Dynamic Correlations and Portfolio Optimization between Islamic and Conventional Equity Indexes: A Vine Copula-Based Approach

Imen Dhaou

Abstract—This study examines conditional Value at Risk by applying the GJR-EVT-Copula model, and finds the optimal portfolio for eight Dow Jones Islamic-conventional pairs. Our methodology consists of modeling the data by a bivariate GJR-GARCH model in which we extract the filtered residuals and then apply the Peak over threshold model (POT) to fit the residual tails in order to model marginal distributions. After that, we use pair-copula to find the optimal portfolio risk dependence structure. Finally, with Monte Carlo simulations, we estimate the Value at Risk (VaR) and the conditional Value at Risk (CVaR). The empirical results show the VaR and CVaR values for an equally weighted portfolio of Dow Jones Islamic-conventional pairs. In sum, we found that the optimal investment focuses on Islamic-conventional US Market index pairs because of high investment proportion; however, all other index pairs have low investment proportion. These results deliver some real repercussions for portfolio managers and policymakers concerning to optimal asset allocations, portfolio risk management and the diversification advantages of these markets.

Keywords—CVaR, Dow Jones Islamic index, GJR-GARCH-EVT-pair copula, portfolio optimization.

I. INTRODUCTION

THE Islamic stock market is steadily rising and has attracted the attention of global financial industry. Their field of application is delimited by the principles of Shariah, the legal code of Islam. The emergence of this industry is the result of increased demand from market participants to both Muslims and non-Muslims, who wish to invest in socially responsible portfolios. This led to index providers to create Islamic indices such as, the Dow Jones Islamic Market Index and the FTSE Global Islamic Index Series, which are keeping track shares in companies whose activities are consistent with Islamic law.

The Islamic equity markets differ fundamentally from conventional equity markets in many ways. The main difference between Islamic and conventional markets is that the former operate in accordance with Shariah compliance. The Shariah principles prohibit all practices including speculation, short selling, margin trading, equity futures and options, all of which are severely restricted or forbidden within an Islamic market [1].

Investment in Islamic stock markets is considered to be less risky compared to conventional stocks. This has been established by Sukmana and Kholid [2] and Setiawan and Oktariza [3] for the Jakarat Islamic stock index (JAKISL) and its conventional counterpart the Jakarta Composite Index (JCI) in the Indonesian stock market. However, with reference to the non-Islamic countries' stock markets, their Shariah stocks were more risky than conventional stocks [4]. The few number of halal-business companies and the lack of diversification opportunities can explain this situation [5].

Many investors often evaluate the diversification benefits around various countries or regions by international diversification. The principle of diversification is based on the one hand on the mean-variance approach, and on the other hand on the dependence structure of financial assets.

It is essential to have exact measures of dependence. According to Markowitz [6], even when the assets are more dispersed, the income will not be higher. Rather, investors' income will rise significantly when the correlation between the international financial assets is weak. Hence, the risks and benefits of international portfolio diversification are closely linked to the concept of correlation or the dependence structure of financial assets. The dependence structure of financial assets is often studied since the information obtained allows investors to make investment decisions and to estimate the risk of the portfolios.

The copula model is a popular model to research dependence in many fields. Copula function allows to link univariate marginal distributions to the joint multivariate distribution function. Reference [7] is the first to present an overview of copula in financial risk management field. Reference [8] provides a complete discussion in the application of copulas to financial times series. Jondeau and Rockinger [9] combine the copula-GARCH approach in order to measure the dependence structure of stock markets. However, in financial applications, the class of multivariate copulas generates many complicated pattern of dependence in both the center and the tails of the distribution.

Joe [10], Bedford and Cooke [11] develop flexible structure model of multivariate distribution based into a cascade of conditional bivariate copulas as building blocks, which they named, vine structure. Reference [12] proposed two sub-families of regular vines, named canonical vines (C-vines) and drawable vines (D-vines). This paper seeks to measure the dependence between innovations of the return series, using vine copula-based GARCH family model. Additionally, we also propose to use the mean-variance approach and to

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investigate optimization problem of Markowitz [6].

In order to optimize a portfolio, VaR has been suggested as better alternative measure of risk. However, VaR has several drawbacks. To avoid such drawbacks, Rockefeller and Uryasev [13] propose a-so called CVaR as an alternative measure of risk.

CVaR have several attractive proprieties, which may be usually used in the optimization of the portfolio. This research suggests to use CVaR as an alternative measure of risk and optimizing portfolio problem with CVaR constraint. Then, we integrate extreme value theory (EVT) to model various tails distribution. On the other hand, the return series exhibit always volatility clustering leptokurtosis and heavy tails; it is unsuitable to use only the EVT to the return series.

In line with the approach developed by McNeil and Frey [14], we will use the volatility model from the GARCH family such as the asymmetric GARCH model. This model is the Glosten-Jagannathan-Runkle [15] (GJR) for forecasting return embedded in the data. In order to take accounts for rare and extreme events contained in the tails of conditional distributions, we will then apply the POT model from the EVT approach for the residuals.

The remainder of this article proceeds as follows. In Section II, we present a global review of Islamic stock market, and Sections III and IV outline each methodology. In Section V, we describe the data. Then we present the empirical estimates about the EVT, C-Vine parameters and analysis in VaR and ES, and optimal portfolio weights. Finally, Section VI, concludes the paper.

II. OVERVIEW

The magnitude of growth of the Islamic capital market has garnered much interest on a global scale, for both Islamic and non-Islamic countries alike. Despite the attention paid to this global phenomenon, there are few empirical studies concerning the equity portfolios that discuss the asset allocation framework and diversification perspectives. However, the issue of asset allocation and diversification should be addressed concurrently, but thought of separately. The analyses of the portfolio choice problem that incorporate the Islamic asset class are still in its infancy compared to the issue of diversification. Umar [16] analyzed the portfolio choice problem of two types of investors. The results of this study suggest that for the conventional investor, the inclusion of conventional equities in the asset menu reduces the desirability of Islamic equities only for the short-run. For long-run investors, investing in conventional equities is more desirable. In addition, the exclusion of conventional equities within a portfolio of Shariah compliant stocks could cause substantial welfare losses.

By employing a mean-variance spanning test in multiple regimes, [17] also found that for intra-asset allocation, investing in a mix of asset classes (Islamic and conventional) is more desirable for both conventional and Islamic investors. However, for inter-asset allocation, conventional institutional investors cannot obtain any profit from Islamic asset classes. In fact, Malaysian Islamic institutional investors can minimize its risk by investing in conventional bonds only in the high term premium regime.

Other empirical studies have focused on the issue of diversification perspectives, where several empirical approaches can be adopted.

The first approach tends to explore the interdependencies of conventional stock indices and Islamic stock indices. Reference [18] found that the interdependence of Islamic stock markets seems to display greater asymmetry across a vast geographical area. The authors determined that the Islamic stock indices of Indonesia and Malaysia, the US and Canada, and Japan and Asia Pacific are strongly correlated, while this is not exactly the case across regional stock markets. The study affirms that the Islamic stock market in the United States has a significant influence on other Islamic stock markets, but not vice versa.

Reference [19] documented that there was no cointegration links between conventional and Islamic stock indices (FTSE, DJ, MSCI, and S&Ps and Jakarta series). Based on daily observations for the period from 2002 to 2014 through an application of the VARMAX procedure and Johansen's cointegration approach, results show that in the long-run, Islamic indices can most likely offer potential diversification benefits for attracting international portfolios. Using a vine copula, [20] find empirical results of risk diversification benefits and downside risk reductions related with Islamic emerging equity markets, for GCC and global investors. In addition, the study reveals that Islamic equities can serve as a better safe haven than investment in gold. In the same way, other studies have also stressed that investment holding periods (e.g. two days, six days, thirty days, etc.) have an impact on the portfolio diversification of stock market returns. This type of research is relatively new, and there are few empirical studies that include time scaling in the interdependence. In this context, using both wavelet techniques discrete and continuous that allow comovement dynamics at different time scales or horizons, Aloui and Hkiri [21] examine the short-term and long-term dependencies between stock market returns for the Gulf Cooperation Council (GCC) countries. This finding indicates that the linkage among the GCC stock markets during the financial crisis increased and the portfolio benefits for short-term investors were improved compared to the long term investors, which note decreasing diversification benefits. Reference [22] used three methodologies namely: M-GARCH-DCC, Continuous Wavelet Transforms (CWT), and Maximum Overlap Discrete Wavelet Transform (MODWT), to assess international portfolio diversification opportunities for investors having heterogeneous investment horizons. They document the presence of effective portfolio diversification opportunities only for short holding periods; otherwise, for periods exceed one year, the markets exhibit high correlation yielding minimal portfolio diversification benefits. Using time-varying and timescale dependent techniques, Rahim and Masih [23] investigate the extent to which Malaysian Shariah investors can benefit from portfolio diversification with the Shariah indices of their major trading partners. The paper

concluded that to benefit from portfolio diversification with their trading partners, the Malaysian Shariah investors should often reassess stock exposures and investment horizons.

The second approach examines the extent of integration between conventional and Islamic stock markets, through more sophisticated techniques, such as Vector autoregression (VAR), Granger causality tests, cointegration test, and the vector error correction modeling (VECM). In this context, Majid and Kassim [24] examine the extent and prowess of market integration among five major Islamic stock markets, namely Malaysia, Indonesia, Japan, the United Kingdom and the US. Using the Auto Regressive Distributed Lag (ARDL) and the Vector Error Correction Model (VECM), the results reveal that investors can gain benefits by diversifying their portfolio in Islamic stock markets in both developed and developing countries. Otherwise, if the investments are only in the emerging Islamic stock markets or only in the developed Islamic stock markets, the benefits of diversification would be limited. The same result was found in Abbes and Trichilli [25], by using the VECM model, the Johansen-Juselius cointegration approach, and the granger causality test. The results of this study also suggest that Shariah compliant stocks provide significant diversification gains by taking into account various economic groupings as well as in developed and emerging countries.

Based on several cointegration techniques, [26] found that diversification benefits can be realized only in Japan, GCC, Indonesia, Malaysia and Taiwan; however, in the US, investing in Islamic equity does not lead to incremental diversification benefits.

Using the VAR estimation technique, [27] study the integration among Islamic stock markets in Malaysia, Indonesia and the World. Empirical results provide that there is no co-integration relationship between these Islamic indices. The study suggests that the Malaysian Islamic stock markets can be a profitable opportunity for both local and international investors who desire to diversify their Islamic investment portfolios.

Based on the earlier research studies, our analysis gives another perspective in the asset allocation framework, by investigating whether Islamic stock indices can provide more diversification benefits relative to their conventional counterparts. Using the vine-copula model based on GARCH-GJR model with POT approach, in order to calculate VaR and CVaR, and to construct the optimal portfolio.

III. VINE COPULAS

Vine copulas are originally presented by Joe [10], and examined in more detail by Bedford and Cooke [11]. Vines are a flexible graphical model for characterizes multivariate copulas built up through a cascade of bivariate copulas, so-called pair-copulas. In regular vines, Kurowicka and Cooke [28] presented two particular cases: the canonical vine (C-vine) and the drawable vine (D-vine). Figs. 1 and 2 show the specifications corresponding to a five-dimensional C-vine and a five-dimensional D-vine, respectively:
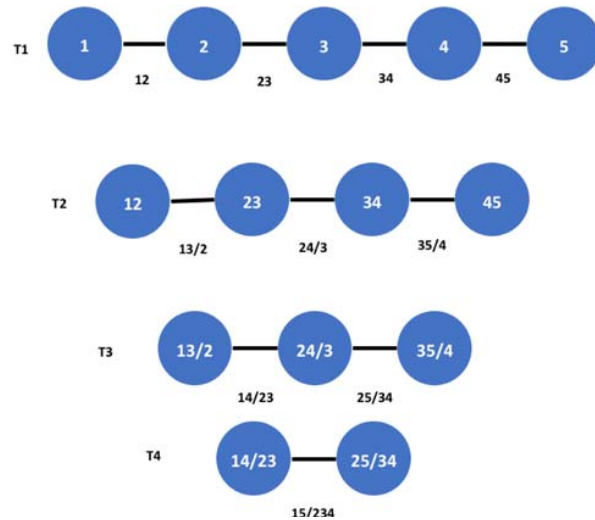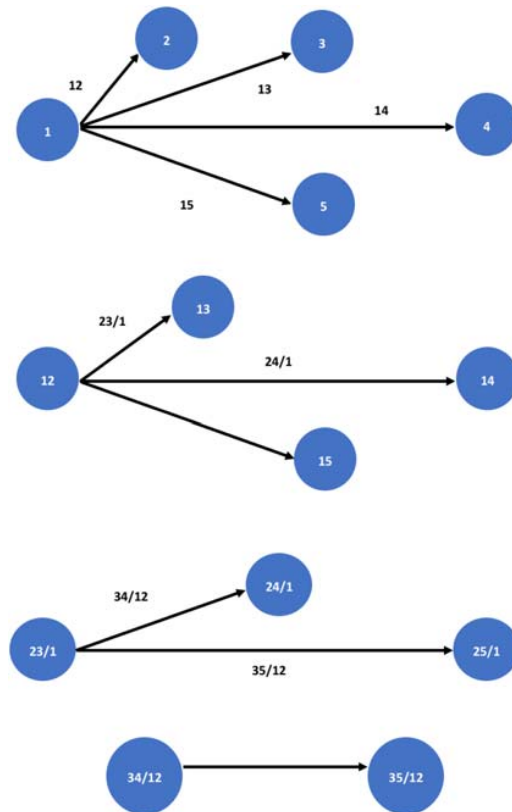

Fig. 1 Five-dimensional C-vine

Fig. 2 Five-dimensional D-vine

Bedford and Cooke [11] illustrate the decomposition of a multivariate density for the C-vine as follows:

$$F(x_1, \ldots, x_d) = \prod_{k}^{d} f(x_k) \prod_{j=1}^{d-1} \prod_{i=1}^{d-j} C_{j,j+1|1} \quad (1)$$
$$\left(F(x_j | x_1, \ldots, x_{j-1}), F(x_{j+1} | x_1, \ldots, x_{j-1}) \right).$$

For the D-vine is written as follows:

$$F(x_1, \ldots, x_d) = \prod_k^d f(x_k) \prod_{j=1}^{d-1} \prod_{i=1}^{d-j} C_{i,i+j|i+1} \quad (2)$$
$$(F(x_i|x_{i+1}, \ldots, x_{i+j-1}), F(x_{i+j}|x_{i+1}, \ldots, x_{i+j-1})$$

In the pair copula construction, $C_{x\upsilon_j|\upsilon_{-j}}$, there are a couple of marginal conditional distributions $F(x|\upsilon)$. From (1) and (2), according to [12] for every j,

$$h(x|\boldsymbol{\upsilon}, \boldsymbol{\theta}) := F(x|\boldsymbol{\upsilon}) \frac{\partial C_{x\upsilon_j|\upsilon_{-j}} \cdot (F(x|\upsilon_{-j}), F(\upsilon_j|\upsilon_{-j})|\boldsymbol{\theta})}{\partial F(\upsilon_j|\upsilon_{-j})} \quad (3)$$

The second parameter of $h(.)$ specifies the conditioning variable. θ indicates the set of parameters for the copula of the joint distribution function of x and υ.

It is feasible to estimate parameters of a multivariate density for a C-vine and D-vine copula using the maximum likelihood method. The log-likelihood for a C-vine can be written as:

$$\ell(\theta_{CV}|\mu) = \sum_{j=1}^{n-1} \sum_{i=1}^{n-j} \sum_{t=1}^{T} \log \left[\begin{pmatrix} C_{j,j+i|1,\ldots,j-1} \\ F(x_{j,t}|x_{1,t}, \ldots, x_{j-1,t}), \\ (F(x_{j+i,t}|x_{1,t}, \ldots, x_{j-1,t}) \end{pmatrix} \right] \quad (4)$$

In regards to D-vine, the log-likelihood can be written as:

$$\ell(\theta_{DV}|\mu) = \sum_{k=1}^{N} \sum_{i=1}^{d-1} \sum_{j=1}^{d-i} \log \left[\begin{pmatrix} C_{j,j+i|(j+1):(j+i-1)} \\ F_{j|(j+1):(j+i-1)}, \\ F_{j+i|(j+1):(j+i-1)}|\theta_{j,j+i|(j+1):(j+i-1)} \end{pmatrix} \right] \quad (5)$$

Estimating the parameters of (4) and (5), we must first estimate the value of the parameters of the copula through an iterative method tree-by-tree for *every* pair copula. After, we should use them as the starting values of parameters for maximization of the log-likelihood in order to obtain the final set of values parameters of the copulas [12].

IV. EXTREME VALUE THEORY

EVT admits two main branches: the theory of the extreme values generalized (GEV) and the approach Peaks over Threshold (POT). In this paper, we use the POT model, which has proven to exploit more information than GEV models from a data set of extreme movements [29].

The POT method is based on the Generalized Pareto Distribution (GPD) presented by Pickands [30]. The GPD estimation requires two steps: the choice of the suitable threshold μ and the parameter estimations for ξ (the shape parameter) and σ (the scale parameter), which can be done using maximum likelihood method. The Peak-Over-Threshold method is usually based on the approximation of the laws of the excess beyond a particular threshold μ, by the Pareto Generalizing law (GPD).

Balkema and de Haan [31], and Pickands [30] indicate that for an appropriately high threshold μ, F(μ) can be approximated by GPD, which is written as:

$$G(y) = \begin{cases} \left(1 + \frac{\xi y}{\sigma}\right)^{-\frac{1}{\xi}}, \text{if } \xi \neq 0 \\ 1 - e^{-y/\sigma}, \text{if } \xi = 0 \end{cases} \quad (6)$$

For y∈[0, $(x_F - \mu)$] if ξ ≥ 0 and y∈ $\left[0, - \left(\sigma/\xi\right)\right]$ if ξ < 0. ξ represent the shape parameter and σ represent the scale parameter for GPD.

Asset returns have frequently non-normal distributions, leptokurtosis, *Volatility clustering*, heavy tails and leverage effect. As a result, it remains important to fit the asset *returns* by a univariate GJR-GARCH (1,1) model to consider the asymmetric responses to negative returns for volatility clustering. Specifically, we estimate the following conditional variance equation:

$$y_{i,t} = \mu_i + \varepsilon_{i,t} \quad (7)$$

$$\varepsilon_{i,t} = \sigma_{i,t} + z_{i,t} \; z_{i,t} \sim i.i.d \quad (8)$$

$$\sigma_t^2 = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2 + \gamma_i \psi \left[\varepsilon_{i,t-1} < 0\right] \varepsilon_{i,t-1}^2 \quad (9)$$

where, ω_i, α_i, $\beta_i \gamma_i$ are parameters, γ_i is the leverage coefficient and $z_{i,t}$ is the innovation process, that must be independently and identically distributed. In the empirical research, we suppose $z_{i,t}$ follows student t distribution.

In the next step, for the marginal asset return distributions, separate GP models fit the lower as well as the upper distribution tails.

$$F_i(Z_i) \begin{cases} \frac{N_{\mu_i^L}}{N}\left(1 + \xi_i^L \frac{\mu_i^L - Z_i}{\beta_i^L}\right)^{-\frac{1}{\xi_i^L}} Z_i < \mu_i^L \\ \varphi(Z_i) \; \mu_i^L < Z_i < \mu_i^R \\ 1 - \frac{N_{\mu_i^R}}{N}\left(1 + \xi_i^R \frac{Z_i - \mu_i^R}{\beta_i^R}\right)^{-\frac{1}{\xi_i^R}} Z_i > \mu_i^R \end{cases} \quad (10)$$

V. CVAR OPTIMIZATION

CVaR is the worst loss expected beyond the VaR at a given confidence level. Rockafellar and Uryasev [13] defined CVaR as:

$$CVaR_\beta(y) = E\left[-y / -y \geq VaR_\beta(y)\right] \quad (11)$$

where y and β are, respectively, the returns of a portfolio and the confidence level. $VaR_\beta(y)$ is the VaR at the β confidence level; however, $CVaR_\beta(y)$ represents the conditional expected losses of the portfolio at the β confidence level. This equation is used in order to present the probability that corresponds to high losses beyond the threshold $VaR_\beta(y)$.

Rockafellar and Uryasev [13] suggested an approach where CVaR is reformulated as a minimizer of the auxiliary function F_α. Their approach is principally pertinent when Monte Carlo simulations express the probability density function, and therefore, the portfolio selection problem can be resolved as a linear programming problem. For a more detailed description

of the optimization problem refer to Rockafellar and Uryasev [13] or Meucci [32]. The resulting optimization problem takes the form:

$$min\ \alpha + \frac{1}{(1-\beta)q} \sum_{k=1}^{q} z^k \qquad (12a)$$

Under the following constraints:

$$X^T Y^k + \alpha + \mu^k \geq 0 \qquad (12b)$$

$$\mu^k \geq 0 \qquad (12c)$$

$$\frac{1}{q} X^T \sum_{k=1}^{q} Y^k \geq \rho \qquad (12d)$$

$$\sum_{i=1}^{n} x_i = 1 \qquad (12e)$$

$$x \geq 0 \qquad (12f)$$

where ρ return is required by investors and z^k can be obtained by applying Monte Carlo simulations by using pair-copula-GARCH-POT model.

VI. EMPIRICAL STUDIES AND RESULTS

A. Data Description

In this article, we select four indexes and their counterpart's in the Shariah-compliant stocks from the Dow Jones stock index universe and for stocks in four principal regions: the Asia pacific, Emergent country, Europe and the United States.

The Dow Jones Islamic Market measures the performance of a global universe of investable equities subject to screening processes in order to remains in compliance with the principles of the Shariah. The companies in this index are screened based in two sets of screens, the first screens exclude any companies engaged in activities which include the following lines of activities: alcohol, pork-related products,

conventional financial services (banking, insurance, etc.), entertainment, tobacco, weapons and defense. A second set of screens utilizes financial ratios to exclude companies whose business includes debt and interest income level. The use of the Dow Jones Islamic Market (DJIM) is due to the fact that it is the most extensively used and most comprehensive representative of Islamic stocks. The data spans the period of April 14, 2007 to July 1, 2016. We use daily closing prices of all the Dow Jones stocks. All data series returns are calculated as the natural log of the closing index values between day t and t-1. We briefly present an overview summary of the statistics. The descriptive statistics for each return are presented in Table I. All series are around zero mean, exhibit negative skewness which indicates an asymmetric distribution (except for the DJIEU index), and show excess kurtosis, which denotes a fat-tailed distribution. The Jarque–Bera statistic shows that all series return is not normally distributed. The Ljung–Box statistic indicates no significant evidence of serial correlation. Finally, the ARCH-LM statistic reveals strong evidence of heteroskedasticity.

B. Marginal Models Estimation

To use C-Vine-EVT for each return series, we apply a GJR-GARCH(1,1) model to treat heteroskedasticity in asset returns. This step gives a filtered residuals and conditional variances from the returns of each index. Then, the POT method is applied to model lower and upper tails of the underlying distributions. Table II summarizes the results of estimated parameters of tails from GPD for all return series. Of lower tails, the DJIEMG index, the DJIMKT index and the DJG index exhibit strong heavy lower tails with $\hat{\epsilon}$ equal to 0.00564, 0.00669 and 0.08370, respectively.

Of upper tails, the strongest heavy one is the W5DOW index, which means that a daily rise of the investor's capital is followed by a greater risk of extreme gains.

TABLE I
DESCRIPTIVE STATISTICS OF RETURNS

|	W5DOW	DJIAP	E1DOW	P1DOW	DJIEU	DJIEMG	DJIMKT	DJG
Mean	0.059	0.034	0.029	0.035	0.031	0.044	0.029	0.026
Standard deviation	1.395	1.2851	1.489	1.305	1.457	1.375	1.099	1.176
Skewness	-0.578	-0.547	-0.060	-0.461	0.023	-0.484	-0.443	-1.534
Kurtosis	10.573	9.541	11.879	9.022	12.889	10.360	14.299	28.658
JarqueBera statistic	4733.761[*]	3547.727[*]	6358.931[*]	2994.607[*]	2994.607[*]	4445.724[*]	10358.97[*]	53864.69[*]

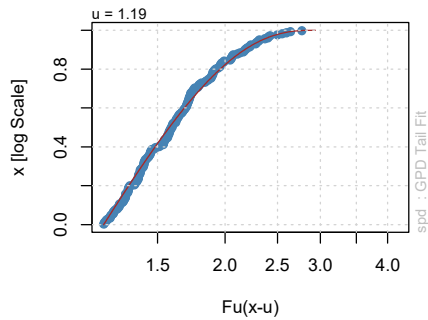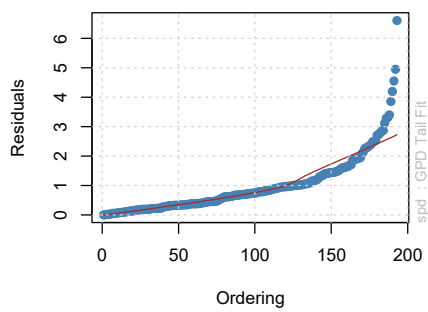Note: * denotes significance at the 1%

TABLE II
ESTIMATED TAILS FROM GPD.

| Lower tails	W5DOW	DJIAP	E1DOW	P1DOW	DJIEU	DJIEMG	DJIMKT	DJG
μ	-1.27758	-1.29026	-1.26591	-1.30868	-1.23851	-1.3257	-1.28479	-1.24178
$\hat{\epsilon}$	-0.07610	-0.06544	-0.01895	-0.06028	-0.11759	0.00564	0.00669	0.08370
$\hat{\beta}$	0.66244	0.69376	0.60652	0.65199	0.70489	0.55728	0.60866	0.59862
Upper tails								
μ	1.1772	1.21284	1.18993	1.1917	1.22415	1.16203	1.15754	1.12335
$\hat{\epsilon}$	0.02865	-0.25975	-0.03008	-0.35953	-0.01254	-0.03166	-0.15227	-0.07816
$\hat{\beta}$	0.47133	0.52767	0.51460	0.62743	0.48906	0.51635	0.60101	0.59330

GPD Fit Assessment-Upper Tail

Fig. 3.1.a Excess Distribution
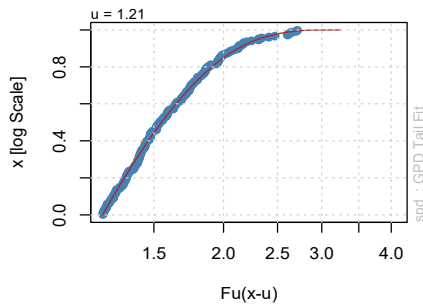
Fig. 3.1.b Tail of Underlying Distribution
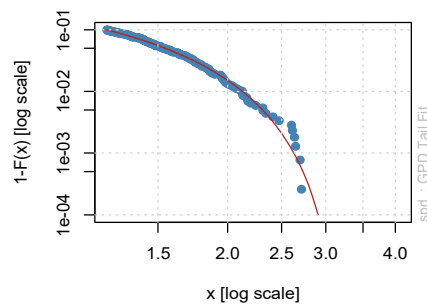
Fig. 3.1.c Scatterplot of Residuals
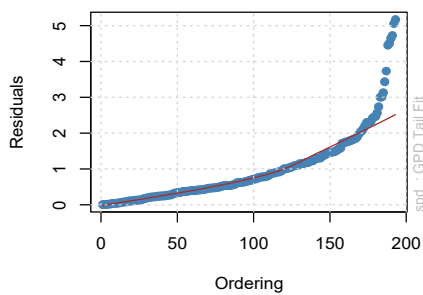
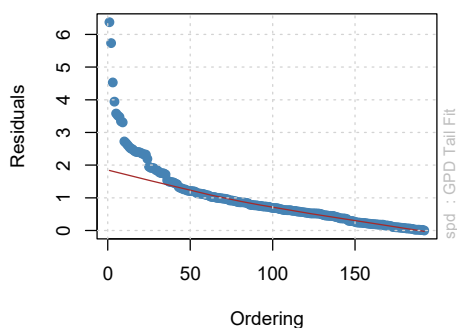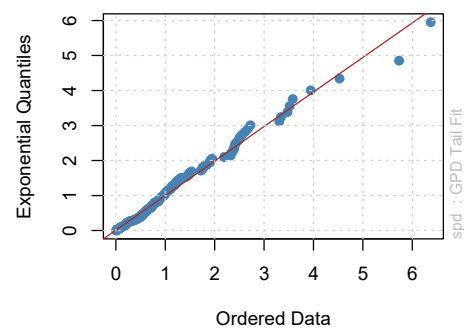**Fig. 3.1.d QQ-Plot of Residuals**
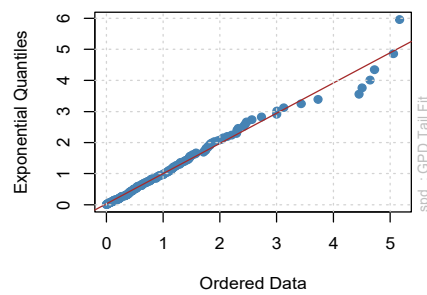
GPD Fit Assessment-Upper Tail

Fig. 3.2.a Excess Distribution
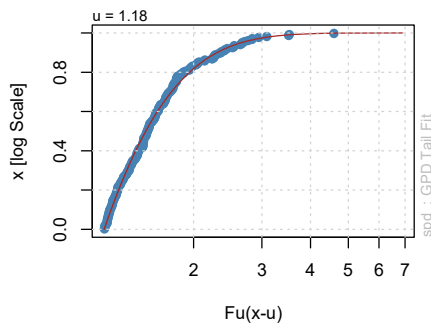
Fig. 3.2.b Tail of Underlying Distribution
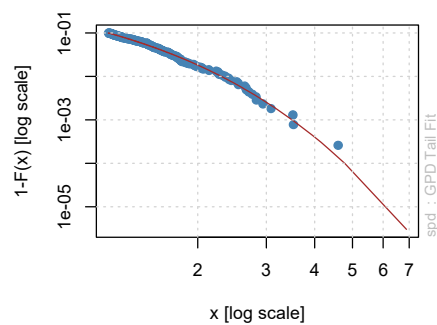
Fig. 3.2.c Scatterplot of Residuals
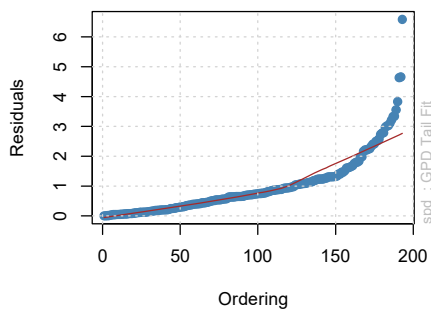
Fig. 3.2.d QQ-Plot of Residuals
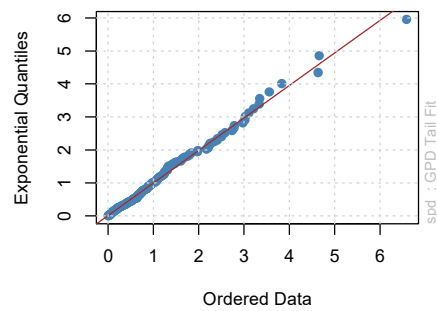
GPD Fit Assessment-Lower Tail

Fig. 3.3.a Excess Distribution
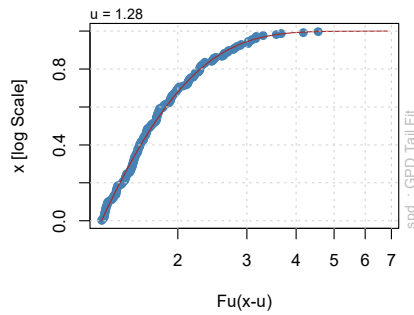
Fig. 3.3.b Tail of Underlying Distribution
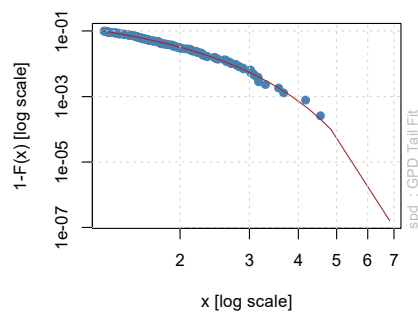
Fig. 3.3.c Scatterplot of Residuals
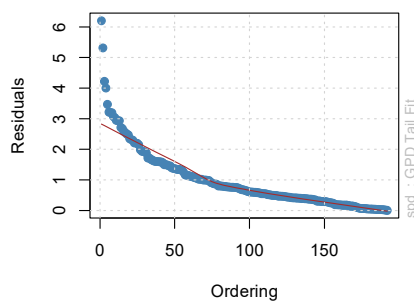
Fig. 3.3.d QQ-Plot of Residuals
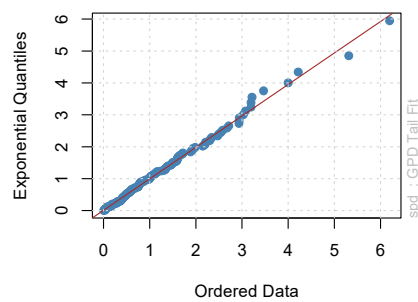
GPD Fit Assessment-Lower Tail

Fig. 3.4.a   Excess Distribution
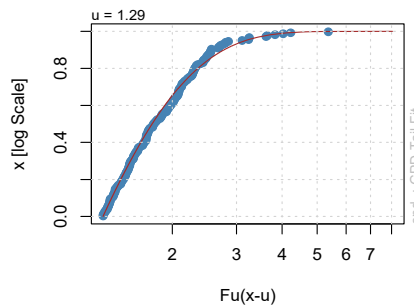
Fig. 3.4.b   Tail of Underlying Distribution
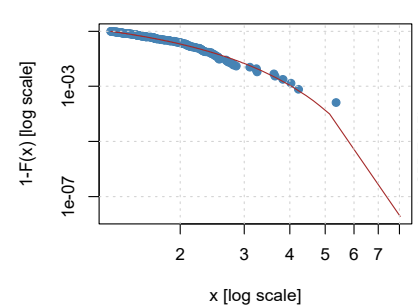
Fig. 3.4.c   Scatterplot of Residuals
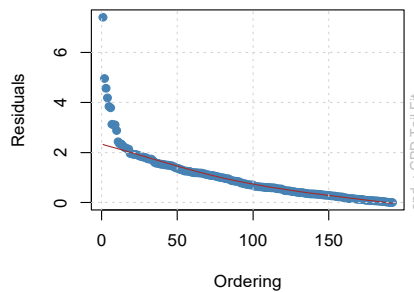
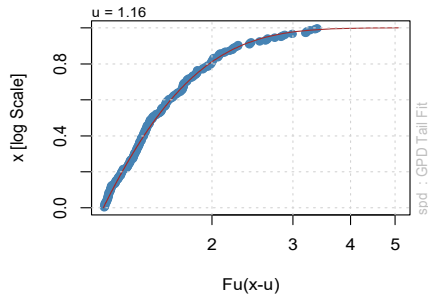**Fig. 3.4.d   QQ-Plot of Residuals**
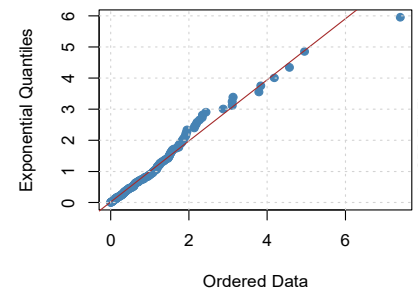
GPD Fit Assessment-Upper Tail

Fig. 3.5.a   Excess Distribution
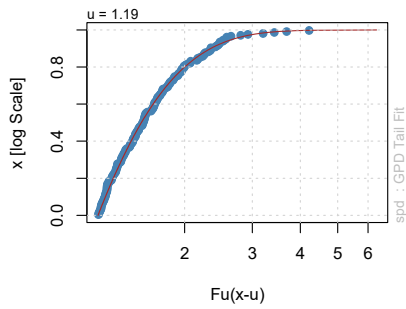
Fig. 3.5.b   Tail of Underlying Distribution
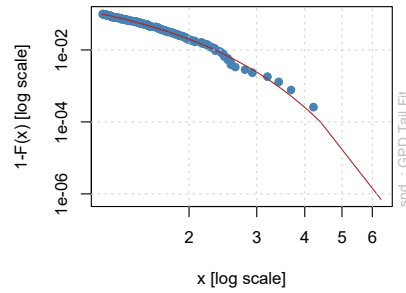
Fig. 3.5.c   Scatterplot of Residuals
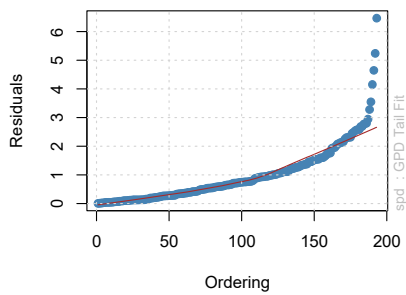
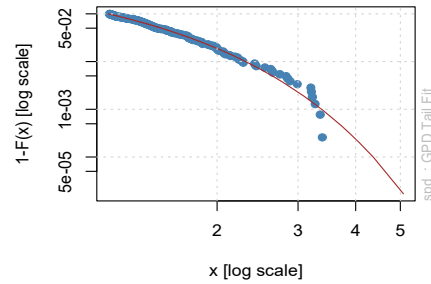**Fig. 3.5.d   QQ-Plot of Residuals**
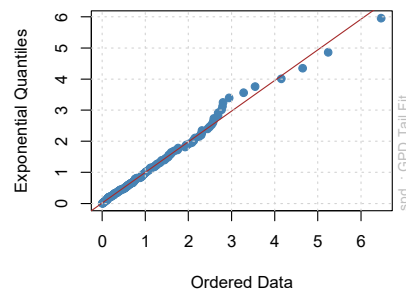
GPD Fit Assessment-Lower Tail

Fig. 3.6.a   Excess Distribution
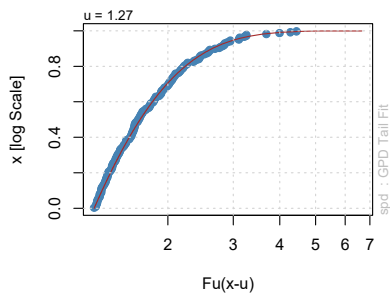
Fig. 3.6.b   Tail of Underlying Distribution
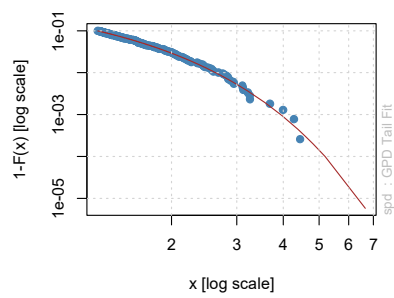
Fig. 3.6.c   Scatterplot of Residuals
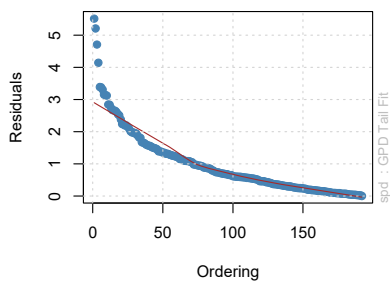
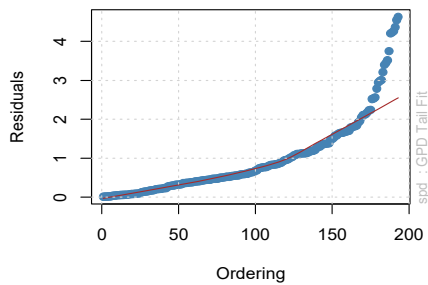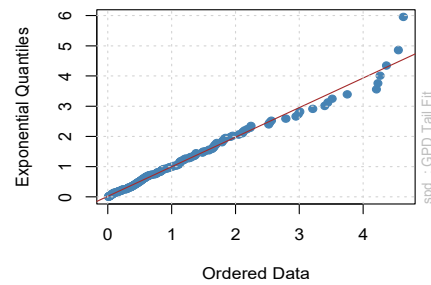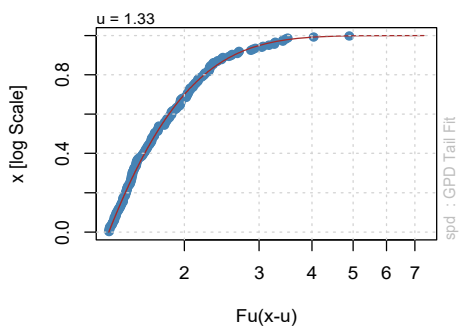**Fig. 3.6.d   QQ-Plot of Residuals**
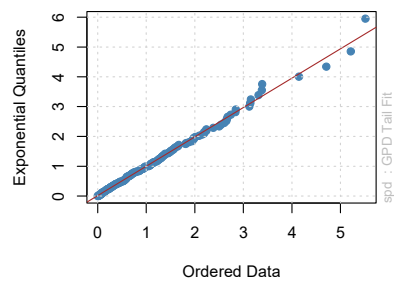
GPD Fit Assessment-Upper Tail

Fig. 3.7.a Excess Distribution   **Fig. 3.7.b   Tail of Underlying Distribution**
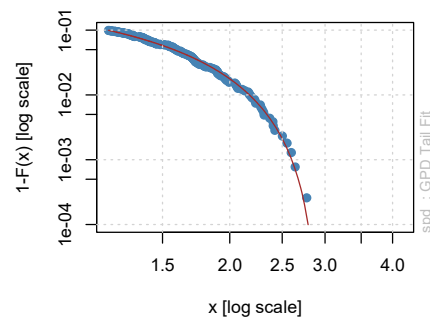
Fig. 3.7.c Scatterplot of Residuals   **Fig. 3.7.d   QQ-Plot of Residuals**
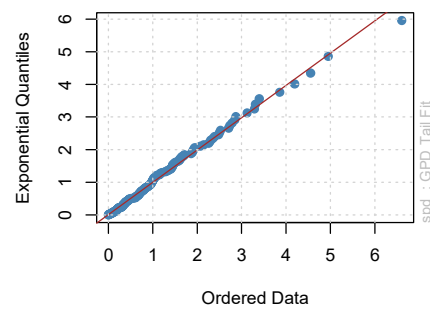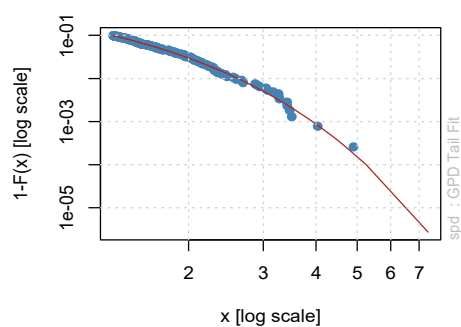

GPD Fit Assessment-Lower Tail

Fig. 3.8.a Excess Distribution   **Fig. 3.8.b   Tail of Underlying Distribution**
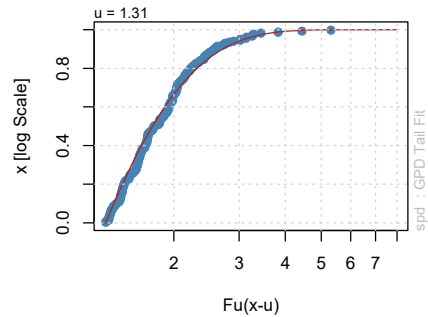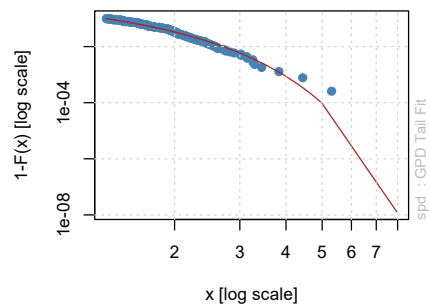
Fig. 3.8.c Scatterplot of Residuals   **Fig. 3.8.d   QQ-Plot of Residuals**
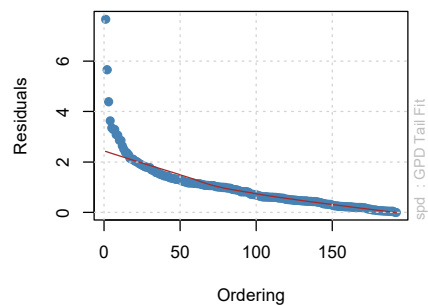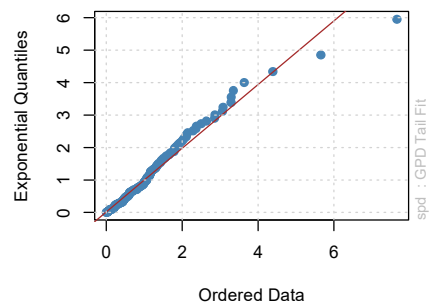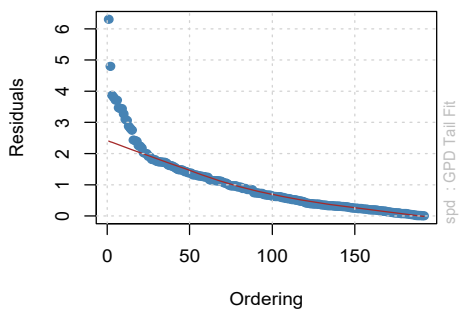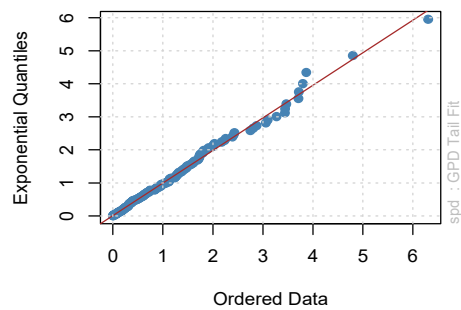
GPD Fit Assessment-Upper Tail

Fig. 3.9.a   Excess Distribution
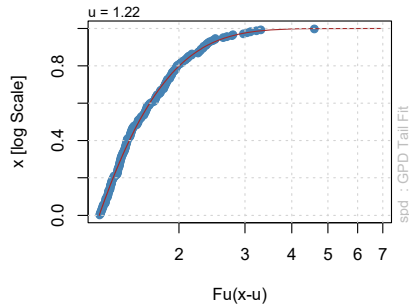
Fig. 3.9.b   Tail of Underlying Distribution
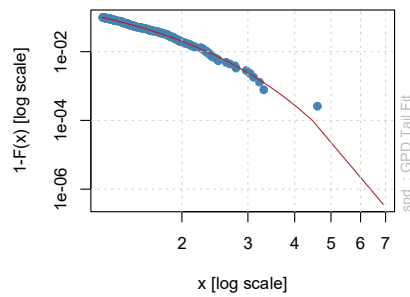
Fig. 3.9.c   Scatterplot of Residuals
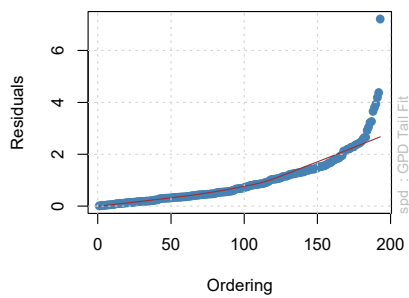
Fig. 3.9.d   QQ-Plot of Residuals
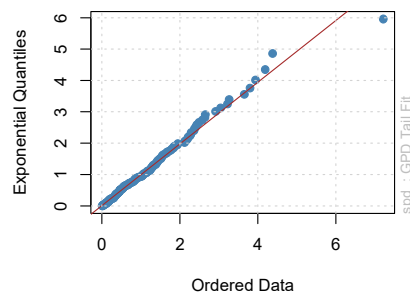
GPD Fit Assessment-Lower Tail

Fig. 3.10.a   Excess Distribution
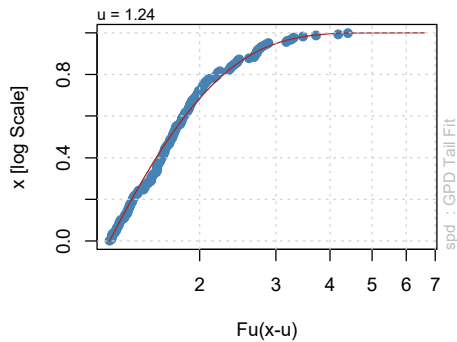
Fig. 3.10.b   Tail of Underlying Distribution
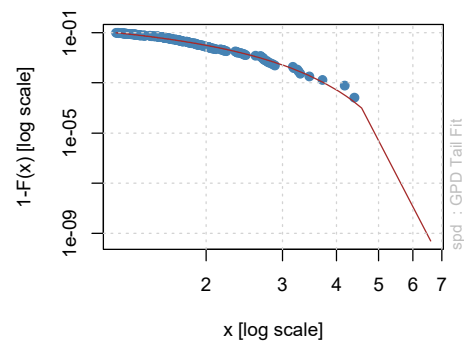
Fig. 3.10.c   Scatterplot of Residuals

Fig. 3.10.d   QQ-Plot of Residuals

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GPD Fit Assessment-Upper Tail

Fig. 3.11.a   Excess Distribution

Fig. 3.11.b   Tail of Underlying Distribution

Fig. 3.11.c   Scatterplot of Residuals

Fig. 3.11.d   QQ-Plot of Residuals

GPD Fit Assessment-Lower Tail

Fig. 3.12.a   Excess Distribution

Fig. 3.12.b   Tail of Underlying Distribution

Fig. 3.12.c   Scatterplot of Residuals

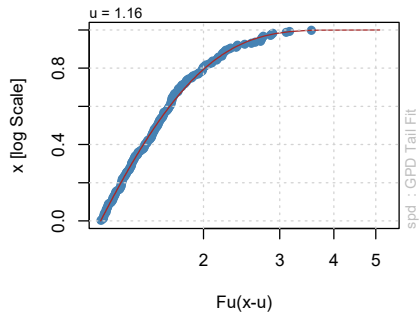**Fig. 3.12.d   QQ-Plot of Residuals**

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GPD Fit Assessment-Upper Tail

Fig. 3.13.a Excess Distribution

Fig. 3.13.b Tail of Underlying Distribution
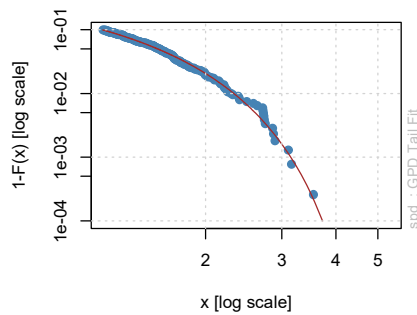
Fig. 3.13.c Scatterplot of Residuals
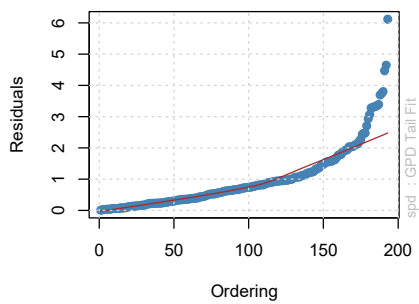
Fig. 3.13.d QQ-Plot of Residuals

GPD Fit Assessment-Lower Tail

Fig. 3.14.a Excess Distribution

Fig. 3.14.b Tail of Underlying Distribution

Fig. 3.14.c Scatterplot of Residuals

Fig. 3.14.d QQ-Plot of Residuals

GPD Fit Assessment-Upper Tail

Fig. 3.15.a Excess Distribution   **Fig. 3.15.b   Tail of Underlying Distribution**
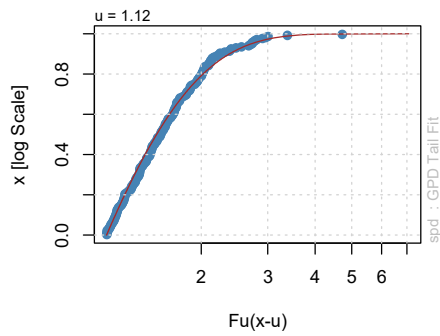
Fig. 3.15.c Scatterplot of Residuals   **Fig. 3.15.d   QQ-Plot of Residuals**
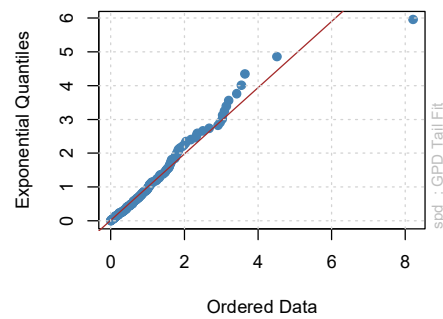
GPD Fit Assessment-Lower Tail

Fig. 3.16.a Excess Distribution   **Fig. 3.16.b   Tail of Underlying Distribution**
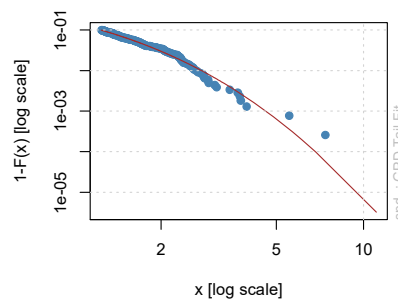
Fig. 3.16.c Scatterplot of Residuals   **Fig. 3.16.d   QQ-Plot of Residuals**
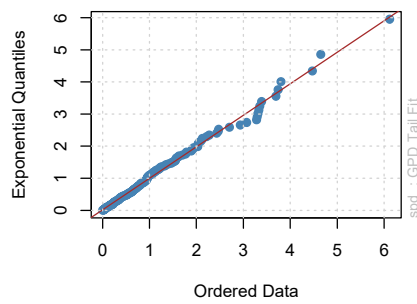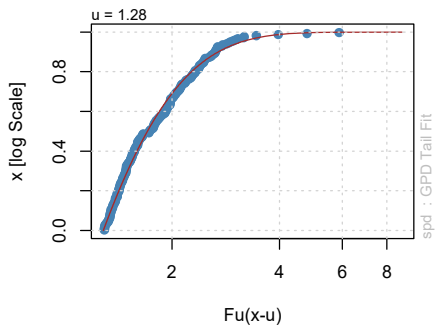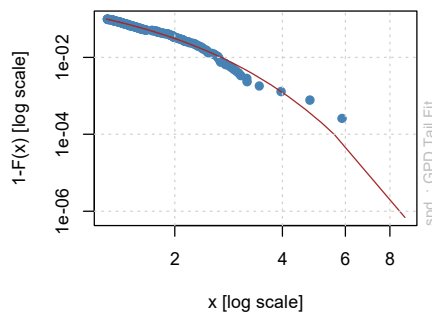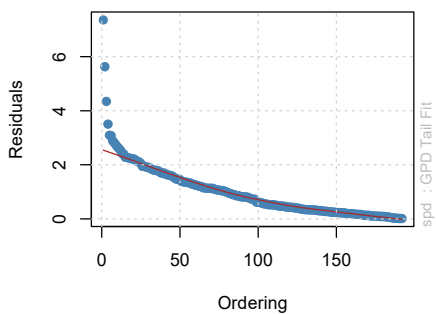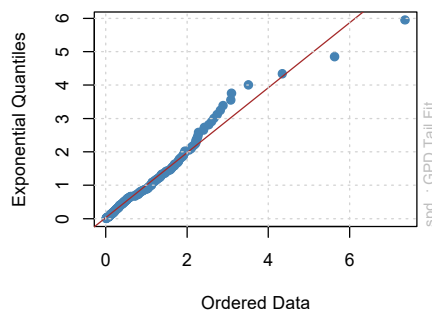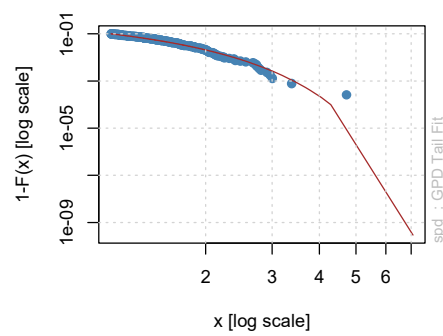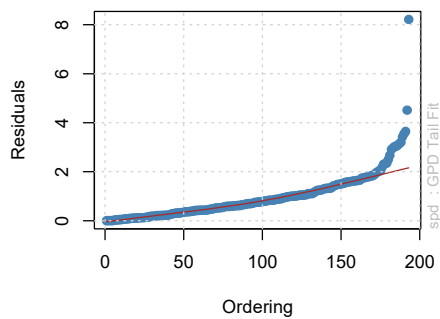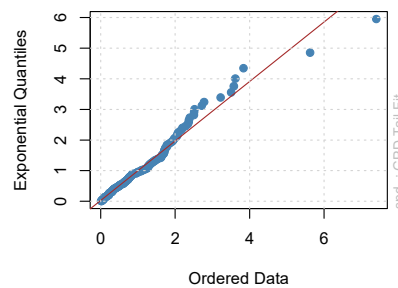
Fig. 3 The q-q plots of excesses over the thresholds against the quantiles of the fitted GPD models

In order to assess the fit of the GPD model in the tails of filtered residuals, we illustrate in Fig. 3, the q-q plots of excesses over the thresholds against the quantiles of the fitted GPD models. Fig. 3 presents marginal distributions curves. In the left column, we show the lower tail fit, while in the right, we show the upper tail of filtered residuals. The curves on the right part of each plot show a good fit in the upper tails as well as in the lower tails of the distributions. These curves fitted the tails of all return series distributions very well. Both Table II and Fig. 3 confirm that the fitted GPD models sufficiently specify filtered residuals of all return series.

C. Pair Copula Decomposition and Parameter Estimation

This research uses the C-vine structure selection criterion described by [33], in order to select a suitable C-vine copula model. Table III illustrates the empirical Kendall's correlation matrix of the copula data and the sum total of their values, noted by $\widehat{S}_l$. Table III shows that the W5DOW index has the maximum sum of the absolute values, $\widehat{S}_l$, and hence, it is positioned as the pilot variable in level 1 of the C-vine structure. For level 2, we calculated the empirical Kendall's matrix of return series, conditioned on W5DOW, and denoted by i* and the sum of the absolute entries in every line. We find that the DJIAP index has the maximum sum of the absolute

values, $\widehat{S}_l$, and hence, it is placed as the pilot variable in level 2 of the C-vine structure.

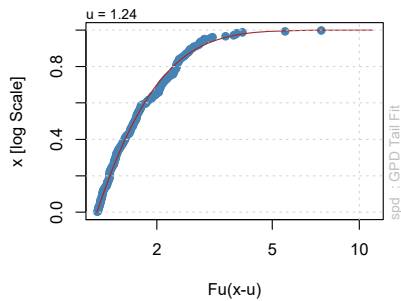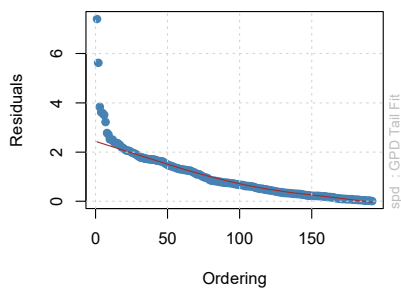We can deduce that the family of every bivariate copula is chosen by the AIC criteria. With respect to copula selection, the majority of the selected copula families correspond to the t copula. This finding concurs with the empirical findings of [34].

In particular, all selected copula families in level 1 belong to the t copula. Student copulas have right and left tail dependencies, which are symmetrical. Thus, coefficients of the upper tail and lower tail are similar. This symmetry suggests that the risk of occurrence of extreme movements is the same in the case of a bull market or a bear market. From the sequential selection, we note that the dependence between the emergent market and the other markets is the strongest and that the dependence between the US market and the other markets is the weakest. We also note that the Emerging market co-movements are affected, to a large extent, by the Islamic market index of the Asia pacific, Emerging markets and Europe.

Next, we will use all these values to plot efficient portfolio and identify the optimal portfolio for the best-expected returns with respect to lowest loss.

TABLE III
ESTIMATED EMPIRICAL KENDALL'S τ MATRIX WITH THE SUM OVER THE ABSOLUTE ENTRIES OF EACH ROW OF INNOVATIONS

	W5DOW	DJIAP	E1DOW	P1DOW	DJIEU	DJIEMG	DJIMKT	DJG	$\widehat{S}_l$
W5DOW	1.000000000	0.56762413	0.18856319	0.49862103	0.46573675	0.807754459	0.52511314	0.006541538	3.867224041
DJIAP	0.567624127	1.00000000	0.25634635	0.78219607	0.31221864	0.580859679	0.35684058	-0.013029450	3.734185408
E1DOW	0.188563194	0.25634635	1.00000000	0.24659121	0.21501490	0.184552542	0.16322312	-0.037140334	2.24000044
P1DOW	0.498621034	0.78219607	0.24659121	1.00000000	0.25221454	0.487085387	0.31000422	-0.014740016	3.462925237
DJIEU	0.465736750	0.31221864	0.21501490	0.25221454	1.00000000	0.444043321	0.58172050	0.029830761	3.285578833
DJIEMG	0.807754459	0.58085968	0.18455254	0.48708539	0.44404332	1.000000000	0.49074824	0.001177214	3.813297883
DJIMKT	0.525113137	0.35684058	0.16322312	0.31000422	0.58172050	0.490748242	1.000000000	0.021128960	3.206111866
DJG	0.006541538	-0.01302945	-0.03714033	-0.01474002	0.02983076.	0.001177214	0.02112896.	1.000000000	1.078471822

D. Portfolio Risk Analysis

We consider an equally weighted portfolio of theses index, Table IV presents the estimated VaR with expected shortfall or CVaR at level of 0.1%, 5%, and 10%, respectively. In period t+1, the estimated CVaR were higher than VaR and converged to -6.949%, -8.525% and -8.525% at the 1%, 5% and 10% level, respectively.

Next, we simulate a set of 10,000 samples applying the Monte Carlo methods, as previously discussed. Finally, we will use CVaR minimization approach at the confidence level of β=95%, in order to get the efficient frontier of an optimal weighted portfolio.

In Fig. 4, each plot in this figure illustrates each portfolio. The plot grants that low risk provide low return. It means that the more return, the more risky it will be. As a recommendation, for risk-aversion investors, they can better select the low return with low risk while risk-lover investors

tend to take high risk, but also for a highest return.

Table V showed few results of optimal weight and the expected returns in the frontier. As can be seen in Plot 1, the optimal tangency portfolio is consists of only the DJIMKT stock. The minimum risk portfolio with 0% target return contains eights stocks: W5DOW 2.15%, DJIAP 1.34%, E1DOW 0%, P1DOW 0.54%, DJIEU 1.11%, DJIEMG 2.56%, DJIMKT 72.73%, DJG 19.57%. However, in general, DJIMKT portfolio shares most of the investment weight, where one should invest the most.

TABLE IV
VAR AND CVAR FOR EQUALLY WEIGHTED PORTFOLIOS.

Portfolio	1%	5%	10%
VaR	-5.742%	-7.721%	-9.511%
CVaR	-6.949%	-8.525%	-9.801%
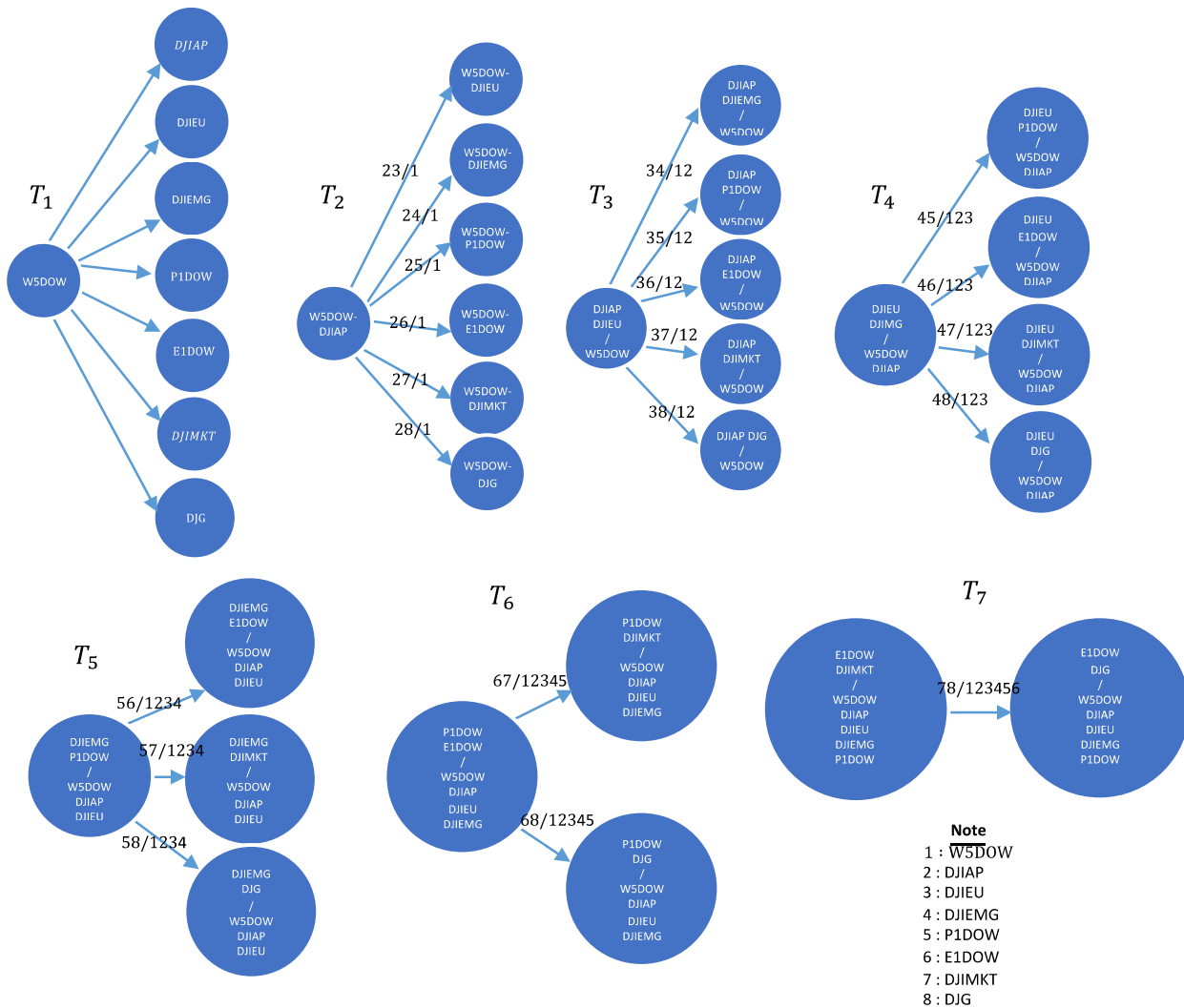
Fig. 4 The efficient frontiers with *CVaR* constraint

TABLE V
OPTIMAL WEIGHTED PORTFOLIOS WITH MINIMUM RISK (ES 5%)

Portfolios	W5DOW	DJIAP	E1DOW	P1DOW	DJIEU	DJIEMG	DJIMKT	DJG	Returns
1	0.047	0.144	0.000	0.043	0.000	0.075	0.462	0.227	1.718%
2	0.041	0.083	0.000	0.000	0.000	0.046	0.476	0.353	5.673%
3	0.021	0.013	0.000	0.005	0.011	0.025	0.727	0.195	1.135%
4	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	2.317%

VII. CONCLUSION

In this paper, we examined the optimum portfolio of Islamic index return of Asia–Pacific, Europe, USA, and their conventional counterpart's markets with C-vine copula based on VaR and Expected Shortfall applying with GARCH-GJR-EVT-Copula model. The study starts with the GJR-GARCH model, next the POT method is applied to estimate the tail distribution of the filtered residual. Then, the dependence structure between all simulated residuals is captured by C-vine copula model. The estimated VaR and ES (CVaR) are calculated based on 10%, 5%, 1% levels, respectively. Finally, we obtained the optimal portfolio weight, which suggests to invest the largest proportion in Islamic-conventional US Market index pairs, which share most of the investment proportion. The smaller share investment proportion is for Islamic-conventional emerging markets index pairs, followed by Islamic-conventional Asia pacific markets index pairs, and finally, we find Islamic-conventional Europe markets index pairs.

This study provides a major contribution relating to the Islamic market index and reveals that practitioners may implement an international diversification strategy by further integrating into their portfolios Islamic financial assets and conceiving optimal portfolio model accordingly.

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