Reliability and Cost Focused Optimization Approach for a Communication Satellite Payload Redundancy Allocation Problem

Mehmet Nefes, Selman Demirel, Hasan H. Ertok, Cenk Sen

***Abstract*—**A typical reliability engineering problem regarding communication satellites has been considered to determine redundancy allocation scheme of power amplifiers within payload transponder module, whose dominant function is to amplify power levels of the received signals from the Earth, through maximizing reliability against mass, power, and other technical limitations. Adding each redundant power amplifier component increases not only reliability but also hardware, testing, and launch cost of a satellite. This study investigates a multi-objective approach used in order to solve Redundancy Allocation Problem (RAP) for a communication satellite payload transponder, focusing on design cost due to redundancy and reliability factors. The main purpose is to find the optimum power amplifier redundancy configuration satisfying reliability and capacity thresholds simultaneously instead of analyzing respectively or independently. A mathematical model and calculation approach are instituted including objective function definitions, and then, the problem is solved analytically with different input parameters in MATLAB environment. Example results showed that payload capacity and failure rate of power amplifiers have remarkable effects on the solution and also processing time.

***Keywords*—**Communication satellite payload, multi-objective optimization, redundancy allocation problem, reliability, transponder.

I. Introduction

COMMUNICATION satellites are essential platforms to provide telecommunication through long distances. The main mission of communication satellites payload systems is to receive and filter the uplink signals from earth stations, apply frequency translation, signal amplification and finally retransmit those signals on the downlink. This function is fulfilled with the operation of transponder and antenna subsystems of the satellite payload which can be classified as mission critical. A transponder can generally be defined as a series of components and units linked each other in order to transfer the received communication signals by satellite [1].

High power amplifiers are core equipment of payload transponders, and their probability of failure is comparably higher than other components due to their physics and nature. Transponders include series and parallel structured subsystems and power amplification stage is the most dominant for total reliability since its high failure rate. There are many technologies for power amplification equipment; however, in the satellite industry reliable and proven technologies are commonly used such as Traveling Wave Tube Amplifier (TWTA), which are used in transponders to provide final output power required to the transmit antenna. Fig. 1 shows an example of power amplification device called Traveling Wave Tube (TWT) which is part of TWTA and consisting of an electron-beam gun.

Fig. 1 Traveling Wave Tube Amplifier Device- Ku Band TWT [2] (courtesy of L-3 Comm. Electron Technologies Inc., CA)

This paper is focused on the power amplifiers redundancy scheme within a communication satellite payload module composed of transponders. The study outlined in this paper aims to solve and analyze a multi-objective optimization problem analytically, whereby optimization of power amplification stage redundancy configuration and reliability of a communication satellite payload is needed due to some limitation such as cost and weight.

The rest of the paper is organized as follows. In section II, background information and general literature review is given with regard to the theory of reliability, RAP and Multi-Objective Optimization (MOO) which have been applied throughout the study. Problem definition and mathematical model to solution are described in Section III. Application processes and simulation results are presented in Section IV. Finally, conclusion is given along with discussions.

II. Background Information

The purpose of this section is to provide theoretical background before defining and solving the studied problem.

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A. Failure Rate, Reliability and Redundancy

The failure rate can be described basically as the frequency of failures under defined conditions and is function of the total number of failures over a particular time period [3]. The Failures-In-Time (FIT) value of a component or unit is the number of failures which occur, as expectation, in one billion (10^9) device-hours of operation.

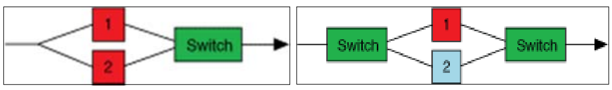
Space equipment FIT values, are provided by equipment manufacturers, which are used to find their reliability figures. Table I lists example FIT values of some equipment used in satellite industry. As shown in Table I, high power amplifiers have higher FIT values.

TABLE I
EXAMPLE FIT VALUES OF SATELLITE PAYLOAD EQUIPMENT

Equipment	FIT Value (10^{-9} failures/hour)
Antenna	1-10
Receiver	250-300
Input Multiplexer	200-250
Channel Amplifier	30-50
High Power Amplifier (exp:TWT)	500-700
Output Multiplexer	50-70
Filter	1-10
Switch	0.5-5
Microwave equipment, Load	0.1-3

Equipment used in system reliability design can be classified as a single point failure (SPF) source if there is no additional and identical one as redundant exists in the same system/subsystem. Alternative to this non-redundancy configuration, some equipment groups structuring active or passive redundancy units can be installed in parallel or series combination inside the satellite, which provides higher reliability figures. During the design process, necessary redundancy schemes supporting with complex reliability calculations should be implemented in order to satisfy system/subsystem reliability requirements.

As in Fig. 2, all identical units work simultaneously within active redundancy (also known as hot redundancy), whereas in passive redundancy (also known as cold redundancy) only nominal units operate, while remaining units (spares) stay non-operating [4]. Switching mechanisms also need to be considered in order to route nominal and redundant unit paths.



(a) (b)

Fig. 2 (a) Active/Hot redundancy structure (b) Passive/Cold redundancy structure

On the contrary of passive or cold redundancy, there is no need to activate or enable the redundant unit to operate in active or hot redundancy. For this reason, hot redundancy increases operational availability of the system; however, it also increases operational cost since this structure requires more power.

If an equipment has a failure rate *λ(t)*, its probability of survival from time 0 to *t*, or reliability *R(t)*, is given by [4], [5];

$$R(t) = exp\left[-\int_0^t \lambda(u)du\right] \quad (1)$$

Equation (1) gives a general form which is independent of the failure rate variation law with time. For reliability prediction, a constant failure rate model with exponential law has been applied for equipment. If the failure rate λ is constant, the non-redundant single point failure unit reliability equation reduces to;

$$R(t) = e^{-\lambda t} \quad (2)$$

Reliability of *m/n* active redundant units can be formulated as;

$$R(t) = \sum_{i=0}^{n-m} \binom{n}{i} \left(1 - e^{-\lambda t}\right)^i \left(e^{-\lambda t}\right)^{n-i} \quad (3)$$

Reliability for passive redundancy (also known as cold redundancy) of *m/n* units can be calculated as;

$$R(t) = e^{-m\lambda t}\left[1 + \sum_{i=0}^{n-m} \frac{(1-e^{-\lambda^* t})^i}{i!} \prod_{j=0}^{i-1}\left(j + m\frac{\lambda}{\lambda^*}\right)\right] \quad (4)$$

where, ***t***: time period in hours (lifetime), $\boldsymbol{\lambda}$: Failure rate in FIT (1 FIT = 10^{-9} Failure / hour), $\boldsymbol{\lambda^*}$: Off state failure rate, ***n***: number of identical parallel units, ***m***: number of operating units.

B. RAP and Multi-Objective Optimization (MOO)

RAP is a significant phenomenon in reliability optimization problems dealt with the design stage of the parallel-series systems, network based systems, and other different formed systems. A generic layout of parallel-series system is shown in Fig. 3 [6].

Solving a RAP of a system should take multiple considerations into account, e.g. an engineer or user hopes to obtain a system with high reliability, while the manufacturer or buyer naturally chooses spending lower cost in the designing stage of the system. For this reason, both the reliability and cost should be optimized along the two objectives which implies that a multi-objective approach is needed.

The MOO problem has a quite different approach compared to single objective one. Only one global optimum is available in single objective case, whereas in MOO there is a set of solutions, called the Pareto-optimal (PO) set, which are assumed to be equally important. All of them form global optimum solutions. While moving from one Pareto solution to another, there is always a certain amount of sacrifice in one objective to achieve a certain amount of gain in the other. Pareto optimal solution sets are often preferred to single solutions because they can be practical when considering real-life problems, since the final solution of the decision maker is always a trade-off between crucial parameters [7].

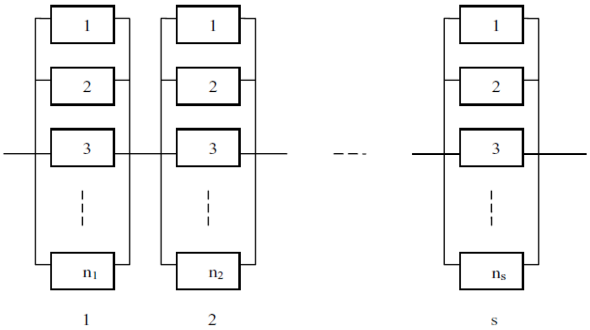


Fig. 3 Generic layout of a parallel-series system

For the multi-objective redundancy allocation problem (MORAP) the ultimate goal is to determine the optimal design configuration that will maximize system reliability and minimize the total cost at the same time. The mathematical formulation of a reliability-cost optimization problem is given in (5):

$$\left\{max[R=\prod_{i=1}^{s} R_i(x_i)], \min\left[C=\sum_{i=1}^{s}\sum_{j=1}^{m_i} c_{ij}\, x_{ij}\right]\right\}$$
$$subject\ to \quad (5)$$
$$I \le \sum_{j=1}^{m_i} x_{ij} \le n_{max,i} \quad \forall\, i = 1,2,\dots,s \ and\ x_{ij} \in \{0,1,2,\dots\}$$

where R and C, is the reliability and cost of the system respectively, s is number of subsystems, x_{ij} is quantity of j^{th} component in subsystem i, $n_{max,i}$ is user defined maximum number of components in parallel used in subsystem i, m_i is total number of available components for subsystem i, $R_i(x_i)$ is reliability of subsystem i, c_{ij}, is cost for the j^{th} available component for subsystem i [8].

III. Problem Definition & Mathematical Model To Solution

In this study, it was formulated a RAP by considering the system reliability and designing cost difference due to redundancy as two objectives, and the resultant MORAP takes both objectives into account simultaneously. The engineering problem is to determine the required number and type of redundant power amplifiers while achieving predefined reliability and capacity constraints for a communication satellite payload.

Number of nominal power amplifiers (PA) as mission of satellite dictates is given as $\boldsymbol{m}$. The purpose is to determine optimum number of redundant PAs and their type whether active ($\boldsymbol{a}$) or passive ($\boldsymbol{p}$) while maximizing the reliability and minimizing the cost of payload system sourced by redundancy. The studied analytical optimization approach searches minimum value of cost function, which is basically defined as sum of redundant units, achieving reliability threshold and select the optimum redundant numbers and type. The studied method calculates all the possible combinations within boundary conditions and then finds out the minimum valued **($\boldsymbol{a,p}$)** pair which provide reliability constraint.

Mathematical model of the engineering problem is described by steps:

1) Table II lists all the parameters used in order to calculate and analyze the mathematical functions in the model. Number of active redundant units ($\boldsymbol{a}$) and number of passive redundant units ($\boldsymbol{p}$) are the decision variables and ultimate outputs.

TABLE II
Parameters of the Mathematical Model

Symbol	Quantity	Description	Relation
a	integer	Number of active redundant units	decision variable
p	integer	Number of passive redundant units	decision variable
t	time period in hours	Satellite lifetime period	input
$\boldsymbol{\lambda}$	10^{-9} failure / hour	Failure rate in FITs	input
$\boldsymbol{\lambda^*}$	10^{-9} failure / hour	Off state failure rate in FITs	λ /10 is assumed nominally
m	integer	Number of nominal operating units	input
tr	integer	Total number of redundant units	$tr = a + p$
n	integer	Total number of identical parallel units (including redundant units)	$n = m + a + p$
R	integer between 0 and 1	Reliability Figure	function of a, p, t, λ and m (objective function)

2) Two objective functions are available in the model, which are reliability figure of communication payload transponder/power amplification redundancy scheme and delta cost function of redundant units. The definitions of the objective functions are given in (6) and (7), respectively:

$$Maximize$$
$$R_{Power\ Amplification}(m,a,p) = \begin{cases} 1-\left[\left(1-R_{active}(m,a)\right)\cdot\left(1-R_{passive}(m,p)\right)\right], & a+p\neq 0 \\ e^{-\lambda t}, & a+p=0 \end{cases} \quad (6)$$

where

$$R_{active}(m,a) = \sum_{i=0}^{a}\binom{m+a}{i}\left(1-e^{-\lambda t}\right)^{i}\left(e^{-\lambda t}\right)^{m+a-i} \text{and}$$
$$R_{passive}(m,p) = \mathrm{e}^{-\mathrm{m}\lambda \mathrm{t}}\left[1+\sum_{i=0}^{p}\frac{(1-e^{-\lambda^* t})}{i!}\prod_{j=0}^{i-1}\left(j+m\frac{\lambda}{\lambda^*}\right)\right]$$

It is assumed that $R_{active}(m,0) = 0\ and\ R_{passive}(m,0) = 0$ if $a+p\neq 0$.

$$Minimize \quad (7)$$
$$\Delta Cost_{PA\ redundant\ units}\,(a,p) = (a+p)$$

3) Objective functions given in (6) and (7) are subject to the following reliability figure and capacity constraints as shown in (8) and (9), respectively:

$$R_{Power\ Amplification}(m,a,p) \quad \ge 0.999\ (99.9\%) \quad (8)$$

Reliability figure constraint indicates the minimum required threshold level, whereby the system design should have a sufficient reliability level.

$$tr \leq m \tag{9}$$

Capacity constraint implies that there is an upper limit for the number of redundant units due to mass, power, allocation, other technical and cost limitations.

IV. Application Process and Simulation Results

Once problem is defined and mathematical model for solution is structured, a simulation script was generated in MATLAB environment. The script is parametric, easy to change inputs, and includes all the required functions dictated by mathematical model. The input parameters are shown on Table III.

TABLE III
APPLIED INPUT PARAMETERS FOR SIMULATION

Symbol	Description	Value
t	Satellite lifetime period (design lifetime)	15 years
λ	Failure rate (λ) of one transponder power amplifier (FITs:**10^{-9}** failures/hour)	$\boldsymbol{\lambda}_1$=500 FITs, $\boldsymbol{\lambda}_2$= 700 FITs
λ^*	Off state failure rate (λ*) for passive redundancy	$\boldsymbol{\lambda}^*_1$=50 FITs, $\boldsymbol{\lambda}^*_2$= 70 FITs
m	Number of nominal operating units	m_1=16, m_2=30
$R(t)$	Minimum (Target) Reliability Figure of communication payload transponder/power amplification subsystem	0.999 (99.9%) at end of design lifetime

As an example and comparative analysis, multi-objective optimization processes were applied for two different scaled payloads (small scaled: m_1=16, mid-scaled m_2=30) and for two different type power amplifiers whose failure in time-FIT values are $\boldsymbol{\lambda}_1$= 500 FITs and $\boldsymbol{\lambda}_2$= 700 FITs. Thanks to the comparative simulations performed in MATLAB, the effect of optimization on these parameters was figured out.

It should be noted that absolute processing durations are definitely related to computer RAM properties; on the other hand, since the purpose is to compare and analyze effects of input parameters, it is considered that a relative viewpoint is sufficient. In order to assure any superiority in terms of processing time, simulations are repeated a few times for stabilization.

Four simulations were conducted via analytic method with different combinations of m and $\boldsymbol{\lambda}$, while other parameters remain the same. For m_1=16 run, there are 289 (17x17) and for m_2=30 run, there are 961 (31x31) different reliability figure results, implying the whole solution spaces. Fig. 4 shows the solution space of m=m_2=30 & $\boldsymbol{\lambda}$ =$\boldsymbol{\lambda}_1$=500 FITs run and 917 of 961 results are higher than 99.9% (0.999) reliability figure threshold (light color/gray), and 44 pairs have lower reliability figure less than threshold (dark color/black).

Analytic method makes search an optimum ***(a,p)*** pair within the light color (gray) area after the all possible solutions are computed. For $\boldsymbol{m}$=30 and $\boldsymbol{\lambda}$=500 FITs, it is obtained that minimum tr value (delta cost function: ***a+p***) achieving 0.999 reliability figure target threshold and complying with capacity constraint is 8. Reliability figure results (R) for ***(a,p)*** pairs in that ***tr***=8 are given in Table IV.

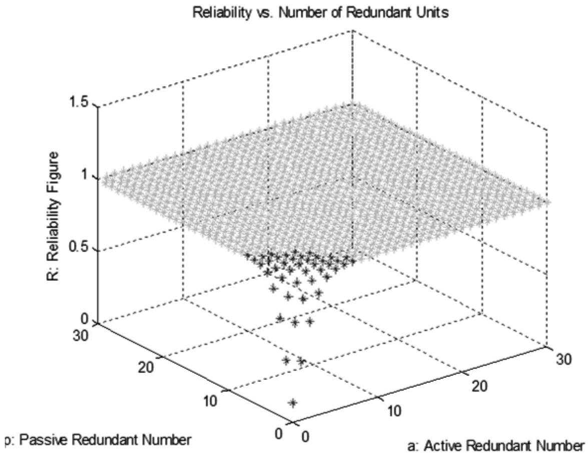


Fig. 4 Reliability figure solutions space for $m=30$ & λ=500 FITs

TABLE IV
RELIABILITY FIGURE RESULTS

Calculated *(a,p)* pair	Resulting Reliability Figure (*R*)
(0,8)	*R*=0.99976 > 0.999
(1,7)	*R*=0.99936 > 0.999
(2,6)	*R*=0.99853 < 0.999
(7,1)	*R*=0.99886 < 0.999
Other *(a,p)* pairs	*R*< 0.999

Two solutions (0,8) and (1,7) are available to achieve target reliability figure and complying with eight redundant units. Although both solutions are optimum, the generated analytic method script maximizes the reliability function and picks up the highest reliability figure pair which is (0,8) and *R*= 0.99976. Fig. 5 shows MATLAB output screen for optimum result. If there is no power limitation, (***a*=1, *p*=7**) solution can also be chosen as optimum since it is advantageous in terms of operational availability.

>>Optimum number of active redundant units(a): **0**
>>Optimum number of passive redundant units(p): **8**
>>Optimum number of total redundant units(a+p) :**8**
>>Optimum Reliability Figure: **0.99976**
>>Process time: **71.286** seconds

Fig. 5 MATLAB output screen of the generated script for the problem with m=30 and $\boldsymbol{\lambda}$=500 FITs

Once the optimum number and type of redundant units are identified, time varying reliability analysis was performed up to 30 years for the optimum solution (***a*=0, *p*=8**) with ***m***=30 transponders. Results are shown on Table V. It was obtained that after 18 years, target reliability figure (0.999) was not achieved and after 20 years, reliability figure decreased rapidly.

For the same problem, mathematical model was applied again with higher failure rate. Failure rate of one transponder power amplifier $\boldsymbol{\lambda}$=$\boldsymbol{\lambda}_2$=700 FIT (10^{-9} failures/hour) is taken this time. MATLAB outputs for $\boldsymbol{\lambda}$=$\boldsymbol{\lambda}_1$=700 FIT values (other parameters are same as previous case) are shown on Fig. 6.

TABLE V
TIME-VARYING RELIABILITY ANALYSIS FOR OPTIMUM (*A*,*P*) PAIR WITH *M*=30

Time (years)	Resulting Reliability Figure (R)
0	1.0000000
5	1.0000000
10	0.9999890
11	0.9999769
12	0.9999551
13	0.9999180
14	0.9998579
15	**0.9997650**
16	0.9996263
17	0.9994264
18	**0.9991463**
19	0.9987642
20	0.9982547
21	0.9975896
22	0.9967376
23	0.9956650
24	0.9943358
25	0.9927121
26	0.9907552
27	0.9884253
28	0.9856827
29	0.9824883
30	0.9788040

>>Optimum number of active redundant units(a): **0**
>>Optimum number of passive redundant units(p): **9**
>>Optimum number of total redundant units(a+p) :**9**
>>Optimum Reliability Figure: **0.99934**
>>Process time: **94.314** seconds

Fig. 6 MATLAB output screen of the generated script for the problem with m=30 and λ=700 FITs

TABLE VI
RAP SOLUTION RESULTS SUMMARY

Input Configuration	Processing time (seconds)	Optimum number of total redundant units	Optimum Reliability Figure
m=16, λ=500 FITs	16.827	5 (0 active, 5 passive)	0.99917
m=16, λ=700 FITs	17.123	6 (0 active, 6 passive)	0.99908
m=30, λ=500 FITs	71.286	8 (0 active, 8 passive)	0.99976
m=30, λ=700 FITs	94.314	9 (0 active, 9 passive)	0.99934

Increase in equipment failure rate from 500 FITs to 700 FITs directly affected the subsystem redundancy scheme. Based on the obtained results, in order to achieve the same target reliability figure, one additional redundant unit is needed compared to 500 FITs case. Also, required processing time in order to obtain the optimum solution was increased compared to lower FIT valued problem.

To sum up, example simulations were run on the studied approach in order to find and compare the output parameters which are processing time, optimum number and type of redundant units and lastly the obtained optimum reliability figure value. Table VI shows the results based on the inputs; two different scaled payloads (small scaled: m=16, mid-scaled m=30 and two different type power amplifiers whose failure in time-FIT values are $\lambda=\lambda_1$= 500 FITs and $\lambda=\lambda_2$=700 FITs.

V. CONCLUSION

In this study, a RAP was analyzed from a reliability engineering perspective, which can be defined as the determination of the required number and type of redundant payload power amplifier units regarding communications satellites achieving a target reliability figure. An analytical optimization approach was applied for the problem. The main advantage of the studied approach is to deal with and process the objective functions simultaneously based on equal priority rather than analyzing them independently and respectively, thus the approach reaches to the optimum results efficiently.

It was observed that that failure rate λ also has a significant effect on the studied engineering problem. Using equipment which has higher failure rate in payload power amplification scheme may result in allocating additional redundant units in order to achieve a predefined reliability target. Also, it is necessary to state that relative processing time increases if payload capacity increases since solution space gets bigger. Constraints and limitations are also important factors to reach the optimum results in terms of processing time and number of redundant units.

As potential future works of this study, wider problem solution spaces, investigating alternative and hybrid approaches including probabilistic techniques, reducing processing time, and also introducing operational availability function which affects redundancy type especially active types may be considered.

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