

Use of Linear Programming for Optimal Production in a Production Line in Saudi Food Co.

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Abstract—Few Saudi Arabia production companies face financial profit issues until this moment. This work presents a linear integer programming model that solves a production problem of a Saudi Food Company in Saudi Arabia. An optimal solution to the above-mentioned problem is a Linear Programming solution. In this regard, the main purpose of this project is to maximize profit. Linear Programming Technique has been used to derive the maximum profit from production of natural juice at Saudi Food Co. The operations of production of the company were formulated and optimal results are found out by using Lindo Software that employed Sensitivity Analysis and Parametric linear programming in order develop Linear Programming. In addition, the parameter values are increased, then the values of the objective function will be increased.

Keywords—Parameter linear programming, objective function, sensitivity analysis, optimize profit.

I. INTRODUCTION

LINEAR programming is a technique used to find the best value by taking many linear inequalities relating to some situation. In real life salutation, linear programming (LP) is very important part in different areas of industry. A large number of companies are using LP to solve several kinds of practical problems. Organization supervisors frequently look with choices identifying with the utilization of restricted resource. These resources may incorporate people, materials and money. Applications include transportation problems, production planning, investment decision problems, blending problems, location and allocation problems, among many others. Sensitivity analysis is one of the most interesting areas in optimization. It is a technique used to determine how different values of an independent variable impact a dependent variable under a given set of assumptions. Sensitivity analysis (or post-optimality analysis) is used to determine how the optimal solution is affected by changes, within specified ranges, in the objective function coefficients or the right-hand side (RHS) values. The strategy is utilized inside particular limits that rely upon at least one information factors, for example, the impact that adjustments in loan fees have on security costs. Sensitivity analysis is used to quantify the effect of changes in the initial data of linear programs on the optimal value.

The plant selected in this project is Saudi Food Company. Saudi Food Company established in 1979 in Riyadh city, Saudi Arabia. It became one of the leading companies in the

industry of milk, juices, ice cream, chocolate, coffee and tea baradians in the Kingdom of Saudi Arabia, which is engaged in the production and marketing of juices and dried fruits, milk and dairy products [1].

In a few years the company has grown significantly and has been able to obtain a significant increase in its market share and rapidly gained widespread fame for its products. The company's annual production capacity is about 250 million liters of juices, beverages and milk. The company markets its products in Saudi Arabia, Cooperation Council Through more than 32,000 outlets in and outside the Kingdom [1]. The company uses the latest technology to produce high-quality products. Saudi owns a state-of-the-art factory, one of the largest factories in the Middle East. It operates the latest high technology in the world with the most modern production lines that are fully compatible with the plans to meet the needs and requirements of the market. The company also has large warehouses and sales offices throughout the Kingdom.

II. LITERATURE REVIEW

There are many researches done in food industry. It is necessary to review some of these works. These days, the most often used decision making tools in industrial is LP which help mangers and administration for various features to deal with production and other industry operations [2]. Balogun et al. worked on to maximize profit with the LP model, they focus on the application of LP Problem to drive the maximum profit from production of soft drink for Nigeria Bottling Company Nigeria, Ilorin plant 2012 [3]. The result they found out that Niger Mills Company Calabar should produce 576 (50 kg) bags of flour per day, and stop presently to produce semolina and wheat offal, because of their non-optimality, to enable the company maximize profit. Saurav and Ashish worked to use sensitivity analysis in order to complete the study effects of parametric variations on the production volume [4].

III. PROBLEM STATEMENT

The Company has produced some kind of natural juice with different flavors, such as pineapple, orange, fruit cocktail, mango, apple, grape, pomegranate, strawberries, guava, and banana as shown in Table I. In addition, the average cost and sealing by dollar for all products are shown in Table II. The available quantity of raw materials in inventory that company used to produce is shown in Table III. There are some insufficient resources available, so the issue depends on the most proficient method to settle on which resources would be assigned to get the best outcome. This outcome may identify

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with profit, cost or both. This company also produces four different brands of mix juice which are watermelon and rose, berry mix, lemon and mint, and lime and kiwi. The data will be formulated into interval objective LP problem. Parametric linear programming LP method is used to solve the problem. The transformation of interval objective LP to parametric includes the parametric LP problems [5]. Parametric LP has many applications in production schedules [6].

TABLE I
THE AMOUNT OF RAW MATERIALS FOR ALL PRODUCTS

Flavours	Sodium (kg)	Carbohydrate (Kg)	Dietary Fibers (Kg)	Sugar (Kg)	Water (Litters)
Pineapple	0.01	0.012	0.005	0.012	6.822
Orange	0.01	0.008	0.005	0.008	7.552
Fruit Cocktail	0.01	0.015	0.01	0.015	4.824
Mango	0.005	0.015	0.005	0.015	7.621
Apple	0.005	0.012	0.01	0.012	6.539
Grape	0.01	0.014	0.005	0.014	7.602
Pomegranate	0.005	0.010	0.01	0.013	6.12
Strawberries	0.01	0.008	0.005	0.010	7.055
Guava	0.005	0.012	0.01	0.012	7.508
Banana	0.01	0.012	0.005	0.008	7.051

TABLE II
AVERAGE COST AND SELLING OF A CRATE OF EACH PRODUCT

Products	Average Cost (SR)	Average Selling (SR)	Profit (SR)
Pineapple	1	2.19	1.19
Orange	1	2.51	1.51
Fruit Cocktail	1	2.51	1.51
Mango	1	2.19	1.19
Apple	1	2.51	1.51
Grape	1	2.41	1.41
Pomegranate	1	2.10	1.10
Strawberries	1	2.19	1.19
Guava	1	2.15	1.15
Banana	1	2.08	1.08

TABLE III
QUANTITY OF RAW MATERIALS AVAILABLE IN STOCK

Raw materials	Quantity available
Sodium (kg)	4,037
Carbohydrate (kg)	23,651
Dietary Fibres (kg)	4,651
Sugar (kg)	46,712
Water (Liters)	16,376,630

IV. RESEARCH OBJECTIVES

LP Technique is considered as one of methods that can derive the maximum profit from production of natural juice for the company, deciding how limited resources, raw materials of company, and shows which items should be produced. One of the company goals is to have efficient and organized production schedules by using a new algorithm which is parametric LP sensitivity analysis.

V. DATA COLLECTION

The data were collected from the company website and management, including the amount of raw materials needed to

produce a bottle of each products. Also, the available quantity of raw materials holds in stock, and the average cost and sealing in Saudi riyal (SR) for all products. Some assumptions have been made for unavailable data.

VI. FORMULATION OF PROBLEM

A. Define the Decision Variables

X_1 = Pineapple
 X_2 = Orange
 X_3 = Fruit Cocktail
 X_4 = Mango
 X_5 = Apple
 X_6 = Grape
 X_7 = Pomegranate
 X_8 = Strawberry
 X_9 = Guava
 X_{10} = Banana

The definition of the decision variable implies:

B. Formulate Objective Function (OF)

The basic formula for profit is revenue minus cost. Thus, Profit = Revenue – Cost, and Profit = $Z_{\max}$. Therefore, both revenue and cost must be defined in relation to this problem. After defining revenue and cost, profit ($Z_{\max}$) can be determined using the formula above. To maximize Z:

$$Z = 1.19 X_1 + 1.51 X_2 + 1.51 X_3 + 1.19 X_4 + 1.51 X_5 + 1.41 X_6 + 1.10 X_7 + 1.19 X_8 + 1.15 X_9 + 1.08 X_{10}$$

C. Formulate Constraints

Constraint 1: The amount of sodium used to produce products in Kg. The amount of the raw material should be less than the 4,037 Kg that is available for production.

$$0.01 X_1 + 0.01 X_2 + 0.01 X_3 + 0.005 X_4 + 0.005 X_5 + 0.01 X_6 + 0.005 X_7 + 0.01 X_8 + 0.005 X_9 + 0.01 X_{10} \leq 4037$$

Constraint 2: The amount of carbohydrate used to produce products in Kg. The amount of the raw material should be less than the 23,651 Kg that is available for production.

$$0.012 X_1 + 0.008 X_2 + 0.015 X_3 + 0.015 X_4 + 0.012 X_5 + 0.014 X_6 + 0.010 X_7 + 0.08 X_8 + 0.012 X_9 + 0.012 X_{10} \leq 23651$$

Constraint 3: The amount of Diet Fibers used to produce products in Kg. The amount of the raw material the raw material should be less than the 4,651 Kg that is available for production.

$$0.005 X_1 + 0.005 X_2 + 0.01 X_3 + 0.005 X_4 + 0.01 X_5 + 0.005 X_6 + 0.01 X_7 + 0.005 X_8 + 0.01 X_9 + 0.005 X_{10} \leq 4651$$

Constraint 4: The amount of sugar used to produce products is in units of Kg. The amount of sugar should be less than the 46,012 Kg that is available for production.

$$0.012 X_1 + 0.008 X_2 + 0.015 X_3 + 0.015 X_4 + 0.012 X_5 + 0.014 X_6 + 0.013 X_7 + 0.010 X_8 + 0.012 X_9 + 0.008 X_{10} \leq 46012$$

Constraint 5: The amount of water used to produce from

products is in units of Liters. The amount of water should be less than the 16,376,630 Liters that is available for production.

$$6.822 X_1 + 7.552 X_2 + 4.824 X_3 + 7.671 X_4 + 6.539 X_5 + 7.602 X_6 + 6.12 X_7 + 7.055 X_8 + 7.508 X_9 + 7.051 X_{10} \leq 16376630$$

D. Solution Using LONDO, Results Analysis and Discussion

LINDO software was used to solve the problems.

$$\begin{aligned} \max \quad & 1.19 X_1 + 1.51 X_2 + 1.51 X_3 + 1.19 X_4 + 1.51 X_5 + 1.41 X_6 + \\ & 1.10 X_7 + 1.19 X_8 + 1.51 X_9 + 1.08 X_{10} \\ \text{S.T.} \end{aligned}$$

$$0.01 X_1 + 0.01 X_2 + 0.01 X_3 + 0.005 X_4 + 0.005 X_5 + 0.01 X_6 + 0.005 X_7 + 0.01 X_8 + 0.005 X_9 + 0.01 X_{10} \leq 4037$$

$$0.012 X_1 + 0.008 X_2 + 0.015 X_3 + 0.015 X_4 + 0.012 X_5 + 0.014 X_6 + 0.010 X_7 + 0.08 X_8 + 0.012 X_9 + 0.012 X_{10} \leq 23651$$

$$0.005 X_1 + 0.005 X_2 + 0.01 X_3 + 0.005 X_4 + 0.01 X_5 + 0.005 X_6 + 0.01 X_7 + 0.005 X_8 + 0.01 X_9 + 0.005 X_{10} \leq 4651$$

$$0.012 X_1 + 0.008 X_2 + 0.015 X_3 + 0.015 X_4 + 0.012 X_5 + 0.014 X_6 + 0.013 X_7 + 0.010 X_8 + 0.012 X_9 + 0.008 X_{10} \leq 46012$$

$$6.822 X_1 + 7.552 X_2 + 4.824 X_3 + 7.671 X_4 + 6.539 X_5 + 7.602 X_6 + 6.12 X_7 + 7.055 X_8 + 7.508 X_9 + 7.051 X_{10} \leq 16376630$$

LP OPTIMUM FOUND AT STEP 3

OBJECTIVE FUNCTION VALUE

1) 1000102.

VARIABLE	VALUE	REDUCED COST
X1	0.000000	2.048100
X2	0.000000	0.550000
X3	0.000000	0.870000
X4	684600.000000	0.000000
X5	122800.000000	0.000000
X6	0.000000	0.650000
X7	0.000000	0.410000
X8	0.000000	0.870000
X9	0.000000	0.000000
X10	0.000000	0.980000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	174.000000
3)	11908.400391	0.000000
4)	0.000000	64.000000
5)	34269.398438	0.000000
6)	10322074.000000	0.000000

NO. ITERATIONS= 3

Fig. 1 LINDO output for objective function and decision variable values

RANGES IN WHICH THE BASIS IS UNCHANGED:

VARIABLE	CURRENT COEF	OBJ COEFFICIENT RANGES	
		ALLOWABLE INCREASE	ALLOWABLE DECREASE
X1	0.011900	2.048100	INFINITY
X2	1.510000	0.550000	INFINITY
X3	1.510000	0.870000	INFINITY
X4	1.190000	0.320000	0.183333
X5	1.510000	0.550000	0.000000
X6	1.410000	0.650000	INFINITY
X7	1.100000	0.410000	INFINITY
X8	1.190000	0.870000	INFINITY
X9	1.510000	0.000000	INFINITY
X10	1.080000	0.980000	INFINITY

ROW	CURRENT RHS	RIGHTHAND SIDE RANGES	
		ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	4037.000000	614.000000	1711.500000
3	23651.000000	INFINITY	11908.400391
4	4651.000000	3423.000000	614.000000
5	46012.000000	INFINITY	34269.398438
6	16376630.000000	INFINITY	10322074.000000

Fig. 2 LINDO output for objective function ranges

After using the parametric LP method, the problem was formulated as shown in Figs. 1 and 2. These figures show that two products that should be produced are mango and apple juice. The company should produce 684,600 crates of mango juice and 122,800 crates of apple juice to produce a maximum profit of SR 1,000,102.

VII. SENSITIVITY ANALYSIS

Fig. 2 shows the output of sensitivity analysis. It provides the following information:

- ❖ Changing the objective function coefficient
- ❖ Determining which variables are basic and which ones are non-basic
- ❖ Changing the RHS of the constraints [7], [8].

The objective function range portion of the LINDO output gives the range of values for an objective function coefficient for which the current basis remains optimal. Currently, $X_4 = 684,600$, $X_5 = 122,800$, and $X_1 = X_2 = X_3 = X_6 = X_7 = X_8 = 0$. The current solution value for X_5 is 807,400 and the current objective function coefficient for X_4 is 1.19. The allowable increase that is shown in Fig. 3 illustrates that the coefficient of X_4 in the objective function lies between 1.006667 and 1.51. The values of the decision variables remain unchanged. Producing 1,000,102 crates of mango juice remains optimal even if the profit on mango juice is not actually 1.19 but lies between 1.006667 and 1.51.

The same considerations can be drawn for $X_2, X_3, \dots, X_{10}$. The variables that are currently zero must be improved before they become non-zero. For example, the non-basic variable X_2 has a reduced cost of SR 0.55. This implies that if the company increases X_2 's objective function coefficient by exactly SR 0.55, then there will be alternative optimal solutions, at least one of which will have X_2 as a basic variable. If the company increases X_2 's objective function coefficient by more than SR 0.55, then any optimal solution to the LP will have X_2 as a basic variable. The company must keep a close watch on X_2 's objective function coefficient, because a slight increase will change the LP's optimal solution. The same considerations can be drawn for $X_1, \dots$ and X_{10} . This company also produces four different brands of mix juice, which are watermelon and rose, berry mix, lemon and mint, and lime and kiwi.

The data will be formulated into interval objective LP problem. The transformation of interval objective LP to parametric includes the parametric LP problems. Parametric LP has many applications in production schedules. This company has some constraints to produce this kind of mixed juice and wants to have efficient and organized production schedules. The method that this company should employ is the parametric LP algorithm for the customer demands [9]. Al Rabie Food Co. requires the production of four bottles of watermelon and rose and one bottle of berry mix will be at most ten bottles. Also, the production of one bottle of watermelon and rose and five bottles of berry mix will result in at most nine bottles of the natural juice. The production of five bottles of watermelon and rose, four bottles of berry mix,

and three bottles of lemon and mint will result in at least twelve bottles of the natural juice. Also, the production of one bottle of lemon and mint minus five bottles of lime and kiwi will result in at most four bottles in total. Finally, the production of one bottle of lemon and mint and one bottle of lime and kiwi will result in at most eight bottles of the natural juice.

The estimated profit per bottle of watermelon and rose is between SR 0.1 and SR 0.13, berry mix is between SR 0.16 and SR 0.19, lemon and mint is between SR 0.14 and SR 0.16, and lime and kiwi is between SR 0.25 and SR 0.27. Determining the production schedule for Al Rabie Food Co. is shown in Table IV. Let X_1 be the number of bottles of watermelon and rose produced, X_2 be the number of bottles of berry mix produced, X_3 be the number bottles of lemon and mint produced, X_4 be the number of bottles of lime and kiwi produced.

A. Mixed Juice

$$\text{Max } Z = (0.1+0.03p)X_1 + (0.16+0.03p)X_2 + (0.14+0.02p)X_3 + (0.25+0.02p)X_4$$

Subject to;

$$\begin{aligned} 4X_1 + X_2 &\leq 10, \\ X_1 + 5X_2 &\leq 9 \\ 5X_1 + 4X_2 + 3X_3 &\geq 12 \\ X_3 - 5X_4 &\leq 4 \\ X_3 + X_4 &\leq 8 \\ X_i &\geq 0 \end{aligned}$$

LINDO software was used to solve problems when interval parametric value (p) was 0.

LP OPTIMUM FOUND AT STEP		0	
OBJECTIVE FUNCTION VALUE			
1)	2.434737		
VARIABLE	VALUE	REDUCED COST	
X1	2.157895	0.000000	
X2	1.368421	0.000000	
X3	0.000000	0.110000	
X4	8.000000	0.000000	
ROW	SLACK OR SURPLUS	DUAL PRICES	
2)	0.000000	0.017895	
3)	0.000000	0.028421	
4)	4.263158	0.000000	
5)	44.000000	0.000000	
6)	0.000000	0.250000	
NO. ITERATIONS= 0			
RANGES IN WHICH THE BASIS IS UNCHANGED:			
VARIABLE	CURRENT COEF	OBJ COEFFICIENT RANGES	ALLOWABLE DECREASE
		ALLOWABLE INCREASE	
X1	0.100000	0.540000	0.068000
X2	0.160000	0.340000	0.135000
X3	0.140000	0.110000	INFINITY
X4	0.250000	INFINITY	0.110000
ROW	CURRENT RHS	RIGHTHAND SIDE RANGES	ALLOWABLE DECREASE
		ALLOWABLE INCREASE	
2	10.000000	26.000000	3.857143
3	9.000000	41.000000	6.500000
4	12.000000	4.263158	INFINITY
5	4.000000	INFINITY	44.000000
6	8.000000	INFINITY	8.000000

Fig. 3 Output of the company when interval is 0

Fig. 3 shows the output of the company when interval is 0, the company should produce three bottles of watermelon and rose, two bottles of berry mix, and eight bottles of lime and

kiwi to get an estimated profit of SR 2.434737 and should stop producing lemon and mint. It is also presented that the coefficient of X_1 in the objective function lies between 0.032 and 0.64. The values of the decision variables remain unchanged. Producing of one bottle of watermelon and rose remains optimal even if the profit on watermelon and rose is not actually 0.1, but lies between 0.032 and 0.64. The same considerations can be drawn for X_2 , X_3 , and X_4 .

The variables which are currently zero must be improved before they become non-zero. For example, the non-basic variable X_3 has a reduced cost of SR 0.11. This implies that if company increases X_3 's objective function coefficient by exactly SR 0.11, then there will be alternative optimal solutions, at least one of which will have X_3 as a basic variable. If company increases X_3 's objective function coefficient by more than SR 0.11, then any optimal solution to the LP will have X_3 as a basic variable. Company must keep a close watch on X_3 's objective function coefficient, because a slight increase will change the LP's optimal solution as shown in Table IV.

LINDO software has been used to solve the parametric LP problem from 0.1 to 1.0 and got the results are shown in Table IV.

TABLE IV THE PARAMETRIC LP PROBLEM FROM 0.1 TO 1					
Parameter (p)	X_1	X_2	X_3	X_4	Max Z SR
0.1	3	2	0	8	2.287
0.2	3	2	0	8	2.311
0.3	3	2	0	8	2.335
0.4	3	2	0	8	2.359
0.5	3	2	0	8	2.379
0.6	3	2	1	8	2.407
0.7	3	2	1	8	2.411
0.8	3	2	1	8	2.432
0.9	3	2	1	8	2.453
1.0	3	2	1	8	2.475

VIII. SUMMARY

It is clear that when the parameter values are increased, the values of the objective function are increased. For example, the company should produce three bottles of watermelon and rose, two bottles of berry mix, and eight bottles of lime and kiwi to get the estimated profit. When the parameter is increased to 1, the company should produce two bottles of watermelon and rose, two bottles of berry mix, one bottle of lemon and mint, and eight bottles of lime and kiwi. In addition, the estimated profit will be increased to SR 2.475. The result shown in Table IV presents the value of objective function (Z) for each interval. It is shown that if the parameter values are increased, then the values of the objective function will be increased. When this problem was solved, it was noticed that this method of LP which is parametric LP would be a good option for any company that wants to have the perfect production schedule.

IX. CONCLUSION

Based on the analysis the data collected from Saudi Food Co., the company should produce more mango juice and apple juice to achieve customer satisfaction and meet customer requirements. In addition, producing more mango juice and apple juice will maximize the company's profit. Moreover, any factory that would like to maintain an organized production schedule should apply the parametric LP algorithm that was used in this paper. The problem was solved using LINDO software for the parametric LP method; however, another developed computer program should be used to identify the production schedule of a company faster and more accurately.

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