

Performance Analysis of a Hybrid DF-AF Hybrid RF/FSO System under Gamma Gamma Atmospheric Turbulence Channel Using MPPM Modulation

Hechmi Saidi, Nouredine Hamdi

Abstract—The performance of hybrid amplify and forward - decode and forward (AF-DF) hybrid radio frequency/free space optical (RF/FSO) communication system, that adopts M-ary pulse position modulation (MPPM) techniques, is analyzed. Both exact and approximate symbol-error rates (SERs) are derived. The random variations of the received optical irradiance, produced by the atmospheric turbulence, is modeled by the gamma-gamma (GG) statistical distribution. A closed-form expression for the probability density function (PDF) is derived for the whole above system is obtained. Thanks to the use of hybrid AF-DF hybrid RF/FSO configuration and MPPM, the effects of atmospheric turbulence is mitigated; hence the capacity of combating atmospheric turbulence and the transmitted signal quality are improved.

Keywords—FSO, RF, hybrid, AF, DF, SER, SNR, GG channel.

I. INTRODUCTION

WIRELESS Optical Communication or Free Space Optic (FSO) is a complementary technology to microwave and optical fiber to meet the growing needs in telecommunications at high data rates. In 2008, the first FSO link with 10 Gbps was introduced on the market, making it the fastest wireless technology available in the market.

A. Related Works

FSO systems have these days received much attention in firstmile access environments owing to their advantages including unregulated bandwidth, high security, and ease of installation [1]. FSO systems are, however, confined to short-haul applications since their reliability is degraded due to the distance dependent atmospheric turbulence and channel loss [2].

Wireless relaying systems have drawn a significant attention in radio frequency (RF) wireless communications due to their strong potential to increase the coverage area and quality-of-service (QoS). Relays can be deployed to extend the coverage of networks to areas where it is either technically or economically unfeasible to set up fully edged base stations. Furthermore, relaying can help to decrease interference and

thereby improve the overall network performance: If the number of available relays is large compared to the number of base stations, mobile users will most probably be closer to one of the relays than to a base station and therefore require less transmit power, leading to less interference. As a result, relaying techniques have already found application in several wireless standards. The relaying systems can be classified into amplify-and-forward (AF) and decode and-forward (DF) relays.

Multi-hop transmission relays the data signal from the source node (S) to destination node (D) through N intermediate terminals called relays (R). By subsequently deploying of smaller multiple hops, more reliable FSO transmission over longer distances can be achieved. Moreover, multi-hop techniques can support an optical connection between two buildings which do not have a line-of-sight. Most of the existing studies focused on conventional electrical relaying [3]-[5], where amplifying or decoding process is done in electrical domain [6]. With the aim of allowing efficient high-speed transmission without the need for complex optoelectronics and electronic processing at each relay, alloptical relaying was first proposed [7].

B. Contributions

In this work, we are going to deal with an asymmetric FSO/RF AF-DF three - hop system using MPPM modulation technique. In addition, background noise and thermal noise at the destination are dominant compared to other terms of noise from the source and the relay. We will only consider the indirect links through relays and neglect direct links between the source and the relay 2, the relay 1 and the destination but also between the source and destination because the source is considered far from the destination and the transmitted power of the source signal small enough that it barely reaches the relay 1. Therefore, there is no diversity at the reception, i.e., the destination receives only the signal from the relay 2. We also consider that each Relay is equipped with one antenna while having full knowledge of the channel. The noise is assumed to be centered Gaussian one. In this paper, both exact and approximate symbol-error rates (SERs) are derived. Besides, a closed-form expression for the PDF of the whole above system is obtained.

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This paper is organised as follows: The system model of AF-DF three-hop MPPM hybrid FSO/RF systems is introduced in Section II. In Section III, we formulate the BER of whole systems. Section IV shows the numerical results and discussion. Finally, Section V concludes the paper.

II. SYSTEM MODEL

A model of hybrid DF-AF hybrid RF/FSO system is adopted.

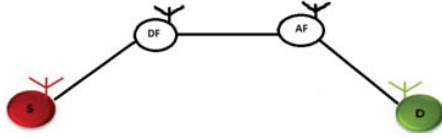


Fig. 1 Hybrid DF-AF RF/ FSO system

A. FSO Link

At the source, look at our model in Fig. 1, input data is first modulated at a PPM modulator, where each block of b bits is mapped to one of M possible symbols ($s_0, s_1, \dots, s_M$) converted into an optical pulse with constant power of P_t by a laser diode. At the relay, these signals are amplified by an optical amplifier with the gain of G and then transmitted to the destination together with ASE noise generated by the optical amplifier. ASE noise can be modeled as an additive zero-mean white Gaussian noise with the optical power given by [8]

$$P_A = h_p f (G - 1) n_{sp} B_0 \quad (1)$$

where h_p is Planck's constant, f is the frequency, n_{sp} is the amplifier spontaneous emission parameter, and B_0 is the optical bandwidth.

At the destination, the total electric field is converted to the photocurrent thanks to a photodetector (PD). Next, integrated photocurrents over M time slots are compared at the PPM demodulator to find the position of the slot with the highest current, which determines the transmitted symbol. Finally, detected symbol is converted to the binary data.

Furthermore, we present the mathematical model of FSO channel with the state of h . In our model, h arises due to two factors: channel loss (h^l) and atmospheric turbulence (h^a). The channel coefficient hence can be described as $h = h^l h^a$.

1) *Turbulence Channel Model:* To model atmospheric turbulence, Gamma-Gamma distribution is the best one: It is, in fact, suitable in both strong and weak turbulence so we will use it in our analysis and its PDF is given by [9]:

$$f(K_s) = \frac{2(\alpha\beta)^{\frac{(\alpha+\beta)}{2}}}{\lambda\Gamma(\alpha)\Gamma(\beta)} \left(\frac{K_s}{\lambda} \right)^{\frac{(\alpha+\beta)}{2}-1} \times K_{\alpha-\beta} \left(2\sqrt{\frac{\alpha\beta K_s}{\lambda}} \right) \quad (2)$$

Where

$$\alpha = \left(\exp \left[\frac{0.49\sigma_R^2}{(1 + 1.11\sigma_R^{\frac{12}{5}})^{\frac{7}{6}}} \right] - 1 \right)^{-1} \quad (3)$$

$$\beta = \left(\exp \left[\frac{0.51\sigma_R^2}{(1 + 0.69\sigma_R^{\frac{12}{5}})^{\frac{5}{6}}} \right] - 1 \right)^{-1} \quad (4)$$

where α and β are the scintillation parameters that are dependent on σ_R^2 , σ_R^2 is unitless Rytov variance, $\Gamma(\cdot)$ is the gamma function, and $K_c(\cdot)$ denotes the c_{th} order modified Bessel function of the second kind. The scintillation index is related to α and β as

$$\chi_{SC} = \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\alpha\beta} \quad (5)$$

2) *Geometric Spreading and Channel Loss:* It is easy to obtain the value of loss due to geometric spreading as a function of the receiver area A and beam divergence angle θ . In addition, the path loss of laser power through the atmosphere can be described by the exponential BeersLambert Law. As a result, the total of geometric spreading and path loss can be formulated as

$$h_l = \frac{A}{\pi(\theta d)^2} \exp(-\beta d), \quad (6)$$

where β is attenuation coefficient. d is the distance.

B. RF Link

1) *General Data Information:* With M-PPM modulation the signaling interval T_s is composed of N_f frames of duration T_f and each frame is divided into M equal slots. The information is conveyed by transmitting one monocycle per frame, always in the same slot over a given symbol interval. The N_f -fold pulse repetition increases the energy per symbol or, for a given symbol energy, it reduces the energy of the individual pulses. Bit error rate (BER), defined as the ratio between the total error bits and the total bits of the data stream in a communication channel, is mainly affected by noise, interference, multipath distortion or bit synchronization errors.

2) *Multipath Channel Distortion:* If we denote the transmitted pulse as $x(t)$ generated by a UWB signal generator and channel impulse response the channel as $h(t)$, the distorted pulse after channel, denoted as $y(t)$, can be derived from the convolution integral as follows:

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t - \tau) d\tau$$

where $*$ represents the convolution operation.

3) *Additive White Gaussian Noise:* Another root cause for BER is noise. AWGN is a fundamental noise model to simulate the random noise in nature. AWGN follows Gaussian distribution, which can be expressed as:

$$f(x, \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

where x is the value of a random variable, μ is the mean of the random variable, and σ is the standard deviation of the random variable. Furthermore, the autocorrelation function of AWGN is a delta function, which means the power spectrum of AWGN is flat (or white).

III. PERFORMANCE ANALYSIS

Our performance evaluation concerned the model of a symmetrical network; we studied the system performance for relatively remote positions of relays between source and destination and thus considering only the relayed signals. We considered a symmetrical network where the best position of the relay 1 is when it is midway between the source and the relay 2 and where the best position of the relay 2 is when it is midway between relay 1 and the destination.

A. PPM Average SEP Analysis in Atmospheric Turbulence Channel Optical AF Dual-Hop FSO System

The destination employs a photodetector to convert the received signal to the photocurrent, which can be written as

$$I_s = \Re P_t H_{sr} G h_{rd}, \quad (7)$$

where $\Re$ is the responsivity of the photodetector. The receiver noises including shot noise and thermal noise are additive Gaussian noise with zero mean. Assuming that B_e is the electrical bandwidth of the photo detector, the total variance of noises in the case of signal slot (σ_s^2) and non-signal slot (σ_n^2) can be expressed as

$$\begin{aligned} \sigma_s^2 &= 2e\Re(P_t h_{sr} G h_{rd} + (P_b G + P_A) h_{rd} + P_b) B_e + \frac{4k_B T B_e}{R_L}, \\ \sigma_n^2 &= 2e\Re[(P_b G + P_A) h_{rd} + P_b] B_e + \frac{4k_B T B_e}{R_L}, \end{aligned} \quad (8)$$

where e is the electron charge; k_B is Boltzmann constant; T is the absolute temperature; R_L is the load resistance. Hence, the electrical signal to noise ratio (SNR) at the destination that uses PPM can be defined that

$$\text{SNR} = \frac{I_s^2}{\sigma_s^2 + \sigma_n^2} \quad (9)$$

In this paragraph, we present the method to calculate the symbol error probability of the all-optical AF dual-hop FSO system using PPM. Denoting P_e^r as the symbol error probability. We assume that the transmitted data is large enough that the probabilities of sending any symbols are the same. Without the loss of generality, we also assume that symbol s_0 is transmitted with received current of I_s . By using union bound technique, the upper bound to the instantaneous symbol error probability can be expressed as

$$\begin{aligned} P_e^r &\leq 1 - \Pr\{I_s > I_n/n \in \{1, \dots, M-1\}, s = s_0\} \\ &\leq \frac{M-1}{2} \int_0^\infty \int_0^\infty f(h_{sr}^a) f(h_{rd}^a) \text{erfc}\left(\sqrt{\frac{\text{SNR}}{2}}\right) dh_{rd}^a dh_{sr}^a \end{aligned} \quad (10)$$

where s represents the transmitted symbol. I_n is the current corresponding to non-signal slot and $\text{erfc}(\cdot)$ is the complementary error function.

The derivation of closed-form expression for SEP taking into account all noises is very complicated. In many previous studies, such as [10], [11], noise is assume as additive white Gaussian noise (AWGN) for the sake of simplicity. In fact, the performance of OAF relaying FSO systems over strong atmospheric turbulence channels has been recently studied [4], [12]-[14]. Nevertheless, due to the analysis complexity, these

studies are limited, case of PPM modulation, to the special case of dual-hop or parallel OAF relaying systems. To the best of authors' knowledge, the performance of generalized case of multihop OAF relaying FSO systems with an arbitrary number of nodes under the impact of strong atmospheric turbulence has not yet been investigated in the literature.

In our analysis we assume that we work in a clear atmosphere conditions and also both the transmitter and receiver are fixed and perfectly aligned. Scintillation or channel fading, however, is considered in our FSO performance derivation.

In what follows, the closed-form expression can be obtained by considering only the effect of noises induced by the destination: we only consider background noise and thermal at the destination. In fact, we can prove that the impact of background noise and thermal noise at the destination is dominant in comparison with other noise terms from the source and the relay. This explains why the performance of systems in general case and our approximation are the same. Neglecting noise terms induced by the source and the relay, SNR can be written as

$$\text{SNR} = \frac{(\Re P_t h_{sr} G h_{rd})^2}{4e\Re P_b B_e + \frac{8k_B T}{R_L} B_e} \quad (11)$$

By substituting (17) in (16), expressing $\text{erfc}(\cdot)$ integrands as Meijer's G-functions [15], and based on [16] (07.34.21.0013.01), a closed-form expression for the symbol error probability is derived as (13).

$$\begin{aligned} P_e^r &\leq \frac{(M-1)2^{(\alpha_1+\beta_1+\alpha_2+\beta_2-5)}}{\pi^{\frac{5}{2}} \Gamma(\alpha_1) \Gamma(\beta_1) \Gamma(\alpha_2) \Gamma(\beta_2)} G_{9,2}^{2,8} \\ &\quad \times \left[\frac{32(\Re P_t G h_{sr}^l h_{rd}^l)^2}{(e\Re P_b B_e + \frac{2k_B T B_e}{R_L})(\alpha_1 \beta_1 \alpha_2 \beta_2)^2} \right] \\ &\quad \left[\frac{1-\alpha_2}{2}, \frac{2-\alpha_2}{2}, \frac{1-\beta_2}{2}, \frac{2-\beta_2}{2}, \frac{1-\alpha_1}{2}, \frac{2-\alpha_1}{2}, \frac{1-\beta_1}{2}, \frac{2-\beta_1}{2}, 1 \right] \\ &\quad \left[0, \frac{1}{2} \right] \end{aligned} \quad (12)$$

As an approximation to simplify following works, we can take case of equality.

Subsequently, we can consider R_1 like an intermediate source for $R_1 R_2 D$ which make $P_{R_1 R_2 D}^e = P_e^r$ calculated in (13).

B. Bit Error Rate for Conventional MPPM for RF link

Our approach for data analysis uses the BER of conventional MPPM and OOK as reference. It is necessary to get theoretical expression for the BER of conventional MPPM/OOK modulation before we start data analysis.

The Gaussian approximation for the conventional MPPM performance, derived in [17], assumes perfect synchronization and frame period is larger than the channel impulse response. The theoretical bit error function can be written as:

$$P(e) = Q([2(N_0 E_S) + 4B\Delta(N_0 E_S)^2]^{-1/2})$$

Where E_S is the received energy per symbol and $Q(x)$ denotes a zero mean, unit variance Gaussian distribution function, and N_0 is the single-sided power spectral density of the AWGN. In the same equation, B is the bandwidth of signals and T is the integration time.

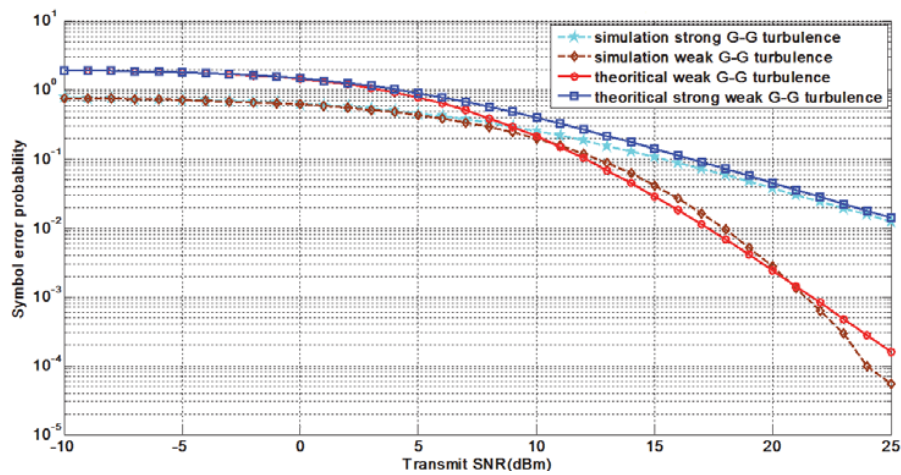


Fig. 2 Strong VS weak turbulence for 8PPM FSO system under gamma gamma turbulence channel

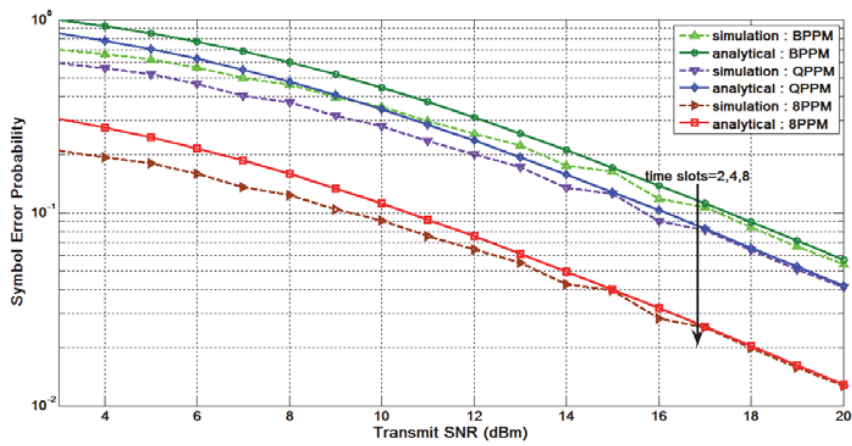


Fig. 3 Comparison of M-ary PPM for three hops AF-DF relaying hybrid RF/FSO system

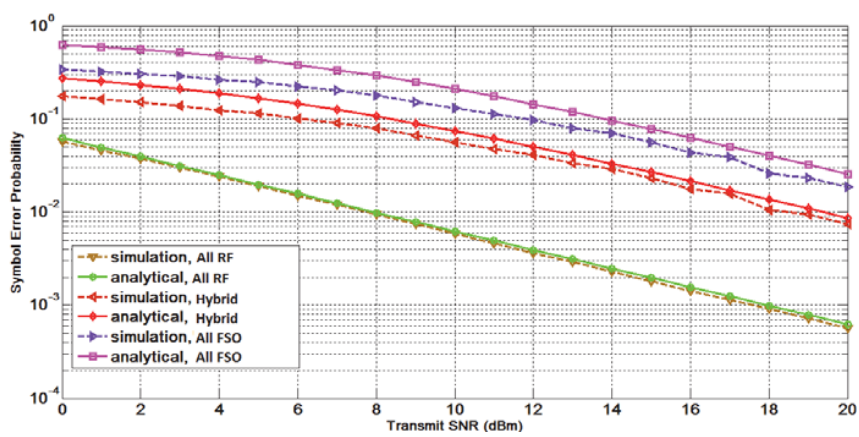


Fig. 4 Comparison between AF-DF relaying only FSO or only RF or hybrid FSO/RF

C. The End to End BER

The hybrid DF-AF hybrid RF/FSO system with is modelled as a concatenation of 2 symmetric channel with an error probability P_i for the i th hop ($i \in \{1, 2\}$). Assuming that each hop is independent, the end-to-end symbol error probability P_e for the i th hop. Assuming that each hop is independent, the end-to-end symbol error probability (P_e^t) is given by [18]:

$$P_e^t = 1 - (1 - P_{SR_1}^e)(1 - P_{R_1R_2D}^e) = 1 - (1 - P(e))(1 - P_r^e) \quad (13)$$

Once the MPPM symbol is detected, it is mapped to a string of $\log_2(M)$ bits through the inverse of the encoding mapping. There are $M/2$ symbol errors that will produce an error in a given bit in the string, and there are $M - 1$ unique symbol errors. Thus, assuming all symbol errors are equally, the resulting BER can be expressed as

$$BER = \frac{M}{2 \times (M - 1)} P_e \quad (14)$$

IV. SIMULATIONS RESULTS

In this section, we numerically investigate the SER of the DF-AF serial relaying hybrid RF/FSO system using PPM under gamma gamma atmospheric turbulence distribution. We also assume that each hop is equidistant. For amplify and forward protocol, we fixed amplifier gain at $G = 10$ dB with $K_b = 1$. For gamma gamma turbulence, $\sigma_R \in \{0.25, 0.75\}$ (respectively weak and strong turbulence). The whole distance between source and destination is $L = 3$ Km.

We see firstly due to Fig. 2 that simulation values and theoretical ones are approximately the same. Besides, when the SNR (or transmitted power per bit) is growing, the SER is decreasing. We conclude also that gamma gamma turbulence describes better the atmospheric turbulence channel which are valid either for weak and strong turbulence. For that reason, we are adopting gamma gamma turbulence in our study to generalization and optimization.

Furthermore we see using Fig. 3 that simulation plots and theoretical ones are approximately the same. Besides, when the SNR is growing, the SER is decreasing. It is shown also that when the number of time per frame M for M -ary PPM is increasing, the average SER is decreasing. We make the comparison between the 8PPM and BPPM ones in term of the required transmitted power per bit. It is seen that the required transmitted-power per bit of BPPM system is about 17 dBm at BER of 10^{-1} . By using 8PPM (the transmitted power per bit is equal to 11dBm when $SER = 10^{-1}$, higher M -ary PPM one can reduce the required transmitted-power per bit to -6 dBm, i.e., the power gain is 9 dB. For that reason, we will adopt for 8PPM in what follows for optimization.

Furthermore, we tried to numerically investigate the BER of 8-PPM hybrid AF-DF hybrid FSO/RF system. As a benchmark, we presented the case where only one of the two networks was used at all links. We further compared between their performances. In each case, we plotted either simulating and analytical results which were nearly similar.

Like it has been showed in Fig. 4 for our hybrid AF-DF hybrid FSO/RF, comparing to the famous AF-DF three-hop all RF or all FSO system, we found that hybrid scheme was more reliable than all FSO links system but the all RF links was the best one with less bit error rate.

V. CONCLUSION

We have formulated a closed-form expression for BER of hybrid AF-DF relaying hybrid RF/FSO systems using MPPM over GG atmospheric turbulence. Thanks to the use of hybrid configuration and MPPM signaling, the effect of many impairments, which were taken into consideration including basically atmospheric attenuation, had been combatted. The numerical results helped to corroborate such result. The benefits of using AF-DF serial-relaying and MPPM in combating atmospheric turbulence and extending the transmission distance could be also seen. Moreover, the integration of multihop (for example All AF) relaying and multiple input/multiple output (MIMO) configuration would be a perspective for this work.

REFERENCES

- [1] D. J. Heatley, D. R. Wisely, I. Neild, and P. Cochrane, "Optical wireless: the story so far," *IEEE communications magazine*, vol. 36, no. 12, pp. 72–74, 1998.
- [2] X. Zhu and J. M. Kahn, "Free-space optical communication through atmospheric turbulence channels," *Communications, IEEE Transactions on*, vol. 50, no. 8, pp. 1293–1300, 2002.
- [3] M. A. Kashani, M. Uysal, and M. Kavehrad, "On the performance of mimo fso communications over double generalized gamma fading channels," in *Communications (ICC), 2015 IEEE International Conference on*. IEEE, 2015, pp. 5144–5149.
- [4] T. V. Pham and A. T. Pham, "Performance analysis of amplify-decode-and-forward multihop bpsk/fso systems using apd receivers over atmospheric turbulence channels,"
- [5] T. A. Tsiftsis, H. G. Sandalidis, G. K. Karagiannidis, and N. C. Sagias, "Multihop free-space optical communications over strong turbulence channels," in *Communications, 2006. ICC'06. IEEE International Conference on*, vol. 6. IEEE, 2006, pp. 2755–2759.
- [6] E. Bayaki, D. S. Michalopoulos, and R. Schober, "Edfa-based all-optical relaying in free-space optical systems," *Communications, IEEE Transactions on*, vol. 60, no. 12, pp. 3797–3807, 2012.
- [7] S. Kazemlou, S. Hranilovic, and S. Kumar, "All-optical multihop free-space optical communication systems," *Lightwave Technology, Journal of*, vol. 29, no. 18, pp. 2663–2669, 2011.
- [8] G. P. Agrawal, "Fiber-optic communication systems, 2002," *A John Wiley &*.
- [9] M. Al-Habash, L. C. Andrews, and R. L. Phillips, "Mathematical model for the irradiance probability density function of a laser beam propagating through turbulent media," *Optical Engineering*, vol. 40, no. 8, pp. 1554–1562, 2001.
- [10] M. Feng, J.-B. Wang, M. Sheng, L.-L. Cao, X.-X. Xie, and M. Chen, "Outage performance for parallel relay-assisted free-space optical communications in strong turbulence with pointing errors," in *Wireless Communications and Signal Processing (WCSP), 2011 International Conference on*. IEEE, 2011, pp. 1–5.
- [11] M. Safari and M. Uysal, "Relay-assisted free-space optical communication," *Wireless Communications, IEEE Transactions on*, vol. 7, no. 12, pp. 5441–5449, 2008.
- [12] P. V. Trinh, A. T. Pham, H. T. Pham, and N. T. Dang, "Ber analysis of all-optical af dual-hop fso systems over gamma-gamma channels," in *Photonics (ICP), 2013 IEEE 4th International Conference on*. IEEE, 2013, pp. 175–177.
- [13] J.-Y. Wang, J.-B. Wang, M. Chen, and X. Song, "Performance analysis for free-space optical communications using parallel all-optical relays over composite channels," *Communications, IET*, vol. 8, no. 9, pp. 1437–1446, 2014.

- [14] L. Yang, X. Gao, and M.-S. Alouini, "Performance analysis of relay-assisted all-optical fso networks over strong atmospheric turbulence channels with pointing errors," *Journal of Lightwave Technology*, vol. 32, no. 23, pp. 4011–4018, 2014.
- [15] I. Gradshteyn, "Gradshteyn and ryzhik's table of integrals, series, and products," 2007.
- [16] Wolfram. (2001) The wolfram function site. <http://functions.wolfram.com>. (Online). Available: •
- [17] C. Carbonelli and U. Mengali, "M-ppm noncoherent receivers for uwb applications," *IEEE Transactions on Wireless Communications*, vol. 5, no. 8, pp. 2285–2294, 2006.
- [18] H. Saidi, K. Tourki, and N. Hamdi, "Performance analysis of psk modulation in df dual-hop hybrid rf/fso system over gamma gamma channel," in *Signal, Image, Video and Communications (ISIVC), International Symposium on*. IEEE, 2016, pp. 213–216.