Improvement of Ride Comfort of Turning Electric Vehicle Using Optimal Speed Control

Yingyi Zhou, Tohru Kawabe

Abstract—With the spread of EVs (electric Vehicles), the ride comfort has been gaining a lot of attention. The influence of the lateral acceleration is important for the improvement of ride comfort of EVs as well as the longitudinal acceleration, especially upon turning of the vehicle. Therefore, this paper proposes a practical optimal speed control method to greatly improve the ride comfort in the vehicle turning situation. For consturcting this method, effective criteria that can appropriately evaluate deterioration of ride comfort is derived. The method can reduce the influence of both the longitudinal and the lateral speed changes for providing a confortable ride. From several simulation results, we can see the fact that the method can prevent aggravation of the ride comfort by suppressing the influence of longitudinal speed change in the turning situation. Hence, the effectiveness of the method is recognized.

Keywords—Electric vehicle, speed control, ride comfort, optimal control theory, driving support system.

I. INTRODUCTION

IN this century, internal-combustion engine vehicles (ICEVs) have been widely spread all over the world. Consequently, the environmental problems are going seriously. One of the key solution of this problem is the development of electric vehicles (EVs). EVs have several good points compared with the ICEVs [1]-[4]. For examples,

- Environmentally friendly. EVs emit no tailpipe pollutants and no air pollutants.
- Energy efficient. Electric motors much better convert the energy to power than internal-combustion engine.
- The input/output response of electric motors is 2 orders magnitude faster than that of internal-combustion engine.
- The accurate magnitude of torque generated by electric motors can be detected.
- The small size car can be realized by using multiple motors placed in each wheels.

From these advantages, EVs can realize the driving with excellent ride comfort by adequate speed control. The ride comfort is important to prevent a motion sickness and an accident. Many researches about the relation of vibration and ride comfort have been done [5]-[7], [14]. However, these are studied about the ride quality referred by the vertical vibration. But, the longitudinal acceleration/deceleration or tuning motion are also affect to the ride comfort.

Therefore, in this paper, to improve the ride comfort during vehicle turning, we proposes new optimal speed control

Y. Zhou is with Honda Motor China Technology Co., Ltd., Guangdong, 510530 China.

T. Kawabe iswith the Division of Information Engineering, Faculty of Engineering, Information and Systems University of Tsukuba, Tsukuba 305-8573 Japan (phone: +81-29-853-5448; e-mail: kawabe @cs.tsukuba.a.jp).

method. In this method, the influence of lateral speed change is considered as well as the influence of longitudinal speed change by using the evaluation index based on longitudinal acceleration, longitudinal jerk and lateral acceleration. The simulations show that the proposed method is effective for the improvement of ride comfort.

II. EVALUATION OF RIDE COMFORT

Since various factors have an influence on ride comfort, the evaluation criteria of ride comfort is generally depend on the sense of individuals. There are several research about the evaluation of the ride comfort with respect to the frequency of vibrations in vertical and horizontal directions about railroad carriage [8]. But, most of these research investigated the ride quality with respect to the sustained vertical vibration of the steady run. But it is important to evaluate the ride comfort with respect to lateral speed change. From this point of view, [9], [10] have investigated the relation between the acceleration/deceleration, the jerk (time derivative of the acceleration) and ride quality. In [9], [10], subjectivity evaluation of the ride comfort tests on a start, a stop, immediate start, a run situations including the hitting the brakes for the resting posture and the reading posture. Then, the linear multiple regression model about the ride comfort index as follows is derived.

$$
\begin{aligned}
d_1(t) \quad = \quad & \beta_0 + \beta_1 a_{p+}(t) + \beta_2 a_{p-}(t) + \beta_3 j_{r+}(t) \\
& + \beta_4 j_{r-}(t) + \varepsilon(t)
\end{aligned}
\tag{1}
$$

where $a_{p+}(t)$, $a_{p-}(t)$, $j_{r+}(t)$, $j_{r-}(t)$ in $T = (t-3, t)$ are given as follows.

$$
a_{p+}(t) = \begin{cases} \max\limits_{t \in T} a(t), & (|\max\limits_{t \in T} a(t)| \ge |\min\limits_{t \in T} a(t)|) \\ 0, & (|\max\limits_{t \in T} a(t)| < |\min\limits_{t \in T} a(t)|) \end{cases}
\tag{2}
$$

$$
a_{p-}(t) = \begin{cases} 0, & (|\max\limits_{t \in T} a(t)| \ge |\min\limits_{t \in T} a(t)|) \\ \min\limits_{t \in T} a(t), & (|\max\limits_{t \in T} a(t)| < |\min\limits_{t \in T} a(t)|) \end{cases}
\tag{3}
$$

$$
j_{r+}(t) = \begin{cases} \sqrt{\frac{1}{3} \int_{t-3}^{t} j^2(\tau) d\tau}, & (\bar{j}(T) \ge 0) \\ 0, & (\bar{j}(T) < 0) \end{cases}
\tag{4}
$$

$$
j_{r-}(t) = \begin{cases} 0, & (\bar{j}(T) \ge 0) \\ \sqrt{\frac{1}{3} \int_{t-3}^{t} j^2(\tau) d\tau}, & (\bar{j}(T) < 0) \end{cases}
\tag{5}
$$

And where parameters in (1) are defined in Table I. Since $d_1(t)$ is the index at the specific time t calculated from the acceleration and the jerk in the real time, we can not derive the

TABLE I
DEFINITION OF RIDE COMFORT PARAMETERS

$d_1(t)$	ride comfort index at time t
$a_{p+}(t)$	peak value of acceleration in $T = (t-3, t)$ (3 seconds just before time t)
$a_{p-}(t)$	peak absolute value of deceleration in T
$\bar{j}(T)$	average value of jerk in T
$j_{r+}(t)$	effective value of jerk in the case of positive value of $\bar{j}(T)$
$j_{r-}(t)$	effective value of jerk in the case of negative value of $\bar{j}(T)$
$\varepsilon(t)$	error term
β_0	constant term
β_k	Partial regression coefficient

TABLE II
DEFINITION OF 2 WHEELED CAR MODEL PARAMETERS

V	velocity of vehicle
M	mass of vehicle
I_z	yaw moment of inertia
β	vehicle sideslip
ω_z	yaw rate
$\alpha_{1,2}$	front/rear tire slip angle
$F_{yf,yr}$	front/rear tire cornering force
$k_{1,2}$	front/rear tire cornering stiffness
δ	front steering angle
$l_{f,r}$	distance from CoG to front/rear axles
l	vehicle length

control input by using $d_1(t)$ directly. Also, $d_1(t)$ is evaluated only the ride quality in longitudinal direction. Because we consider turning vehicle in this research, we also need to evaluate the ride quality in lateral direction. Therefore, we need other index for speed control with improving the ride comfort. In [9], [10], the value of deceleration, the value of jerk in deceleration, the value of jerk in acceleration and the value of acceleration have affect on the ride quality by this order. Then, we see the acceleration/deceleration and the jerk is important factors for the ride comfort and we can develop the speed control method based on these factors. Specifically, we focus on longitudinal acceleration, jerk and lateral acceleration in turning motion.

III. PROPOSED SPEED CONTROL METHOD

A. Problem Formulation

In this research, the vehicle motion is assumed to be restricted to the longitudinal direction to simplify the approximated two-wheel model as shown in Fig. 1. Parameters in this figure are defined in Table II. From Fig. 1 and Table
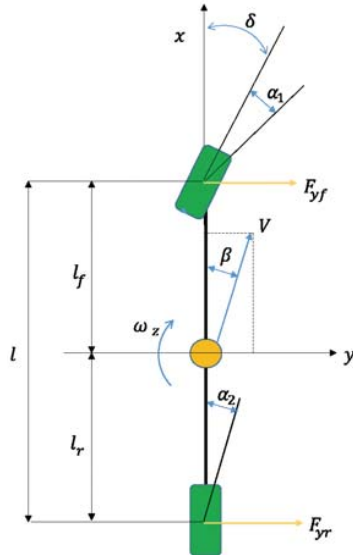

Fig. 1 Approximated 2 wheel car model

II, we can derive the following dynamical equations of turning vehicle.

$$Ma_x = M\dot{V} - MV\beta\omega_z \tag{6}$$

$$Ma_y = MV\dot{\beta} + M\dot{V}\beta + MV\omega_z = F_{yf} + F_{yr} \tag{7}$$

$$I_z\dot{\omega}_z = l_f F_{yf} - l_r F_{yr} \tag{8}$$

Furthermore, we can assume the tire slip angles are relatively small. Then, the cornering forces of vehicle are assumed to be approximated as shown (9) and (10).

$$F_{yf} = k_1\alpha_1 \tag{9}$$

$$F_{yr} = k_2\alpha_2 \tag{10}$$

where

$$\alpha_1 = \delta - \beta - l_f\frac{\omega_z}{V} \tag{11}$$

$$\alpha_2 = -\beta + l_r\frac{\omega_z}{V} \tag{12}$$

B. Proposed Method

From (6)-(8), we can obtain the following state space equation model of turning 2-wheel car.

$$\underbrace{\begin{pmatrix} \dot{x} \\ \dot{V} \\ \dot{a} \\ \dot{\beta} \\ \dot{\omega}_z \end{pmatrix}}_{\dot{x}} = \underbrace{\begin{pmatrix} V \\ a \\ u \\ a_1\frac{\beta}{V} + a_2\frac{\omega_z}{V^2} - \omega_z + a_3\frac{\delta}{V} + a_4\frac{a\beta}{V} \\ b_1\beta + b_2\frac{\omega_z}{V} + b_3\delta \end{pmatrix}}_{f(x,u,t)} \tag{13}$$

where x is vehicle position, a is vehicle acceleration and j is jerk. Please take notice here that the control input is the jerk $u = j$ in this case. The reason is that we need to take into account of the influence that the jerk gives to ride quality. And where $a_{1,2,3,4}$ and $b_{1,2,3}$ are constant parameters derived from vehicle parameters as follows.

$$a_1 = -\frac{k_1 + k_2}{M}, \quad a_2 = \frac{k_2 l_r - k_1 l_f}{M}, \quad a_3 = \frac{k_1}{M}, \quad a_4 = -\frac{1}{M}$$

$$b_1 = \frac{k_2 l_r - k_1 l_f}{I_z}, \quad b_2 = -\frac{k_1 l_f^2 + k_2 l_r^2}{I_z}, \quad b_3 = \frac{k_1 l_f}{I_z}$$

Moreover, from (6)-(8), we can obtain the longitudinal acceleration (a_x), the longitudinal jerk (j_x) and the lateral acceleration (a_y) are derived as follows.

$$a_x = \dot{V} - V\beta\omega_z \tag{14}$$

$$j_x = \ddot{V} - \dot{V}\beta\omega_z - V\dot{\beta}\omega_z - V\beta\dot{\omega}_z \tag{15}$$

$$a_y = V\dot{\beta} + \dot{V}\beta + V\omega_z \tag{16}$$

where $\dot{V} = a\ddot{V} = j$.

From these equations, the evaluation index of ride comfort is defined as (17).

$$
\begin{aligned}
J &= \int_0^{t_f} \left[q^2 j_x^2 + p^2 a_x^2 + k^2 a_y^2 \right] dt \\
&= \int_0^{t_f} \left[q^2 (u - a\beta\omega_z - V\dot{\beta}\omega_z - V\beta\dot{\omega}_z)^2 + p^2 (a - V\beta\omega_z)^2 \right. \\
&\quad \left. + k^2 (V\dot{\beta} + a\beta + V\omega_z)^2 \right] dt \qquad (17)
\end{aligned}
$$

where t_f is terminal time and, qp and k are weighting constants.

In this way, we can derive the optimal control input which reduces the influence on ride comfort by the longitudinal and lateral accelerations at the same time by using this J.

Next, let's derived the optimal control input (u). From state space equation (13) and evaluation index (17), Hamiltonian is obtained by using Lagrange multiplier (λ) as follows.

$$
\begin{aligned}
H &= \frac{1}{2} [q^2 (u - a\beta\omega_z - V\dot{\beta}\omega_z - V\beta\dot{\omega}_z)^2 \\
&\quad + p^2 (a - V\beta\omega_z)^2 + k^2 (V\dot{\beta} + a\beta + V\omega_z)^2] \\
&\quad + \lambda^T f(x, u, t) \qquad (18)
\end{aligned}
$$

Then, in this case, the stationary equation is described based on the optimal control theory [11] as follows.

$$
0 = \frac{\partial H}{\partial u} = q^2 (u - a\beta\omega_z - V\dot{\beta}\omega_z - V\beta\dot{\omega}_z) + \lambda_3 \qquad (19)
$$

The costate equation is $-\dfrac{\partial H}{\partial x} =$

$$
\begin{aligned}
\begin{pmatrix} \dot{\lambda}_1 \\ \dot{\lambda}_2 \\ \dot{\lambda}_3 \\ \dot{\lambda}_4 \\ \dot{\lambda}_5 \end{pmatrix} &= \begin{pmatrix} 0 \\ q^2 j_x (\dot{\beta}\omega_z + \dot{\omega}_z\beta) + p^2 a_x\beta\omega_z - k^2 a_y(\dot{\beta} + \omega_z) \\ q^2 j_x\beta\omega_z - p^2 a_x - k^2 a_y\beta \\ q^2 j_x (a\omega_z + \dot{\omega}_z V) + p^2 a_x V\omega_z - k^2 a_y a \\ q^2 j_x (a\beta + \dot{\beta}V) + p^2 a_x V\beta - k^2 a_y V \end{pmatrix} \\[6pt]
&+ \begin{pmatrix} 0 \\ -\lambda_1 + \lambda_4 \left(\dfrac{a_1\beta + a_3\delta + a_4 a\beta}{V^2} + \dfrac{2a_2\omega_z}{V^3} \right) + \dfrac{\lambda_5 b_2\omega_z}{V^2} \\ -\lambda_2 - \lambda_4 \dfrac{a_4\beta}{V} \\ -\lambda_4 \left(\dfrac{a_1 + a_4 a}{V} \right) - \lambda_5 b_1 \\ -\lambda_4 \left(\dfrac{a_2}{V^2} - 1 \right) - \lambda_5 \dfrac{b_2}{V} \end{pmatrix} \qquad (20)
\end{aligned}
$$

The state equation is also expressed as

$$
\dot{x} = \frac{\partial H}{\partial \lambda} = f(x, u, t) \qquad (21)
$$

From these 3 equations, we can obtain the optimal control input in theoretically. However, these equations are nonlinear and hard to be solved analytically. Therefore, in this paper, we derived the optimal control input numerically by using the conjugate gradient method [11].

Additionally, values of weighting constants qpk in J of (17) are decided for making trial and error so that the effective value of ride quality described below becomes as small as possible under the condition $q + p + k = 1$.

The index $d(t)$ described in (22) is employed as evaluation criteria of ride comfort in this paper.

$$
d(t) = d_1(t) + q_r d_2(t) \qquad (22)
$$

where q_r is weighting constant and where $d_1(t)$ is the index evaluating the longitudinal ride quality in (1), $d_2(t)$ is the index evaluating the lateral ride quality defined as follows [12].

$$
d_2(t) = \beta_{02} + \beta_a a_y(t) + \beta_j j_y(t) + \varepsilon_2(t) \qquad (23)
$$

Then, smaller the value of $d(t)$, the ride quality against longitudinal and lateral directions is better. But, $d_(t)$ is the index at the specific time t calculated from the acceleration and the jerk in the real time as same as $d_1(t)$ as we mentioned in Section II. Namely, we can not decide values of q, p, k from $d_1(t)$. Therefore, we use the effective value of $d(t)$ as shown in (24). We decide values of q, p, k to minimize $d_{rms}(t)$ by trial and error.

$$
d_{rms}(t) = \sqrt{\frac{1}{t_f - t_0} \int_{t_0}^{t_f} d^2(\tau) d\tau} \quad (t_f > t_0) \qquad (24)
$$

IV. Simulation Results

To confirm the effectiveness of the proposed method, we compared the method with the conventional one in some simulations. Reference [13] is used as the conventional method here. Reference [13] proposes a control method of the yaw rate to suppress the maximum value of the lateral jerk.

A. Simulation I (Step Input)

As an example, we perform the simulation under the situation that a car straight running with 60[km/h] is start to turn by step-like input of steering angle at time $t = 1$ as shown in Fig. 2. Table III shows values of parameters used in
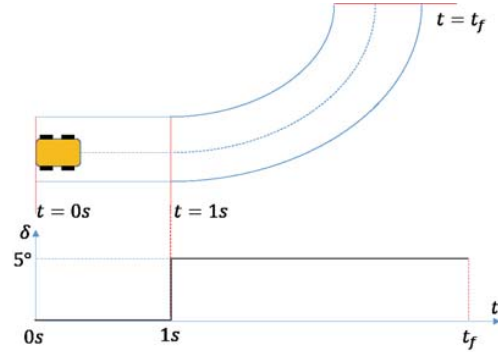

Fig. 2 The step input of front steering angle

this simulation. Values of weighting constants of J in (17) are

simulation time	$0 - 5[s]$
M:mass of vehicle	$1100[kg]$
I_z: yaw moment of inertia	$1600[kg \cdot m^2]$
k_1:front tire cornering stiffness	$32000[N/rad]$
k_2:rear tire cornering stiffness	$32000[N/rad]$
l_f:distance from CoG to front axle	$1.15[m]$
l_r:distance from CoG to rear axle	$1.35[m]$
l:vehicle length	$2.5[m]$

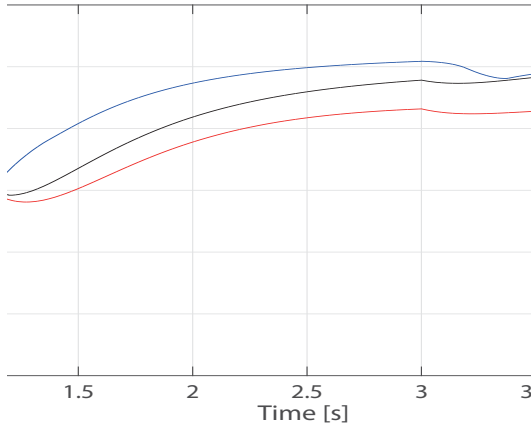obtained as $q = 0.65$, $p = 0.34$ and $k = 0.01$ respectively by

Fig. 3 Time response of $d(t)$. In the whole simulation time, the value of $d(t)$ of the proposed method is smaller than that of the case without control. And the value of $d(t)$ of the proposed method is smaller than that of the conventional method since $1.05s$

minimizing $d_{rms}(t)$ in (24). Time responses of evaluation index of ride quality ($d(t)$) is shown in Fig. 3. In this simulation, effective and maximum values of $d(t)$ by proposed method, conventional method and no-control are shown in Table IV.

TABLE IV
THE EFFECTIVE AND MAXIMUM VALUES OF $d(t)$

	effect. value of $d(t)$	max. value of $d(t)$
No control	4.497	4.983
Conventional method	4.742	5.089
Proposed method	3.978	4.322

From these results, we can see that the proposed method can improve the ride comfort compared with the conventional method.

B. Simulation II (Ramp Input)

The situation in simulation II is assumed that a car straight running with 60[km/h] is start to turn by rump-like input of steering angle at time $t = 1$ as shown in Fig. 4. Values of parameters are same as simulation I (shown in Table III).
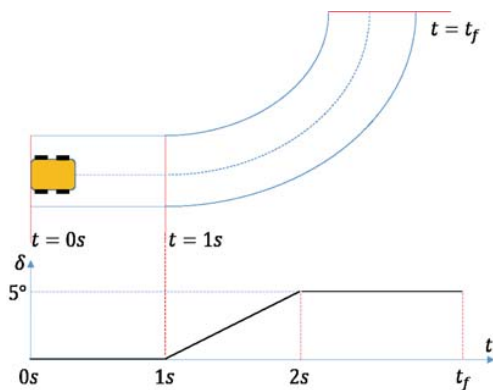

Fig. 4 The ramp input of front steering angle

In this simulation, values of weighting constants are obtained as $q = 0.62$, $p = 0.36$ and $k = 0.02$ respectively. Time responses of $d(t)$ in this simulation is shown in Fig. 5.
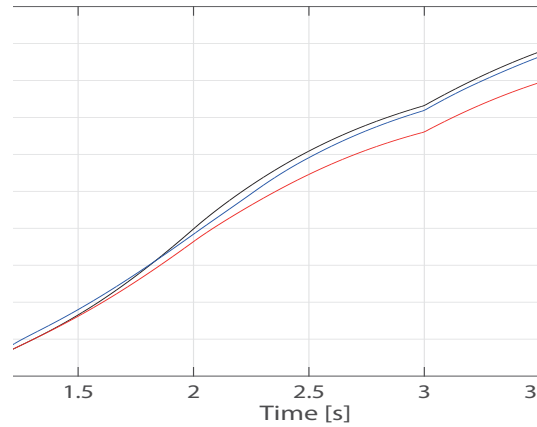

Fig. 5 Time response of $d(t)$. In the whole simulation time, the value of $d(t)$ of the proposed method is smaller than that of both the conventional method and the case without control

Effective and maximum values of $d(t)$ in this simulation are shown in table V.

TABLE V
THE EFFECTIVE AND MAXIMUM VALUES OF $d(t)$

	effect. value of $d(t)$	max. value of $d(t)$
No control	3.647	4.953
Conventional method	3.613	4.951
Proposed method	3.242	4.297

From these results, we also see that the proposed method shows good performance.

C. Simulation III (Change Input Twice)

Let's consider the situation that the steering angle change twice in the simulation as shown in Fig. 6. Other situations and values of parameters are same as simulation I and II. In
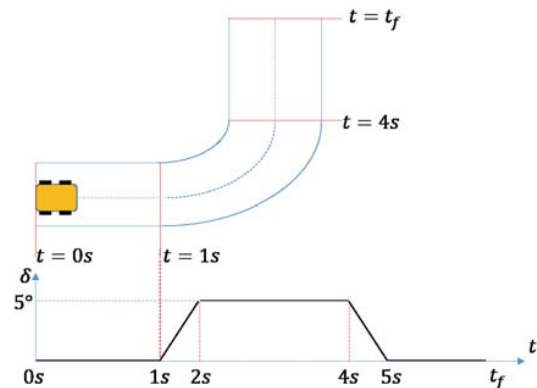

Fig. 6 The ramp input of front steering angle

this simulation, values of weighting constants are obtained as $q = 0.94$, $p = 0.05$ and $k = 0.01$ respectively. Time responses
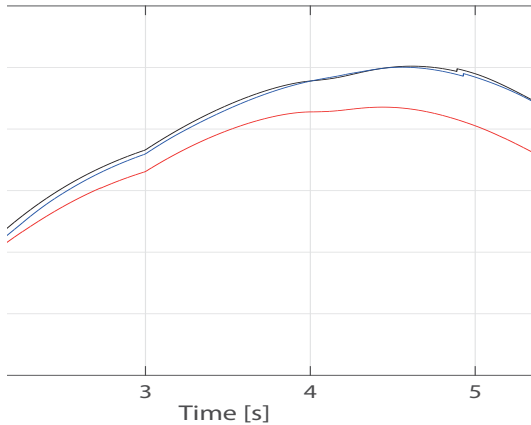
Fig. 7 Time response of $d(t)$. In the whole simulation time, the value of $d(t)$ of the proposed method is smaller than that of both the conventional method and the case without control

of $d(t)$ in this simulation is shown in Fig. 7. Effective and maximum values of $d(t)$ in this simulation are shown in Table VI.

TABLE VI
THE EFFECTIVE AND MAXIMUM VALUES OF $d(t)$

	effect. value of $d(t)$	max. value of $d(t)$
No control	4.236	5.142
Conventional method	4.187	5.076
Proposed method	3.524	4.303

From this figure, we also see that the proposed method shows good performance. In addition, it can be predicted that we can improve the ride comfort more by searching the most suitable values of weighting constant whenever the steering angle changes.

V. CONCLUSION

For improving the ride comfort at the vehicle turning situation, the speed control method that considered the influence of the longitudinal and the lateral speed change at the same time based on the optimal control theory has been proposed. In addition, by numerical simulations, we have showed the effectiveness of the proposed method compared with the conventional one.

The proposed method is able to make the acceleration and deceleration appropriately according to the response of the vehicle, and to prevent aggravation of the ride comfort by suppressing the influence of longitudinal speed change in the turning situation. Furthermore, the proposed method reduces the influence of the lateral speed change by setting the values of weighting constants in evaluation index appropriately. From these facts, we can see that the effective and maximum values of ride comfort index are suppressed small by the proposed method compared with the conventional one.

As future works, we need to investigate the simple and quick searching method for values of weighting constant in evaluation index, and examine the effectiveness by actual experimental car.

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