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Performance of the Strong Stability Method in the Univariate Classical Risk Model

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Abstract—In this paper, we study the performance of the strong stability method of the univariate classical risk model. We interest to the stability bounds established using two approaches. The first based on the strong stability method developed for a general Markov chains. The second approach based on the regenerative processes theory . By adopting an algorithmic procedure, we study the performance of the stability method in the case of exponential distribution claim amounts. After presenting numerically and graphically the stability bounds, an interpretation and comparison of the results have been done.

Keywords—Markov Chain, regenerative processes, risk models, ruin probability, strong stability.

I. Introduction

THE stochastic processes are used in ruin theory to model the insurance company surplus and to evaluate its ruin probability [4], that is to say, the probability that the total claim amounts exceeds its reserve. This characteristic is a much studied risk measure in the literature. In general, this measure is very difficult or even impossible to evaluate explicitly. Thus, different approximation methods have been proposed to estimate this characteristic (see Asmussen and Albrecher, 2010 [3]; Grandell, 1990 [7]).

The classical risk model in the one-dimensional situation can be stated as

$$R(t) = u + c\,t - S(t), \qquad t \geq 0 \tag{1}$$

where $R(t)$ is the surplus of an insurance company at time $t \geq 0$, and the process $\{S(t), t \geq 0\}$ is called the aggregate claims process and is given by

$$S(t) = \sum_{i=1}^{N(t)} X_i,$$

$\{X_i, i \geq 1\}$ is a sequence of independent and identically distributed random variables, representing the claim amounts of distribution function denoted by $F(x)$ and mean claim size denoted by m, $u \geq 0$ the initial surplus, c the rate at which the premiums are received, and $\{N(t), t \geq 0\}$ being a Poisson process with parameter λ, representing the number of claims.

The relative security loading η is defined by

$$\eta = \frac{c - \lambda m}{\lambda m}.$$

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We further assume that $c > \lambda m$ the expected payment per unit of time.

Ruin theory for the univariate risk model characterized by the surplus process $R(t)$ has been discussed extensively in the literature. Many results are summarized in the books authored by Asmussen (2000) [2], Rolski and al. (1999) [11], Dickson (2005) [5], Willmot and Lin (2001) [12] or Asmussen and Albrecher (2010) [3].

The risk process can be related to a dual process $\{V_n\}$ like in queuing and storage. In risk theory this process is known as a the reversed process since it is constructing by a certain time-reversing procedure (for such constructions see [6]).

The reversed process satisfies the relation

$$\Psi_n(u) = \mathrm{P}(V_n > u)$$

and

$$\Psi(u) = \lim_{n\to\infty} \mathrm{P}(V_n > u).$$

where u is the initial reserve.

II. Stability Bounds of the Univariate Classical Risk Model

The application of the strong stability in risk theorie has been widely discussed in Kalashnikov [9], in this article the author determines a stability bounds in the univariate classical risk models using the strong stability [1], [10], where it used the analysis of the stability of limit distributions of general Markov chains, and another bound based on the regenerative processes theory [8].

Let us collect all the parameters of a risk model to a vector valued governing parameter a. The stability approach consists of identifying the ruin probability $\Psi(u)$ associated to the risk model governed by a vector parameter $a = (c, \lambda, m)$, with the stationary distribution of the reversed process $\{V_n\}_{n\geq 0}$ [9].

Let $a' = (c', \lambda', m')$ be the vector parameter governing another univariate risk model defined as above, its ruin probability and its reversed process being respectively $\Psi'(u)$ and $\{V'_n\}_{n\geq 0}$.

In what follows, we are interested to the stability bounds of the univariate classical risk model established by Kalashnikov (2000) [9] using two approaches. The first based on the strong stability method developed for a general Markov chains [1], [10] summarized in Theorem 1. The second approach based on the regenerative processes theory [8] summarized in Theorem 2.

Theorem 1 (Kalashnikov 2000 [9]):
Consider a univariate classical risk model governed by a vector parameter a. Then, there exists $\epsilon > 0$ such that the reversed

process $\{V_n\}_{n\geq 0}$ (Markov chain) associated to this model is strongly stable with respect to the weight function $v(u) = e^{\epsilon u}$ ($\epsilon > 0$), $u \in \mathbf{R}^+$. In addition, if

$$\mu(a, a') < (1 - \rho(\epsilon))^2, \tag{2}$$

then we obtain the margin between the transition operators $\mathbf{P}$ and $\mathbf{P}'$ of the Markov chains $\{V_n\}_{n\geq 0}$ and $\{V'_n\}_{n\geq 0}$:

$$\|P - P'\|_v \leq 2\ \mathbf{E}\ e^{\epsilon Z}|\ln \frac{\lambda c'}{\lambda' c}| + \|F - F'\|_v, \tag{3}$$

where,

$$\mu(a, a') = 2\ \mathbf{E}\ e^{\epsilon Z}|\ln \frac{\lambda c'}{\lambda' c}| + \|F - F'\|_v,$$

$$\rho(\epsilon) = \mathrm{E}(\ exp\ \{\epsilon(Z - c\theta)\})$$

$$\|F - F'\|_v = \int_0^\infty v(u)|d(F - F')|(u) = \int_0^\infty v(u)|f - f')|(u)du.$$

Moreover, we have the deviation between the ruin probabilities:

$$\|\Psi - \Psi'\|_v \leq \frac{\mu(a, a')}{(1 - \rho(\epsilon))\ \ ((1 - \rho(\epsilon))^2 - \mu(a, a'))} \tag{4}$$

Theorem 2 (Kalashnikov 2000 [9]):
Let $\Psi_n(u)$ and $\Psi'_n(u)$ ruin probabilities associated to the reversed processes $\{V_n\}_{n\geq 0}$ and $\{V'_n\}_{n\geq 0}$ respectively. For $v(u) = e^{\epsilon u}$, $\xi = Z - c\theta$, $u \geq 0$ and $\epsilon > 0$,

Then, under assumption:

$$\mathrm{E}(\ exp\ \{\epsilon(Z - c\theta)\}) \leq \rho(\epsilon) < 1 \tag{5}$$

$$\mathrm{E}\ (\ exp\ \{\epsilon Z\}) < \infty. \tag{6}$$

we have the deviation between the ruin probabilities:

$$\sup_{n\geq 0} |\Psi_n - \Psi'_n|_v \leq \frac{\gamma(\epsilon)\ \ \mu}{1 - \rho(\epsilon)} \tag{7}$$

where,

$$\gamma(\epsilon) = \sup_n\ \mathrm{E}\ (\ e^{\epsilon V_n}) < \infty,$$

$$\mu = \sup_{-\infty < x < \infty} e^{\epsilon x}|F_\xi - F_{\xi'}|(x)$$

and

$$\rho(\epsilon) = \mathrm{E}(\ exp\ \{\epsilon(Z - c\theta)\})$$

III. Simulation Based Study

In the following sections, we are interested to the stability bounds defined in Theorems 1 and 2 by the relations (4) and (7). We want to study the performance of the strong stability method in the univariate classical risk model in the case of exponential distribution claim amounts. To do this, we have developed the following algorithms:

A. Algorithms Construction

***Algorithm 1*: The strong stability approach**

1) Introduction the parameters (c, λ, m) of the ideal model, and (c', λ', m') of the perturbed model.
2) Verify the positivity of the relative Security loadings η and η'.
   If Yes, go to step 3
   else return to step 1
3) Generate a value of ϵ betwen 0 et $min\{a, b\}$ such that $0 < \rho(\epsilon) < 1$ and Γ_{MC} be minimal,
   with $a = \frac{1}{m}$, $b = \frac{c - \lambda m}{cm}$,

   and test $\mu(a, a') < (1 - \rho)^2$
   If yes (we can deduce the strong stability inequality) go to step 4;
   else return to step 3.
4) Compute the bound Γ_{MC} and the deviation D_{MC} such that:

$$D_{MC} = \|\Psi - \Psi'\|_v \leq \frac{\mu(a, a')}{(1 - \rho(\epsilon))\ \ ((1 - \rho(\epsilon))^2 - \mu(a, a'))} = \Gamma_{MC}.$$

***Algorithm 2*: The regenerative process approach**

1) Introduction the parameters (c, λ, m) of the ideal model, and (c', λ', m') of the perturbed model.
2) Verify the positivity of the relative Security loadings η and η'.
   If Yes, go to step 3
   else return to step 1
3) Generate a value of ϵ betwen 0 et $min\{a, b\}$ such that $0 < \rho(\epsilon) < 1$ and Γ_{PR} be minimal,
   with $a = \frac{1}{m}$, $b = \frac{c - \lambda m}{cm}$
4) Compute the bound Γ_{PR} on the deviation D_{RP} such that:

$$D_{RP} = \sup_{n\geq 0} |\Psi_n - \Psi'_n|_v \leq \frac{\gamma(\epsilon)\ \ \mu}{1 - \rho} = \Gamma_{PR}.$$

The construction of the steps of this algorithms is based on verification of the positivity condition of the relative security loadings, order to avoid certain ruin. Its implementation will allow us to calculate the stability bounds Γ_{MC} and Γ_{PR} given by the relations (4) and (7) respectively.

B. Numerical and Graphical Results

This section is devoted to present the different numerical and graphical results obtained when studying the performance of the strong stability method in the univariate classical risk model, by considering the exponential distribution claim amounts.

1) The Strong Stability Approach:

2) The Regenerative Process Approach:

3) Comparison of the Two Approaches:

C. Discussion of Results

1) Variation of the Norm Parameter ϵ : Note, according to Fig. 1 (a) (respectively Fig. 3 (a)) that the diviation D_{MC}

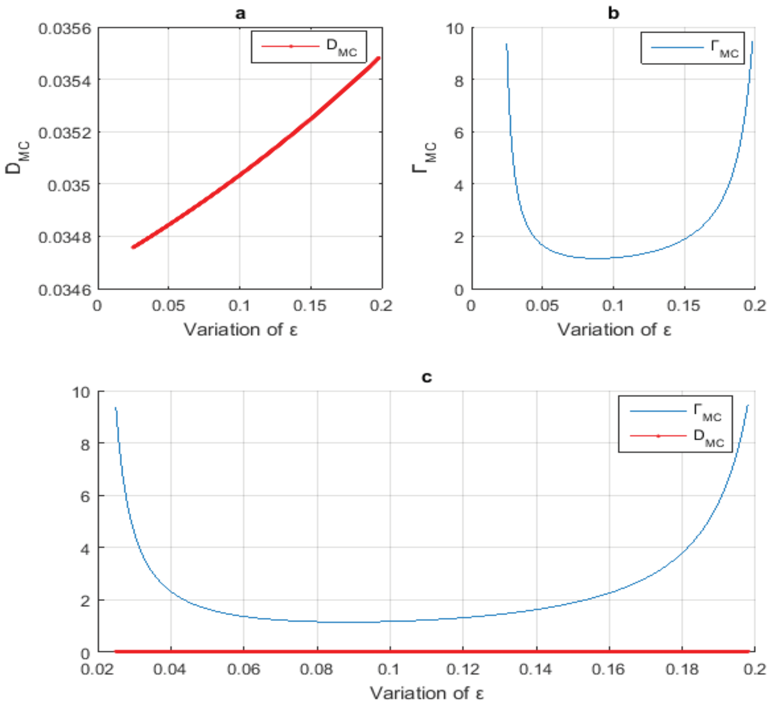

Fig. 1 Variation of the stability bounds Γ_{MC} in function of the norm parameter ϵ

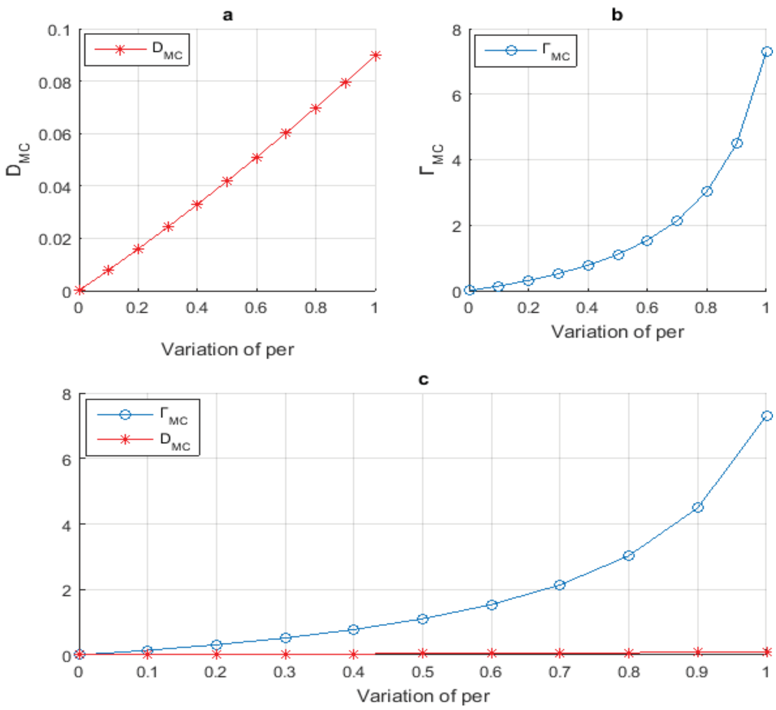

Fig. 2 Variation of the stability bounds Γ_{MC} with respect to the perturbation of the average claim amounts m

(respectively the diviation D_{RP}), increases in function of the ϵ.

We notice also that the stabilitie bounds Γ_{MC} and Γ_{RP} increases speedily in the neighborhood of the boundary of the epsilon variation domain (see Figs. 1 (b) and 3. (b)), that is to say where ϵ is close to its bounds, Γ_{MC} and Γ_{RP} take larger values and move away from the deviation of the ruin probabilitie D_{MC} and D_{RP} respectively.

From the results given by the Fig. 5, we note that, the epsilon variation domain is wider in The regenerative process

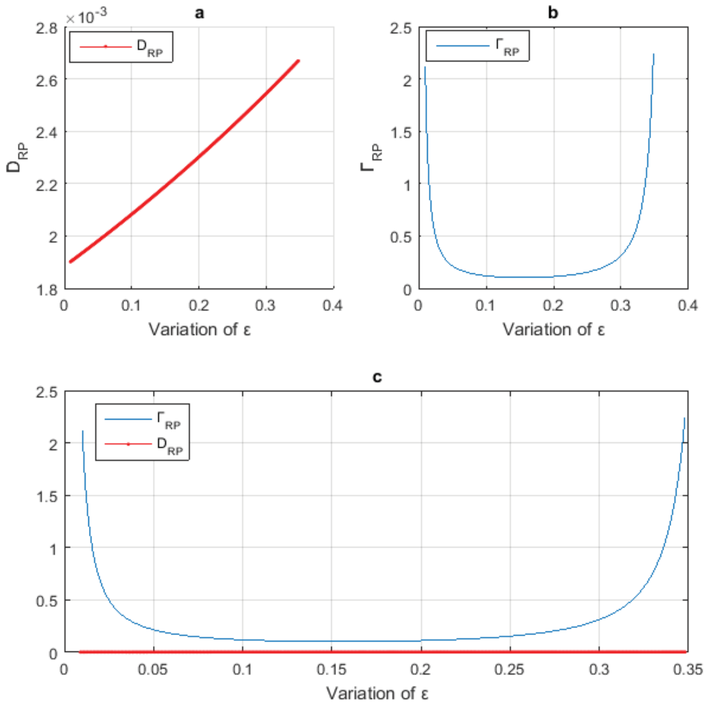

Fig. 3 Variation of the stability bounds Γ_{RP} in function of the norm parameter ϵ

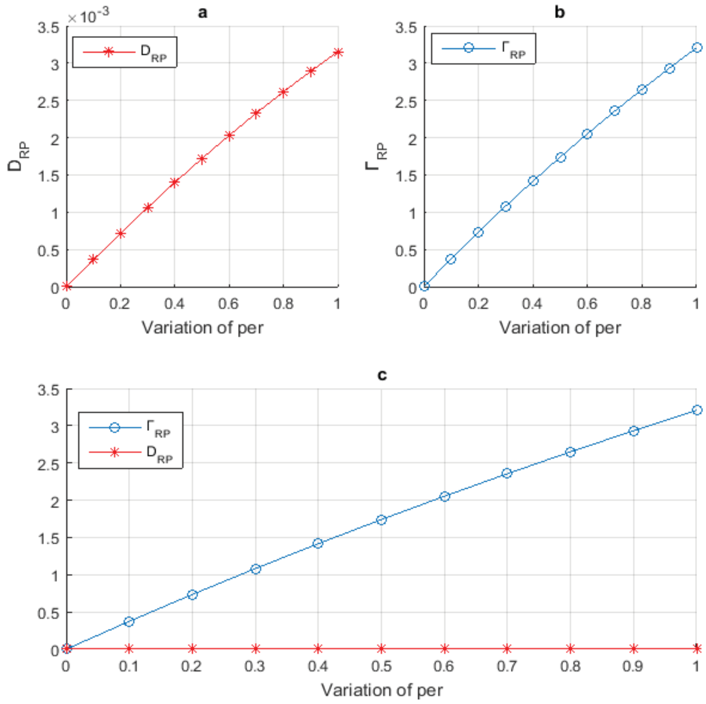

Fig. 4 Variation of the stability bounds Γ_{PR} with respect to the perturbation of the average claim amounts m

approach and too small for that obtained The strong stability approach, this is due to the sensitivity of the stability bound Γ_{MC} with respect to the variation of epsilon and to the condition given by the relation 2 that delimits the variation domain.

2) Perturbation of the Average Claim Amounts m : We note that the error due to the approximation is proportional to the perturbation of the average the claim amounts. In other words, by perturbing the average claim amounts m, the deviations D_{MC} (respectively the diviation D_{RP}) of the probability of

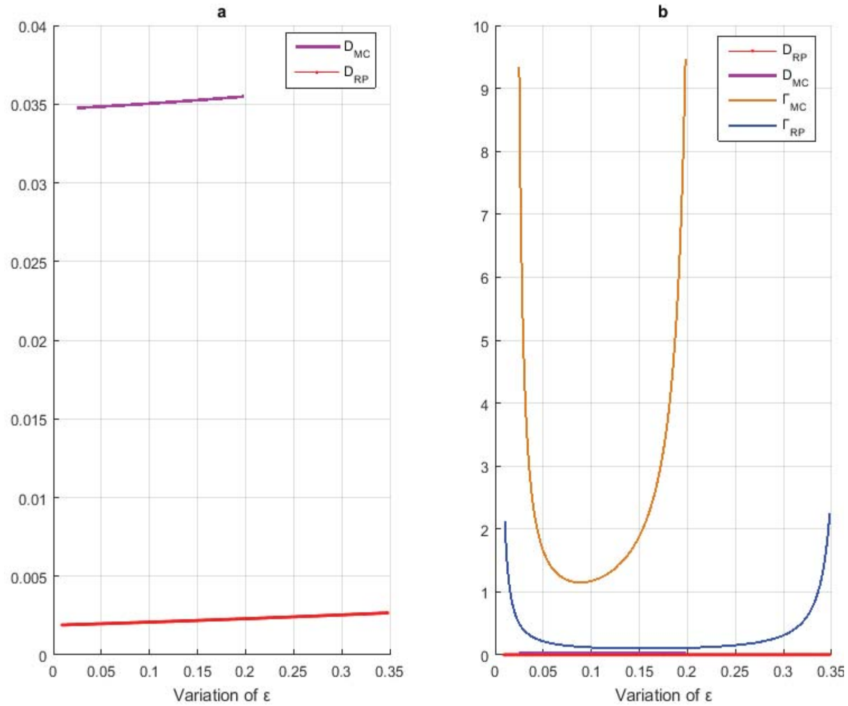

Fig. 5 Variation of the stability bounds Γ_{MC} and Γ_{PR} in function of the norm parameter ϵ

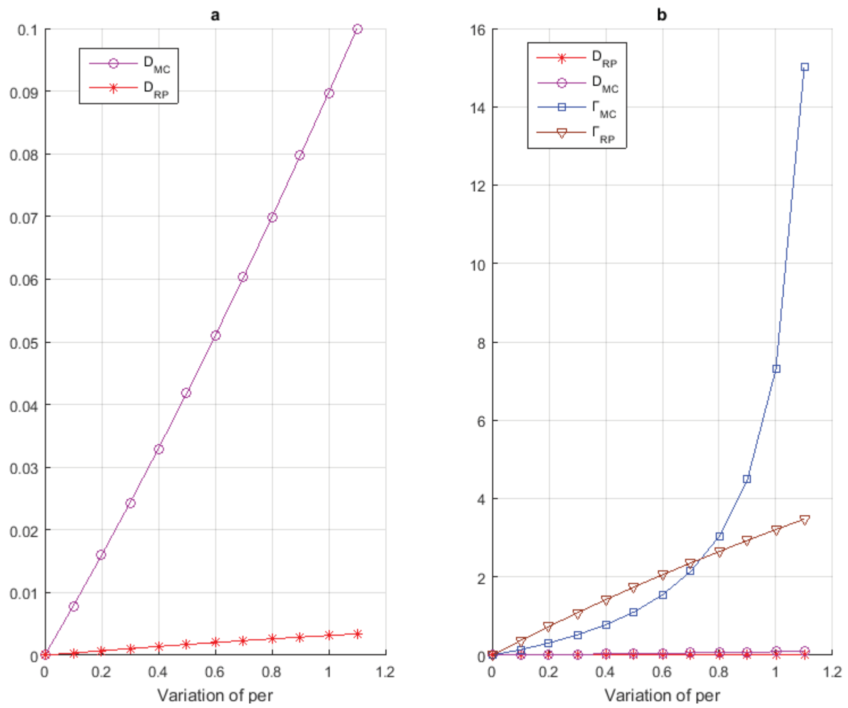

Fig. 6 Variation of the stability bounds Γ_{MC} and Γ_{PR} with respect to the perturbation of the average claim amounts m

ruin and the stabilitie bounds Γ_{MC} (respectively the diviation Γ_{RP}) increases at the same time (see Table I and respectively Figs. 2 and 4). However, we note in this case that the quality of the stabilitie inequality decreases.

We notice also that the two bounds grow gradually as $per = |m - m'|$ increases. Having said, Γ_{MC} grows faster than Γ_{RP} (see Table I and Fig. 6).

The stability bound Γ_{MC} is much more sensitive to perturbations of the average claim amounts than the stability bound Γ_{RP} (see Table I and Fig. 6).

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TABLE I
DISRUPTION OF THE AVERAGE CLAIM AMOUNTS m

per	m	m'	D_{MC}	Γ_{MC}	D_{RP}	Γ_{RP}
0	2.5000	2.5000	0	0	0	0
0.1000	2.5000	2.6000	0.0078	0.1422	0.0004	0.3743
0.2000	2.5000	2.7000	0.0160	0.3121	0.0007	0.7346
0.3000	2.5000	2.8000	0.0243	0.5188	0.0011	1.0820
0.4000	2.5000	2.9000	0.0330	0.7758	0.0014	1.4169
0.5000	2.5000	3.0000	0.0418	1.1041	0.0017	1.7406
0.6000	2.5000	3.1000	0.0510	1.5384	0.0020	2.0531
0.7000	2.5000	3.2000	0.0603	2.1406	0.0023	2.3559
0.8000	2.5000	3.3000	0.0699	3.0319	0.0026	2.6486
0.9000	2.5000	3.4000	0.0797	4.4882	0.0029	2.9326
1.0000	2.5000	3.5000	0.0897	7.2997	0.0032	3.2077

IV. CONCLUSION

By adopting an algorithmic procedure, we have implemented the approach based on the strong stability and the approach based on the regenerative processes theory. Thereafter, we have determined numerically the stability bounds of the ruin probability of the univariate classical risk model in the case of exponential distribution claim amounts. We have interpreted and compared subsequently the results obtained by the two approaches

From the results obtained, We noticed that the bounds obtained by the approach based on the theory of regenerative processes are much more affine because they are closer to the deviation of the ruin probability and this is due to their lack of sensitivity to disturbances of various parameters.

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