A Robust Optimization Model for the Single-Depot Capacitated Location-Routing Problem

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paper, Abstract—In this the single-depot capacitated location-routing problem under uncertainty is presented. The problem aims to find the optimal location of a single depot and the routing of vehicles to serve the customers when the parameters may change under different circumstances. This problem has many applications, especially in the area of supply chain management and distribution systems. To get closer to real-world situations, travel time of vehicles, the fixed cost of vehicles usage and customers' demand are considered as a source of uncertainty. A combined approach including robust optimization and stochastic programming was presented to deal with the uncertainty in the problem at hand. For this purpose, a mixed integer programming model is developed and a heuristic algorithm based on Variable Neighborhood Search(VNS) is presented to solve the model. Finally, the computational results are presented and future research directions are discussed.

Keywords—Location-routing problem, robust optimization, Stochastic Programming, variable neighborhood search.

I. Introduction

THIS Location-routing problem (LRP) is a combination of two well-known problems, facility location and vehicle routing problems(VRP). In recent decades, this problem has attracted the attention of many researchers because of its wide applications in supply chain management and distribution systems. In these problems, the decisions relating to the location of depots simultaneously are made with the routing decisions. In fact, in many supply chain cases, location and routing decisions are closely interrelated. This dependency comes from the fact that in some applications, the trip of vehicles from depot to the customers or vice versa is not necessarily directly and usually involved a tour. Salhi and Rand [23] showed that the ignoring the routing decisions when locating depots might lead to the suboptimal results.

On the other hand, modeling real-world problems is usually affected by several factors which are often impossible to find the exact amounts of them. Hence, considering the uncertainty is an important issue in the optimization problems. Two different criteria are widely used to deal with the uncertainty in the literature of this area: the minimization of the expected cost and the minimization of the worst-case cost or regret. For this purpose, normally one of two approaches, stochastic programming or robust optimization is employed for modeling these problems. Clearly, both of these approaches have some disadvantages and using of them might lead to a solution that, under some circumstances, be very inappropriate. In one hand, the stochastic models try to solve the problem with the expected value among all scenarios. Thus, the optimal solution obtained from this approach may be appropriate for

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some scenarios, but for others it is significantly weak. On the other hand, the robust approach try to find the min-max cost or min-max regret solution that is close or almost optimal for any realization of the uncertainty in a given set and no matter which scenario is likely to happen. The investigated robustness measure incorporates the advantages of both the min-expected-cost and min-max regret measures by seeking the least-cost solution (in the expected value) that bounds the regret by a pre-specified limit. Therefore, in this study, the single depot LRP is investigated when the problem parameters, (i.e. vehicles travel time, customers demand, and vehicles cost) are uncertain and given by a set of scenarios with known probability. A combined stochastic robust optimization is also used to formulate the problem.

The stochastic p-robust approach were defined as follow by Snyder and Daskin [25]. For a given set of scenarios S, let x be a feasible solution of the deterministic LRP under scenario s, $Z_s(x)$ be the objective value of this solution and Z_s^* be the optimal objective value of scenario s. Therefore, by defining a non-negative constant $(p \ge 0)$, x is called a p-robust solution if for all $s \in S$:

$$\frac{Z_s(x) - Z_s^*}{Z_s^*} \le \rho \tag{1}$$

In addition, if scenario s will occur with a certain probability q_s . Hence, in the general form, stochastic p-robustness measure that combining p-robustness measure with a min-expected-cost objective function was introduced as follow. χ gives the feasible solution space of the problem.

$$\min = \sum_{s \in S} q_s Z_s(x) \tag{2}$$

s.t.

$$Z_s(x) \le (1+\rho)Z_s^* \quad \forall s \in S$$
 (3)

$$x \in \chi$$
 (4)

The problem studied in this research is a network with nodes consisting of customers site and potential locations for the construction of a distribution warehouse. Among the candidate sites, one location is chosen to open the depot and the serving tours of vehicles from the depot to the costumers are determined.

In the following, a concise review of the literature related to LRP is presented. A mixed-integer linear programming formulation for the studied problem is given in Section II. In the next section, a solution algorithm based on variable neighborhood search (VNS) is proposed and the computational results are given. Finally, concluding remarks are discussed in Section IV.

The studies in the area of LRP date back to the late 1970s and early 1980s, where researchers developed the first models to implement on some real world applications like distribution systems [7], rubber industry [18], regional blood banking [19], newspaper delivery system [10] or proposed solution algorithms to solve the problem [15], [9], [16]. After these, many studies have been devoted to the LRP in the literature. Recently, three survey papers have reviewed the various aspects of this problem including modeling strategies, optimization algorithms and its applications [21], [3], [24].

Decision-making under uncertainty is one of the principal research themes of the field of LRP that has been only rarely addressed in the literature. Laporte et al. [14] presented a stochastic LRP with random supplies as the first research that has been conducted in this area. A stochastic multi-period LRP was investigated by Klibi et al. [13] that characterized by multiple transportation options, multiple demand periods, and stochastic demands. Ghaffari-Nasab et al. [6] studied some stochastic programming approaches to model the capacitated LRP with probabilistic travel times as bi-objective mathematical programming formulations.

In addition, to the best our knowledge there is no published work on using robustness approach to cope with uncertainty in the LRP. However, robust optimization has been studied in both facility location and vehicle routing problems, separately. Sungur et al. [26] introduced a robust optimization approach to solve the capacitated VRP with demand uncertainty. Adulyasak and Jaillet [1] investigated both stochastic and robust optimization approaches for the VRP with deadlines under travel time uncertainty and developed an exact algorithm based on a branch-and-cut framework to solve the problems. In another paper, an efficient heuristic based on adaptive large neighborhood search was proposed to solve the VRP with uncertain travel times and time windows [2].

The combined stochastic robust optimization approach has been already used to investigate uncertainty in some problems related to facility location and network design problems (i.e., classical facility location problems [25], single allocation p-hub median problem [5], closed-loop supply chain network design [12]).

II. MATHEMATICAL MODELLING

The LRP can be considered under different circumstances. In this study, the following assumptions are made to model the stochastic p-robust single depot LRP: (1) the travel times, customers demand, and vehicles usage cost parameters are given by a set of scenarios; (2) the probability of the occurrence of each scenario is already known; (3) the demand of each customer is satisfied only by one vehicle; (4) each tour starts at the opened depot and finishes at the same depot. According to these assumptions, the aim of the problem is to minimize the total summation of the depot fixed cost, vehicles usage cost and the transportation cost.

The used notation of the problem at hand is introduced in bellow.

A. Notation and Assumptions

The index sets, input parameters and decision variables are described as follows:

Index sets:

I: Set of potential nodes to open the single depot, $i \in I$

J: Set of demand nodes(customers), $i \in J$

K: Set of vehicles, $k \in K$

V: Set of depots and customers, $v \in V$, $V = \{I \cup J\}$

S: Set of scenarios, $s \in S$.

Input parameters:

n: The number of customers, i.e. n = |J|

 t_{ij}^s : Travel time between nodes i and j in scenario s

 o_i : Fixed cost of opening the depot in node i

 f_s : Fixed cost of the vehicle usage in scenario s

 p_s : Occurrence probability of scenario s

 d_i^s : Demand of customer j in scenario s

q: Vehicle capacity.

Decision variables:

Three sets of variables are used to made decisions in two stages. The location variables are independent of the possible scenarios, whereas the routing decisions are not. In the first stage, the location decisions related to the depot location are determined. In the second stage, the routing decisions are taken for each possible scenario subject to the obtained depot location of the first stage.

 $X_{ijk}^s = 1$ if vehicle k travels directly between node i and node j in scenario s, 0 otherwise

 $Y_i = 1$ if candidate location i is opened as depot, 0

 $R_{ij}^s = 1$ if node j assigned to depot i in scenario s U_{jk} : auxiliary variable to eliminate the subtour in route k

$$\min Z = \sum_{i \in I} o_i Y_i + \sum_{s \in S} p_s (\sum_{i \in V} \sum_{j \in V} \sum_{k \in K} t^s_{ij} X^s_{ijk}$$

$$+ \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} f_s X^s_{ijk})$$

$$(5)$$

s.t.

$$\sum_{i \in I} Y_i = 1 \tag{6}$$

$$\sum_{i \in V} \sum_{k \in K} X_{ijk}^s = 1 \quad \forall j \in J, s \in S$$

$$\sum_{i \in V} \sum_{j \in J} d_j^s X_{ijk}^s \le q \quad \forall k \in K, s \in S$$
(8)

$$\sum_{i \in V} \sum_{j \in I} d_j^s X_{ijk}^s \le q \quad \forall k \in K, s \in S$$
 (8)

$$U_{rk} - U_{jk} + nX_{rjk}^s \le n - 1$$

$$\forall r, j \in J, k \in K, s \in S \tag{9}$$

$$\forall r, j \in J, k \in K, s \in S$$

$$\sum_{j \in V} X_{ijk}^{s} - \sum_{j \in V} X_{jik}^{s} = 0$$

$$\forall i \in V, k \in K, s \in S$$

$$\forall i \in V, k \in K, s \in S \tag{10}$$

$$\sum_{i \in I} \sum_{j \in J} X_{ijk}^s \le 1 \quad \forall k \in K, s \in S$$
 (11)

$$-R_{ij}^{s} + \sum_{m \in J} X_{imk}^{s} + \sum_{m \in V \setminus \{J\}} X_{mjk}^{s} \le 1$$

$$\forall i \in I, j \in J, k \in K, s \in S \qquad (12)$$

$$\sum_{i \in I} o_{i} Y_{i} + \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} t_{ij}^{s} X_{ijk}^{s} +$$

$$\sum_{i \in I} \sum_{j \in J} \sum_{k \in K} f_{s} X_{ijk}^{s} \le (1 + \alpha) Z_{s}^{*} \forall s \in S \quad (13)$$

$$Y_i \in \{0, 1\} \forall i \in I \tag{14}$$

$$X_{ijk}^s \in \{0,1\} \forall i \in I, j \in J, k \in K, s \in S$$
 (15)

$$\begin{split} X_{ijk}^s &\in \{0,1\} \forall i \in I, j \in J, k \in K, s \in S \\ F_{ij}^s &\in \{0,1\} \quad \forall i \in I, j \in J, s \in S \end{split} \tag{15}$$

$$U_{jk} \ge 0 \quad \forall j \in J, k \in K \tag{17}$$

In this formulation, the objective function of the problem is given by (5) that minimizes the problem costs. The first term is used to define the fixed facility cost which is not dependent to the scenarios. Furthermore, the second and third terms are related to the scenarios and calculate the transportation and the vehicle usage costs, respectively. Equation (6) states that only one depot should open in this problem. The single assignment of each customer to the tours for each scenario is defined in (7). Each vehicle can only handle a limited number of shipments and this capacity is reflected in (8). Equation (9) is the subtour elimination constraint and do not let to constitute the tours without the depot. Equations (10) and (10) are to ensure the flow conservation of vehicles when visit a network node. These equations, for each scenario, ensure the continuity of each route and a return to the depot of origin. Constraints (10) determine that a customer can be assigned to the depot only if a route linking them is activated. The p-robust constraint is given by (13). According to this constraint, the cost of each scenario will not higher than $(1 + \alpha)$ percent of its optimal value. Finally, (14)-(17) are integrality constraints.

III. COMPUTATIONAL RESULTS

In the following, the solution algorithm with experiment results are presented.

A. Solution Algorithm

Both facility location and vehicle routing problems are belong to the combinatorial optimization problems. Hence, the location routing problem is also NP-hard. Preliminary experiments showed that the execution time of CPLEX solver to solve the stochastic p-robust LRP is highly larger than that the deterministic problem with a single scenario. Hence, developing effective algorithms to deal with this problem is a challenging work. In this paper, besides the CPLEX solver, a heuristic based on the VNS was investigated to solve the problem in the various dimensions.

Variable neighborhood search is a classical meta-heuristic aimed to solve the optimization problems. This algorithm systematically exploits the idea of neighborhood change, both in descent to local minima and in escape from the valleys which contain them. This algorithm was first introduced by Mladenovic and Hansen [17] in solving the traveling salesman problem as a special case of the vehicle routing problem. Since then, this algorithm has been further developed both in its methods and its applications. Several variants of this algorithm efficiently were used for solving both location and routing problems. Jabalameli and Ghaderi [8] used the VNS to solve the multi-source Weber problem. A hybrid VNS was proposed to solve the budget-constrained dynamic uncapacitated facility location-network design problem by Ghaderi et al. [4]. In addition, VNS is also applied on different contexts of routing problems, e.g. ([22], [27]). On the other hand, Jarboui et al. [11] proposed different versions of VNS to solve the location routing problem with multiple capacitated depots and one uncapacitated vehicle per depot. Therefore, VNS is shown to be relevant to solve the location-routing problem and related problems.

As mentioned earlier, two different decisions consisting location and routing should be made in the problem. The pseudocode of the VNS algorithm is presented at the end of this subsection.

In order to represent a solution, a vector consisting of the depot location, customers and vehicles number is constructed. For instance, if we want to select a depot among three candidate locations {1,2,3}, and service five customers $\{1,\ldots,5\}$ by three available vehicles $\{6,7,8\}$. Then, vector [2,3,2,7,6,1,4,5] denotes a solution such that the first number shows the candidate location 2 is selected to open the depot. In addition, the first serving tour is extracted by selecting the numbers between the depot number and the first vehicle number (i.e., 7). Hence, the first vehicle starts at the depot and visits the customers 3 and 2, respectively, and finally returns back to the depot. Similarly, the customers between the vehicles number form the other routes. As a results, the following tours are extracted from the given vector.

 $\{[2,3],[1,4,5]\}$ The VNS algorithm starts with an initial solution X and then evokes the related serving tours. The feasibility of the generated solution is checked and its objective function is calculated. After that, the main phase of the algorithm executes until a termination condition is met. In each iteration, a new solution is generated from the current neighborhood by using different neighborhood structures. Generally, the VNS algorithm sets the neighborhood structure at 1 and increases its value step-by-step after any nonimprovement. The process continues until the neighborhood rank reaches to a predefined threshold(i.e., k_{max}) and then restarts again from k = 1. In this study, we let to the VNS, explore more the solution space with the neighborhood of rank k=1 when there is no improvement in the solution(i.e., t_{max}). Whenever the algorithm leads to improvement in the current solution, the new solution is replaced and searching process continues with k=1. Note that a repairing strategy are used to repair the infeasible solutions within the framework of the algorithm.

proposed **VNS** Main framework of the algorithm:

Input the data of problem and the VNS algorithm:

- 1) Select the set of neighborhood structures, N_k : k = 1, ..., 5;
- 2) Generate initial solution X;
- 3) Extract the related serving tours;
- 4) Check the solution feasibility and repair it if necessary;
- 5) Calculate the objective value of the solution;

6) Set $k \leftarrow 1$ and $t \leftarrow 1$.

Until the stopping criteria are met repeat the following steps:

- Neighborhood searching: generate a new solution \acute{X} from the neighborhood of rank k,
- Tours extracting and solution feasibility checking: extract the serving tours of \acute{X} and check the solution feasibility and repair it if necessary,
- Solution evaluation: calculate the objective value of \dot{X} ,
- Move to the improved solution:

If the generated solution \acute{X} is better than X, replace $X \leftarrow \acute{X}, k \leftarrow 1, t \leftarrow 1;$

Else if $t \leq t_{max}$, $k \leftarrow 1, t \leftarrow t + 1$;

Otherwise, increase the neighborhood rank $k \leftarrow k + 1$:

• If $k > k_{max}$ return to the neighborhood of rank 1, $k \leftarrow 1, t \leftarrow 1$.

Return the improved solution

B. Experimental Results

In this section, an experimental analysis to test the performance of the proposed VNS algorithm is performed. For this purpose, eight test problems were solved with the standard mathematical programming software GAMS 24.7.1 and solver CPLEX 12.6.3.0, on a computer with Core i5 3.1GHz processor and 8 GB RAM. Python 2.7 was also used to code the VNS.

Table I illustrates the characteristics of generated instances. In order to generate the problem data, some parts of the instances introduced by Prodhon [20] are used. For the instances with the smaller dimensions, only some potential depots and customers were used as the first scenario in our model. Furthermore, for each test instance, we generated additional scenarios by multiplying scenario-1 data to a random number drawn uniformly from [0.5, 1.5]. For the sake of tractability, we also assume each scenario has the same probability to occur (i.e., $p_s = \frac{1}{|S|}$, $\forall s \in S$) without losing generality.

TABLE I THE CHARACTERISTICS OF GENERATED TEST PROBLEMS

THE CHARGETERISTICS OF CENERALES TEST PROBLEMS					
TestProblem	#of customers	#of candidate depot	#of vehicles		
TP1	3	2	4		
TP2	6	2	4		
TP3	7	3	4		
TP4	9	3	4		
TP5	11	4	4		
TP6	15	5	4		
TP7	20	5	4		
TP8	25	5	4		

Table II shows the experimental results of the CPLEX with $\alpha=0.5$ consisting the optimal solution of each scenario, Z_s^* , the optimal or best found solution of the robust model, Z_{robust} and the reported gap (or running time) of CPLEX. As we can see, except for the two smallest instances, CPLEX could not reach to the optimal solution in 5 hours as a time limit. Whereas, finding the optimal solution of each scenario also takes much time which not reported here.

The computational results of the proposed algorithm are also given in Table III. Note that the algorithm was executed 4 time and the best found solution, Z_{best} , average fitness in these

TABLE II
THE CPLEX RESULTS

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TP	Z_1^*	Z_2^*	Z_3^*	Z_4^*	Z_{robust}	Gap(Cpu)	
TP1	11627	11975	11458	11604	11627.5	(1.4)	
TP2	14068	13249	13439	13946	13702.5	(17451)	
TP3	9479	7620	8101	8661	8465.25	26.9	
TP4	17548	16673	14616	19946	17214.25	17.8	
TP5	11044	13522	10130	11370	11495.25	44.5	
TP6	17294	17480	17663	18863	17882.5	41.7	
TP7	15856	13504	17612	17586	16148.5	62.7	
TP8	18150	20365	12935	16229	18123.25	68.13	

runs, Z_{Av} , average running time, CPU_{Av} , the calculated GAP of the best found solution with the CPLEX are reported. The numerical experiments show the effectiveness of the proposed algorithm in comparation with CPLEX. For the two first instances, VNS converges to the optimal solution in a shorter time compared to CPLEX. For the other instances, the quality of the solutions is close or better than the CPLEX with a very small proportion of the time.

TABLE III
THE SUMMARY RESULTS OF VNS SLGORITHM

THE SUMMART RESULTS OF			VIVO SEGORITIM		
Z_{best}	Z_{Av} .	CPU_{Av} .	Gap		
11627.5	11627.5	0.5	0		
13702.5	13702.5	1.6	0		
8701.25	8784.68	200	0.02		
17034.5	17109.87	240	-0.01		
11159.5	11334.81	300	-0.03		
19251.5	19836.93	400	0.07		
17156	20050.06	500	0.05		
18301	18579.87	600	0.009		
	Z _{best} 11627.5 13702.5 8701.25 17034.5 11159.5 19251.5 17156	$\begin{array}{c cccc} Z_{best} & Z_{Av}. \\ \hline 11627.5 & 11627.5 \\ 13702.5 & 13702.5 \\ 8701.25 & 8784.68 \\ 17034.5 & 17109.87 \\ 11159.5 & 11334.81 \\ 19251.5 & 19836.93 \\ 17156 & 20050.06 \\ \hline \end{array}$	11627.5 11627.5 0.5 13702.5 13702.5 1.6 8701.25 8784.68 200 17034.5 17109.87 240 11159.5 11334.81 300 19251.5 19836.93 400 17156 20050.06 500		

For one iteration, the improvement trend of the proposed algorithm for the test problem TP5 is also given in Figure 1. This shows that the VNS was able to improve the initial solution from 12654 to 11188 in 20 times.



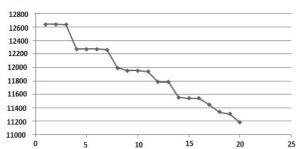


Fig. 1 The improvement trend of the algorithm in solving TP5

IV. CONCLUSION

In this paper, an optimization model for the single depot location-routing problem under parameters uncertainty was presented. In this model, a combined approach consisting stochastic programming and robust optimization was investigated. The p-robustness condition is incorporated in constraints in order to make sure that our solution in each scenario would not increase more than $(1+\rho)\%$ of the optimal

solution of that scenario. The proposed model is a developed version of classical LRP and is very difficult to solve. Hence, a solution procedure based on variable neighborhood search was also developed to solve the problem. To test this algorithm, we generated 8 test instances and compared the algorithm's result with the obtained results by CPLEX. The results demonstrated apparently that our algorithm outperforms CPLEX in some instances in terms of CPU time and solution quality. Since this research is the first to study the robust optimization in the LRP and due to the breadth and importance of the problem applications, further research is required both to model this approach on the LRP with other assumptions and developing efficient algorithms to solve the developed models.

REFERENCES

- Adulyasak, Y. and Jaillet, P. (2015), 'Models and algorithms for stochastic and robust vehicle routing with deadlines', *Transportation Science* 50(2), 608–626.
- [2] Braaten, S., Gjønnes, O., Hvattum, L. M. and Tirado, G. (2017), 'Heuristics for the robust vehicle routing problem with time windows', *Expert Systems with Applications* 77, 136–147.
- [3] Drexl, M. and Schneider, M. (2015), 'A survey of variants and extensions of the location-routing problem', European Journal of Operational Research 241(2), 283–308.
- [4] Ghaderi, A., Boland, N. and JabalAmeli, M. S. (2012), 'Exact and heuristic approaches to the budget-constrained dynamic uncapacitated facility location-network design problem', Centre for Optimal Planning and Operations, The University of Newcastle.
- [5] Ghaderi, A. and Rahmaniani, R. (2016), 'Meta-heuristic solution approaches for robust single allocation p-hub median problem with stochastic demands and travel times', *The International Journal of Advanced Manufacturing Technology* 82(9-12), 1627–1647.
- [6] Ghaffari-Nasab, N., Jabalameli, M. S., Aryanezhad, M. B. and Makui, A. (2013), 'Modeling and solving the bi-objective capacitated location-routing problem with probabilistic travel times', *The International Journal of Advanced Manufacturing Technology* pp. 1–13.
- [7] Harrison, H. (1979), 'A planning system for facilities and resources in distribution networks', *Interfaces* 9(2-part-2), 6–22.
- [8] Jabalameli, M. S. and Ghaderi, A. (2008), 'Hybrid algorithms for the uncapacitated continuous location-allocation problem', The International Journal of Advanced Manufacturing Technology 37(1), 202–209.
- [9] Jacobsen, S. K. and Madsen, O. B. (1980), 'A comparative study of heuristics for a two-level routing-location problem', *European Journal* of Operational Research 5(6), 378–387.
- [10] Jacobsen, S. and Madsen, O. (1978), On the location of transfer points in a two-level newspaper delivery system—a case study, in 'international symposium on locational decisions', pp. 24–28.
- [11] Jarboui, B., Derbel, H., Hanafi, S. and Mladenović, N. (2013), 'Variable neighborhood search for location routing', *Computers & Operations Research* 40(1), 47–57.
- [12] Keyvanshokooh, E., Ryan, S. M. and Kabir, E. (2016), 'Hybrid robust and stochastic optimization for closed-loop supply chain network design using accelerated benders decomposition', *European Journal of Operational Research* 249(1), 76–92.
- [13] Klibi, W., Lasalle, F., Martel, A. and Ichoua, S. (2010), 'The stochastic multiperiod location transportation problem', *Transportation Science* 44(2), 221–237.
- [14] Laporte, G., Louveaux, F. and Mercure, H. (1989), 'Models and exact solutions for a class of stochastic location-routing problems', *European Journal of Operational Research* 39(1), 71–78.
- [15] Laporte, G. and Nobert, Y. (1981), 'An exact algorithm for minimizing routing and operating costs in depot location', *European Journal of Operational Research* 6(2), 224–226.
- [16] Madsen, O. B. (1983), 'Methods for solving combined two level location-routing problems of realistic dimensions', European Journal of Operational Research 12(3), 295–301.
- [17] Mladenović, N. and Hansen, P. (1997), 'Variable neighborhood search', Computers & operations research 24(11), 1097–1100.
- [18] Nambiar, J. M., Gelders, L. F. and Van Wassenhove, L. N. (1981), 'A large scale location-allocation problem in the natural rubber industry', *European Journal of Operational Research* 6(2), 183–189.

- [19] Or, I. and Pierskalla, W. P. (1979), 'A transportation location-allocation model for regional blood banking', AIIE transactions 11(2), 86–95.
- [20] Prodhon, C. (2006), Le Problme de Localisation-Routage(Location-routing problem, PhD thesis, Universit de Technologie de Troyes, Troyes, France. An optional note.
- [21] Prodhon, C. and Prins, C. (2014), 'A survey of recent research on location-routing problems', European Journal of Operational Research 238(1), 1–17.
- [22] Salhi, S., Imran, A. and Wassan, N. A. (2014), 'The multi-depot vehicle routing problem with heterogeneous vehicle fleet: Formulation and a variable neighborhood search implementation', *Computers & Operations Research* 52, 315–325.
- [23] Salhi, S. and Rand, G. K. (1989), 'The effect of ignoring routes when locating depots', European Journal of Operational Research 39(2), 150 – 156
- [24] Schneider, M. and Drexl, M. (2017), 'A survey of the standard location-routing problem', *Annals of Operations Research* pp. 1–26.
- [25] Snyder, L. V. and Daskin, M. S. (2006), 'Stochastic p-robust location problems', *IIE Transactions* 38(11), 971–985.
- [26] Sungur, I., Ordónez, F. and Dessouky, M. (2008), 'A robust optimization approach for the capacitated vehicle routing problem with demand uncertainty', *IIE Transactions* 40(5), 509–523.
- [27] Wei, L., Zhang, Z., Zhang, D. and Lim, A. (2015), 'A variable neighborhood search for the capacitated vehicle routing problem with two-dimensional loading constraints', *European Journal of Operational Research* 243(3), 798–814.

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