{b}^2 - \bar{f}}{2\bar{b}^2\sqrt{\bar{f}}} \quad ; \quad \dfrac{\partial^2\zeta}{\partial\bar{b}^2} = \dfrac{\sqrt{\bar{f}}}{\bar{b}^3} > 0 \\ \dfrac{\partial\zeta}{\partial\bar{f}} = \dfrac{\bar{f} - \bar{b}^2}{4\bar{b}\bar{f}^{3/2}} \quad ; \quad \dfrac{\partial^2\zeta}{\partial\bar{f}^2} = \dfrac{3\bar{b}^2 - \bar{f}}{8\bar{b}\bar{f}^{5/2}} \end{cases} \tag{24}$$

From derivatives (24), one observes that the minimal value of the damping ratio $\zeta_{\min} = 1$ is attained for $\bar{b}^2 = \bar{f}$ (see Figs. 1 and 2). Supplementary, an inflexion point is to be expected for $\bar{f} = 3\bar{b}^2$ (see Fig. 2).

Additionally, from (22), values of the damping ratio when $\bar{b}$ and $\bar{f}$ tend to zero and infinity can be determined as:

$$\begin{cases} \lim\limits_{\bar{b}\to 0}\zeta = \infty \quad ; \quad \lim\limits_{\bar{b}\to\infty}\zeta = \infty \\ \lim\limits_{\bar{f}\to 0}\zeta = \infty \quad ; \quad \lim\limits_{\bar{f}\to 0}\zeta = \infty \end{cases} \tag{25}$$

IV. Results and Discussions

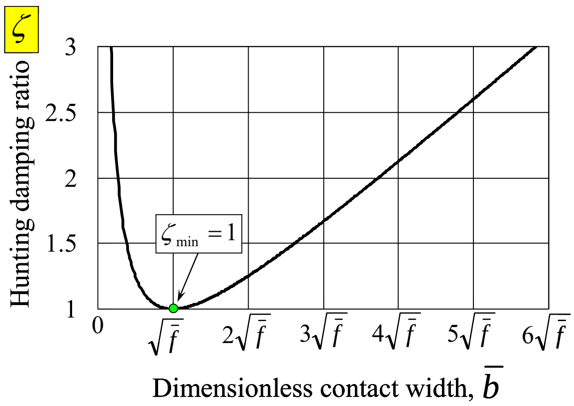

Fig. 1 Variation of the hunting damping ratio versus the dimensionless contact width, for a wheel axle equipped with cylindrical wheel treads

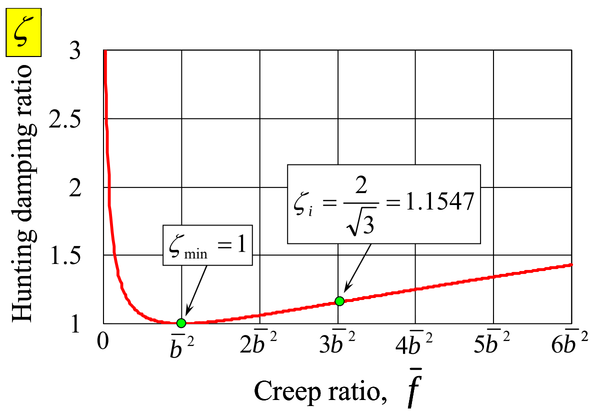

Fig. 2 Variation of the hunting damping ratio versus the creep ratio, for a wheel axle equipped with cylindrical wheel treads

Damping ratio associated to the hunting motion of the wheel axle, furnished with cylindrical wheel treads, is depending only on two parameters. One is a tribological parameter, describing the contact of the wheels with the rails through the creep ratio $\bar{f}$, and the other is a geometrical parameter $\bar{b}$, expressing the relationship between the span of the rails and the yawing diameter of gyration of the railway wheel axle.

Fig. 1 shows the variation of the damping ratio ζ versus the dimensionless contact width $\bar{b}$, and Fig. 2 displays the variation of the damping ratio ζ against the creep ratio $\bar{f}$.

As expected from (22)-(25), damping ratio nonlinearly decreases for $\bar{b} \in (0;\sqrt{\bar{f}})$ on Fig. 1, and $\bar{f} \in (0;\bar{b}^2)$ on Fig. 2. After reaching its minimal value ($\zeta_{\min} = 1$) for $\bar{b}^2 = \bar{f}$, the damping ratio monotonically increases for larger values of the dimensionless contact width (Fig. 1) and creep ratio (Fig. 2). Expectedly, an inflexion point can be observed on Fig. 2 for $\bar{f} = 3\bar{b}^2$ that corresponds to a damping ratio of $\zeta_i = 2/\sqrt{3}$, but the change of convexity is quite mild.

Natural circular frequency ω_n (see (22)) is depending inversely proportionally to the mass m and the speed V of the wheel axle, but it varies proportionally to the dimensionless contact width $\bar{b}$, as well as to the geometrical mean of the longitudinal f_1 and lateral f_2 creep coefficients.

A. Case of the Critically-Damped Hunting Motion

Next, for a wheel axle designed under the condition $\bar{b}^2 = \bar{f}$, a critically-damped ($\zeta_{\min} = 1$) hunting motion is achieved (see (23) and the points of minimal damping ratio on Figs. 1 and 2). In such circumstances, under the following initial conditions:

$$\delta(t=0) = \delta_0 \quad ; \quad \dot{\delta}(t=0) = \dot{\delta}_0 \tag{26}$$

the time depending lateral perturbation δ can be obtained as:

$$\delta(t) = [\delta_0 + (\dot{\delta}_0 + \frac{f_2\delta_0}{mV})t]\exp(-\frac{f_2 t}{mV}) \tag{27}$$

Since $\lim\limits_{t\to\infty}\delta(t) = 0$, one confirms that indeed the dynamical hunting mode is stable. Note that, in dimensionless form, the perturbation (27) can be written as:

$$\Delta(t) = \frac{\delta(t)}{\delta_0} = [1 + (1+\Gamma)\frac{f_2 t}{mV}]\exp(-\frac{f_2 t}{mV}) \tag{28}$$

where Γ is a dimensionless parameter defined as:

$$\Gamma = \frac{mV\dot{\delta}_0}{f_2\delta_0} \geq 0 \tag{29}$$

Analyzing the first and second derivatives of $\Delta(t)$, which

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can be computed as:

$$\begin{cases} \dot{\Delta}(t) = \dfrac{f_2}{mV}[\Gamma - (1+\Gamma)\dfrac{f_2 t}{mV}]\exp(-\dfrac{f_2 t}{mV}) \\ \ddot{\Delta}(t) = -(\dfrac{f_2}{mV})^2[1 + 2\Gamma - (1+\Gamma)\dfrac{f_2 t}{mV}]\exp(-\dfrac{f_2 t}{mV}) \end{cases} \quad (30)$$

one concludes that the graph of the perturbation against time has a point of maximum at $(t_e;\Delta_{max})$ and an inflexion point at $(t_i;\Delta_i)$, as shown by Fig. 3. Values of t_e, t_i, Δ_{max} and Δ_i are given by:

$$\begin{cases} t_e = \dfrac{mV}{f_2}\dfrac{\Gamma}{1+\Gamma} \\ t_i = \dfrac{mV}{f_2}\dfrac{1+2\Gamma}{1+\Gamma} = t_e + \dfrac{mV}{f_2} > t_e \\ \Delta_{max} = (1+\Gamma)\exp(-\dfrac{\Gamma}{1+\Gamma}) \geq 1 \\ \Delta_i = 2(1+\Gamma)\exp(-\dfrac{1+2\Gamma}{1+\Gamma}) \end{cases} \quad (31)$$

It can be proved that $\Delta_{max} = 1$ for $\Gamma = 0$, and $\Delta_{max} > 1$ for $\Gamma > 0$. On the other hand, $\Delta_i \leq 1$ for $\Gamma \in [0;1.461]$, and $\Delta_i > 1$ for $\Gamma > 1.461$.

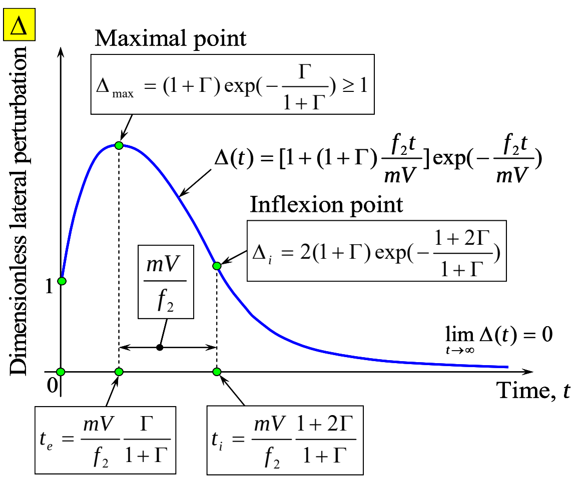

Fig. 3 Variation of the perturbation versus time, obtained for a critically-damped wheel axle equipped with cylindrical treads

B. Case of the Over-Damped Hunting Motion

Next, for a wheel axle designed under the condition $\bar{b}^2 \neq \bar{f}$, an over-damped ($\zeta > 1$) hunting motion is achieved (see (22), Figs. 1 and 2). In this case, under the same initial conditions (26), the dimensionless perturbation $\Delta(t) = \delta(t)/\delta_0$ can be written as:

$$\Delta(t) = \frac{\bar{b}^2 + \Gamma\bar{f}}{\bar{b}^2 - \bar{f}}\exp(-\frac{f_2 t}{mV}) - \frac{\bar{f} + \Gamma\bar{f}}{\bar{b}^2 - \bar{f}}\exp(-\frac{\bar{b}^2}{\bar{f}}\frac{f_2 t}{mV}) \quad (32)$$

Again, since $\lim_{t\to\infty}\Delta(t) = 0$, one reaffirms that the dynamical hunting mode is stable.

Considering the first and second derivatives of $\Delta(t)$, which can be calculated as:

$$\dot{\Delta}(t) = \frac{f_2}{mV}[\frac{\bar{b}^2 + \Gamma\bar{b}^2}{\bar{b}^2 - \bar{f}}\exp(-\frac{\bar{b}^2}{\bar{f}}\frac{f_2 t}{mV}) - \frac{\bar{b}^2 + \Gamma\bar{f}}{\bar{b}^2 - \bar{f}}\exp(-\frac{f_2 t}{mV})] \quad (33)$$

$$\ddot{\Delta}(t) = (\frac{f_2}{mV})^2[\frac{\bar{b}^2 + \Gamma\bar{f}}{\bar{b}^2 - \bar{f}}\exp(-\frac{f_2 t}{mV}) - \frac{\bar{b}^4}{\bar{f}}\frac{1+\Gamma}{\bar{b}^2 - \bar{f}}\exp(-\frac{\bar{b}^2}{\bar{f}}\frac{f_2 t}{mV})] \quad (34)$$

one arrives to the conclusion that the graph of the perturbation against time has a point of maximum at $(t_e;\Delta_{max})$ and an inflexion point at $(t_i;\Delta_i)$, as illustrated by Fig. 4. Values of t_e, t_i, Δ_{max} and Δ_i are given by:

$$\begin{cases} t_e = \dfrac{mV}{f_2}\dfrac{\bar{f}}{\bar{b}^2 - \bar{f}}\ln\dfrac{\bar{b}^2(1+\Gamma)}{\bar{b}^2 + \Gamma\bar{f}} \\ t_i = \dfrac{mV}{f_2}\dfrac{\bar{f}}{\bar{b}^2 - \bar{f}}\ln\dfrac{\bar{b}^4(1+\Gamma)}{\bar{f}(\bar{b}^2 + \Gamma\bar{f})} \\ \ln\Delta_{max} = \dfrac{\bar{b}^2\ln(1+\Gamma\frac{\bar{f}}{\bar{b}^2}) - \bar{f}\ln(1+\Gamma)}{\bar{b}^2 - \bar{f}} \\ \ln\Delta_i = \ln\Delta_{max} + \ln(1+\dfrac{\bar{f}}{\bar{b}^2}) - \bar{f}\dfrac{\ln\bar{b}^2 - \ln\bar{f}}{\bar{b}^2 - \bar{f}} \end{cases} \quad (35)$$

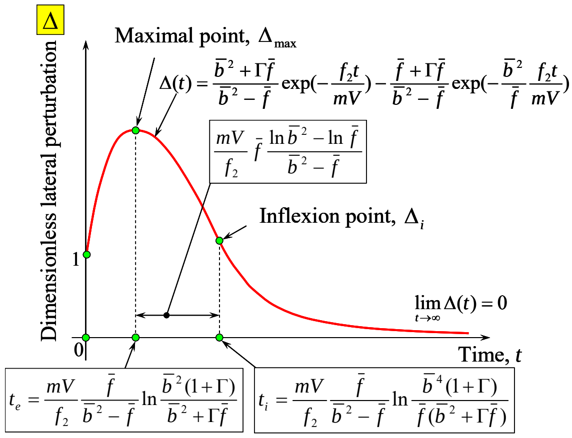

Fig. 4 Variation of the perturbation versus time, obtained for an over-damped wheel axle equipped with cylindrical treads

From the first and second lines of (35), one observes that the time t_i corresponding to inflexion point, exceeds the time t_e of the maximal point:

$$t_i = t_e + \frac{mV}{f_2}\,\bar{f}\,\frac{\ln \bar{b}^2 - \ln \bar{f}}{\bar{b}^2 - \bar{f}} > t_e \qquad (36)$$

V. Conclusions

In this paper, one evaluated the damping and stability of the dynamical hunting motion associated to the wheel axle of bullet trains, supplied with cylindrical treads. Damping ratio and natural circular frequency were identified, without imposing geometrical and tribological limitations into the vibration model. Variation of the hunting lateral perturbation against time was explicitly illustrated both for the critically-damped and over-damped cylindrical wheels.

Main conclusions inferred from the performed theoretical analysis, can be summarized as follows:

1) Geometrical hunting mode of the wheel axle associated to conical treads appears as irrelevant for cylindrical wheels, since the geometrical circular frequency becomes zero.
2) Although the dynamical hunting mode for conical wheels is inherently unstable, it stabilizes for cylindrical wheels. This can be explained by the fact that the characteristic quartic equation, derived for conical wheels, simplifies to a quadratic equation, of positive coefficients, for cylindrical wheels.
3) Damping ratio depends only on the creep ratio, which is a tribological parameter, and on the dimensionless contact span, which is a geometrical parameter. Minimal damping is gained when the creep ratio equals the dimensionless contact span.
4) Natural frequency depends inversely proportionally to the mass and velocity of the wheel axle of cylindrical treads, but it varies proportionally to the dimensionless contact span, as well as, to geometrical mean of the longitudinal and lateral creep coefficients.

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References

[1] S. Iwnicki, *Handbook of Railway Vehicle Dynamics*. New York: CRC Press, Taylor & Francis, 2006, pp. 21–38.
[2] A. H. Wickens, *Fundamentals of Rail Vehicle Dynamics*. New York: Swets & Zeitlinger Publishers, 2003, pp. 101–123.
[3] A.H. Wickens, "The Dynamic Stability of Railway Vehicle Wheelsets and Bogies having Profiled Wheels," *International Journal of Solids and Structures*, 1(3), pp. 319–341, 1965.
[4] B. Suciu, "Clarifications on the Damping Mechanism Related to the Hunting Motion of the Wheel Axle of a High-Speed Railway Vehicle," *International Journal of Aerospace and Mechanical Engineering*, 12(1), pp. 1–8, 2018.
[5] U. Olofsson, and R. Lewis, *Tribology of the Wheel-Rail Contact*. New York: Taylor & Francis, 2012, pp. 121–141.
[6] A. Kapoor, D. I. Fletcher, F. Schmid, K. J. Sawley, and M. Ishida, *Tribology of Rail Transport*. New York: CRC Press, 2001, pp. 161–202.
[7] D. J. Inman, and R.J. Singh, *Engineering Vibration*. New York: Prentice Hall, 2001.
[8] H. Benaroya, and M. L. Nagurka, *Mechanical Vibration: Analysis, Uncertainties, and Control*. London: CRC Press, 3rd ed., 2010.
[9] T. Makino, and H. Sakai, "Fatigue Property of Railway Axles for Shinkansen Vehicles," *Technical Report*, 395, pp. 56–63, 2013 (in Japanese).
[10] D. Yamamoto, "Study on the Running Vibration Characteristics of Railway Vehicle. 1st Report: Estimation of Creep Coefficient between Actual Wheel and Rail," *Journal of System Design and Dynamics*, 4(6), pp. 823–836, 2010.
[11] H. Sakai, "A Consideration to Hunting of Wheel Set," *Proceedings of JSME TRANSLOG*, 1101, pp. 1–10, 2016 (in Japanese).
[12] F. W. Carter, "On the Action of a Locomotive Driving Wheel," *Proceeding of the Royal Society London*, A112, pp. 151–157, 1926.
[13] J. J. Kalker, "Wheel-Rail Rolling Contact Theory," *Wear*, 144, pp. 243–261, 1991.
[14] H. Sakamoto, and M. Yamamoto, "Effect on Nonlinear Creep Force on Railway Truck Dynamics," *Transactions of JSME*, 52(473), pp. 302–309, 1986 (in Japanese).
[15] K. Yokose, M. Igarashi, and J. Takayanagi, "Basic Investigation on the Hunting Motion of the Railway Truck by Considering the Nonlinear Characteristics of the Creep Force," *Transactions of JSME*, 51(466), pp. 1198–1208, 1985 (in Japanese).
[16] T. Matsudaira, N. Matsui, S. Arai, and K. Yokose, "Problems on Hunting of Railway Vehicle on Test Stand," *Journal of Engineering for Industry*, 91(3), pp. 879–885, 1969.
[17] M. Smith, "Train Suspension System," *Patent No. US 9403543*, pp. 1–13, 2016.
[18] B. Suciu, and R. Kinoshita, "Investigation on the Bogie Pseudo-Hunting Motion of a Reduced-Scale Model Railway Vehicle Running on Double-Curved Rails," *International Journal of Mechanical, Aerospace, Industrial, Mechatronic and Manufacturing Engineering*, 11(1), pp. 39–46, 2017.
[19] E. J. Routh, *A Treatise on the Stability of a Given State of Motion: Particularly Steady Motion*. London: MacMillan, 1877, pp. 51–62.
[20] R.C. Dorf, and R.H. Bishop, *Modern Control Systems*. New York: Prentice Hall, 2001, pp. 41–74.
[21] D. E. Newland, *Mechanical Vibration Analysis and Computation*. London: Longman Scientific and Technical, 1989, pp. 125–144.
[22] C. Hoen, "An Engineering Interpretation of the Complex Eigen-Solution of Linear Dynamic Systems," *Proceedings of IMAC XXIII*, pp. 1–6, 2005.

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