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Power and Wear Reduction Using Composite Links of Crank-Rocker Mechanism with Optimum Transmission Angle

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Abstract**—**Reducing energy consumption became the major concern for all countries of the world during the recent decades. In general, power saving is currently the nominal goal of most industrial countries. It is well known that fossil fuels are the main pillar of development of world countries. Unfortunately, the increased rate of fossil fuel consumption will lead to serious problems caused by an expected depletion of fuels. Moreover, dangerous gases and vapors emission lead to severe environmental problems during fuel burning. Consequently, most engineering sectors especially the mechanical sectors are looking for improving any machine accompanied by reducing its energy consumption. Crank-Rocker planar mechanism is the most applied in mechanical systems. Besides, it is one of the most significant parts of the machines for obtaining the oscillatory motion. The transmission angle of this mechanism can be considered as an optimum value when its extreme values are equally varied around 90°. In addition, the transmission angle plays an important role in decreasing the required driving power and improving the dynamic properties of the mechanism. Hence, appropriate selection of mechanism links lengthens, which assures optimum transmission angle leads to decreasing the driving power. Moreover, mechanism's links manufactured from composite materials afford link's lightweight, which decreases the required driving torque. Furthermore, wear and corrosion problems can be treated through using composite links instead of using metal ones. This paper is dealing with improving the performance of crank-rocker mechanism using composite links due to their flexural elastic modulus values and stiffness in addition to high damping of composite materials.

Keywords**—**Composite material, crank-rocker mechanism, transmission angle, design techniques, power saving.

I. Introduction

THE four bar planar mechanism is the most common part of machines in mechanical systems. Crank-Rocker (CR) mechanism is the most famous type of planar mechanisms for producing the oscillatory motion. Many of research works were dealing with the CR mechanism's design over the last recent decades. 50 years ago, the design of CR planar mechanism considering unit time ratio and minimum transmission angle was presented in [1], while recently CR mechanism synthesis dealing with the human knee exoskeleton was proposed in [2].

The transmission angle of CR mechanism plays an important role in improving its dynamic characteristics. Furthermore, the appropriate transmission angles have significant effects for driving the CR mechanism with smooth motion. Transmission angle is the smaller angle confined between the coupler link and the follower link. Moreover, transmission angle can be defined as the angle between the two velocities vectors of coupler driving link and rocker link at the connection pivot as discussed in [3]-[8].

Actually, all mechanism designers are looking for low driving torque fluctuation, appropriate transmission angle, smooth transmission motion and low pressing forces in bearing in addition to low vibrations. Thus, optimal transmission angle usage can treat most of the CR mechanism's problems as discussed in [5]. Very large or very small values of transmission angle of CR mechanism are always associated with undesirable noisy as discussed in [9]. Also, the low variation of the needed driving torque cannot be assured via large transmission angle.

Unluckily, mechanism linkages optimizing process is a complicated problem especially via considering the transmission characteristics in addition to concurrently synthesizing the mechanism's transmission angle. But fortunately, this problem may be significantly simplified via assuming a restricted range of transmission angle bounded by two specific values as mentioned [10].

It is easy to note that, both values of minimum and maximum transmission angle of CR mechanism are depending on the oscillation angle of the rocker link. Furthermore, reasonable transmission angle's variation from 90^o is significant for guarantee mechanism's smooth motion with acceptable vibration level at high speed as discussed in [5], [11]. While the large variation values of transmission angle lead to reducing the transmission's effectiveness. Besides, undesired noise and jerk can appear at high speed with CR mechanisms which have a large deviation limits of transmission angle around 90^o as discussed in [2], [12].

Selecting an appropriate mechanism's linkage associated with the reasonable small range of transmission angle is the main goal of designers, especially in the presence of high friction in joints as mentioned in [3], [5].

Many of recent research works were presented concerning with synthesizing the maximum and minimum transmission

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angles with definite values. For example, analytical synthesis for obtaining definite values of minimum and maximum transmission angles of crank rocker mechanisms dealing with the movement between two certain small positions was presented in [13]. Also, synthesizing the four bar mechanism's linkages using polynomial function generation technique for obtaining definite values of desired transmission angles limits was presented in [14]. Furthermore, four-bar mechanism synthesizing process was presented in [15], dealing with three successive coupler positions which is associated with a definite transmission angles range less than 17°. Moreover, optimizing path synthesis of crank rocker mechanisms using shape optimization technique concerning the optimal transmission angle is introduced in [16].

New optimization method for designing four-bar mechanism using new sense called mini-max was presented in [17], where mini-max is the optimum transmission angles when their limits are equally around 90°. Likewise, other research works have adopted the same mini-max sense as [18], [19]. Furthermore, definite transmission angle ranges as 35°-145° and 30°-150° are recommended in [11], [20].

Some of the recent publications are concerned with the graphical techniques for selecting the desired transmission angles of CR mechanisms. An example, design charts are presented in [21] for selecting the desired minimum transmission angle of CR mechanism. Transmission angle's selection using these charts is depending upon angle's value between the rocker dead-center positions and the two corresponding angles of the crank link through assuming the dimensions of three mechanism's linkages as in [21], [22]. Moreover, design nomogrames for directly selecting the desired transmission angle's range of CR mechanism are presented in [18]. These nomogrames are only depending upon selecting only two values. The first one is the desired rocker link's angle which is corresponding to the desired minimum transmission angle. The second one is selecting the desired range of transmission angles.

This research work is presented for improving the CR mechanism performance using mechanism's linkages manufactured from lightweight composite materials with little rates of wear and corrosion, where appropriate composite materials can be used for fabricating the mechanism's linkages for improving its performance where these materials have many advantages related to their values of stiffness and flexural elastic modules in addition to good damping especially at high speeds.

Some of the research works are dealing with the suitable composite materials usage for manufactured the mechanism's linkages discussed in [23]-[28]. The composite coupler link's dynamic characteristics were discussed in [23]. Also, the design of the four-bar composite linkages considering kinematic and dynamic balancing was discussed in [24]. Furthermore, four bar mechanism's performance evaluation was introduced in [25] considering linkages elasticity and stiffness of joints. Moreover, a rigid body modeling of composite four-bar mechanism's linkages was introduced in [26]. Besides, swing-up control for composite links of robot arms was presented in [27]. Also, composite long reach robotic arm's development was introduced in [28].

II. Analysis and Modeling

A. Transmission Angle and Its Limitations

The planar crank rocker mechanism's transmission angle (μ) is shown in Fig. 1, where crank, coupler, rocker and fixed links lengths are L_2, L_3, L_4 and L_1, respectively. Moreover, the transmission angle (μ) can be expressed as follows;

$$\mu = \cos^{-1}\left(\frac{L_4^2 + L_3^2 - L_2^2 - L_1^2 + 2L_1L_2\cos\theta_2}{2L_4L_3}\right) \tag{1}$$

where μ_{max} is the maximum value of transmission angle occurs at 180° of crank angle, while (μ_{min}) is the minimum value of transmission angle occurs at 0° of crank angle as in [14]. Maximum and minimum transmission angles of crank rocker mechanism geometry are shown in Fig. 2. Hence, both values of μ_{max} and μ_{min} can be formulated as in [14], [18], [29] as follows;

$$\mu_{max} = \cos^{-1}\left(\frac{L_4^2 + L_3^2 - L_2^2 - L_1^2 - 2L_1L_2}{2L_4L_3}\right) \tag{2}$$

$$\mu_{min} = \cos^{-1}\left(\frac{L_4^2 + L_3^2 - L_2^2 - L_1^2 + 2L_1L_2}{2L_4L_3}\right) \tag{3}$$

Also, the rocker link angle (θ_4) can be calculated as formulated in (4) as introduced in [19] related to the crank rocker mechanism geometry as shown in Fig. 3.

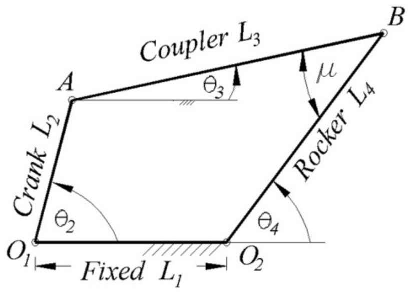

Fig. 1 Four-bar transmission angle

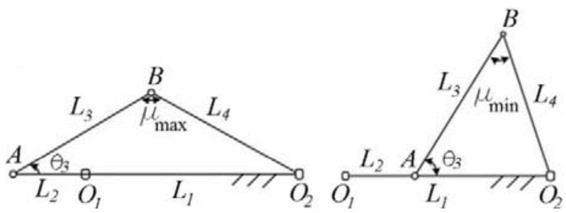

Fig. 2 Maximum and minimum transmission angles

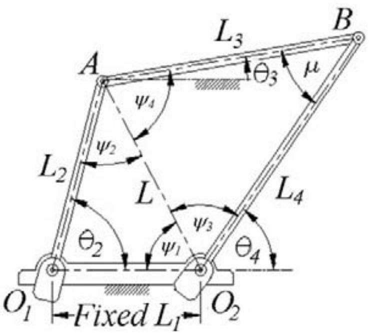

Fig. 3 Crank rocker mechanism geometry

$$\theta_4 = \begin{cases} \pi + \theta_1 - \psi_1 - \psi_3 \text{ , where } \ \theta_1 \le \theta_2 < (\pi + \theta_1) \\ \pi + \theta_1 + \psi_1 - \psi_3 \text{ , where } \ (\pi + \theta_1) \ge \theta_2 \ge (2\pi + \theta_1) \end{cases} \tag{4}$$

where (θ_1) is the fixed link's angle, the length (L) in addition to angles ($\psi_1, \psi_2, \psi_3, \psi_4$ and μ) can be formulated as follows;

$$\left.\begin{aligned} L &= \sqrt{L_1^2 + L_2^2 - 2L_1L_2\cos\theta_2} \\ \psi_1 &= \cos^{-1}\left(\frac{L_1^2 + L^2 - L_2^2}{2L_1L}\right) \\ \psi_2 &= \cos^{-1}\left(\frac{L_2^2 + L^2 - L_1^2}{2\,L_2L}\right) \\ \psi_3 &= \cos^{-1}\left(\frac{L_4^2 + L^2 - L_3^2}{2\,L_4L}\right) \\ \psi_4 &= \cos^{-1}\left(\frac{L_3^2 + L^2 - L_4^2}{2\,L_3L}\right) \\ \mu &= \cos^{-1}\left(\frac{L_3^2 + L_4^2 - L^2}{2L_3\,L_4}\right) \end{aligned}\right\} \tag{5}$$

Obviously, angular velocity ($\theta_4^{'}$) and acceleration ($\theta_4^{''}$) of rocker link are significant factors of mechanism. The angular velocity ($\theta_4^{'}$) and acceleration ($\theta_4^{''}$) of rocker link can be found by derivation of (4).

Moreover, the coupler link's angle (θ_3) can be calculated as;

$$\theta_3 = \begin{cases} \psi_2 + \psi_4 + \theta_2 - \pi \text{ , where } \ \theta_1 \le \theta_2 < (\pi + \theta_1) \\ \psi_4 + \theta_2 - \psi_2 - \pi \text{ , where } \ (\pi + \theta_1) \ge \theta_2 \ge (2\pi + \theta_1) \end{cases} \tag{6}$$

B. Design Technique for Selecting Transmission Angle Range

Understandably, the value of angle (μ) can be considered as the optimal transmission angle if this angle has mini-max values are equally around 90^o as discussed in [17]. Hence, a simple design technique for selecting the desired range of (μ) can depend on the previous sense of mini-max of the angle (μ) as introduced in [19]. Regarding this technique, the summation of μ_{max} in addition to μ_{min} is equal to (π). Thus, the summation of $\cos(\mu_{min})$ in addition to $\cos(\mu_{max})$ is equal to zero which leads to the following relation;

$$R_3^2 + R_4^2 - R_2^2 = 1 \tag{7}$$

where ratios R_2, R_3 and R_4 can be expressed as follows:

$$R_2 = L_2 / L_1 \text{ , } R_3 = L_3 / L_1 \text{ and } R_4 = L_4 / L_1 \tag{8}$$

Fortunately, this technique for selecting the range of angle (μ) will only depending upon selecting only two values. The first one is the desired angle (θ_{4i}) of rocker link which associated with the minimum transmission angle at (θ_2=0). The second one is the desired range of the transmission angle ($\mu=90^0\pm\delta$) where the angle (δ) can be selected in degrees.

Sine rule [30] can be applied for two triangles of transmission angle's limits which are shown in Fig. 2, as;

$$\frac{1 - R_2}{\sin\mu_{min}} = \frac{R_3}{\sin\theta_{4i}} = \frac{R_4}{\sin(\theta_{4i} - \mu_{min})} \tag{9}$$

Thus, the previous equation can be rewritten as two new deduced equations as;

$$R_3 = M(1 - R_2) \quad \text{where} \quad M = \frac{\sin\theta_{4i}}{\sin\mu_{min}} \tag{10}$$

$$R_4 = N(1 - R_2) \quad \text{where} \quad N = \frac{\sin(\theta_{4i} - \mu_{min})}{\sin\mu_{min}} \tag{11}$$

Equation (7) can be rewritten in new form through substituting R_3 and R_4 of the two previous equations as;

$$(M^2 + N^2 - 1)R_2^2 - 2(M^2 + N^2)R_2 + (M^2 + N^2 - 1) = 0 \tag{12}$$

Thus;

$$R_2 = \frac{M^2 + N^2}{M^2 + N^2 - 1} - \frac{\sqrt{2M^2 + 2M^2 - 1}}{M^2 + N^2 - 1} \tag{13}$$

The plus sign of the previous square root was rejected because the value $(M^2+N^2)/(M^2+N^2-1)$ is always greater than one. Additionally, R_2 is always less than one because the crank length of CR mechanism is always shorter than its fixed link length.

Now, the values of R_2, R_3, and R_4 can be calculated by selecting the desired values of (θ_{4i}) of rocker link in addition to the range of transmission angle ($\mu=90^0\pm\delta$).

C. Finite Element Analysis of Crank Rocker Mechanism

In general, the mathematical model of crank rocker mechanism which has elastic linkages must collect the appropriate mass and stiffness characteristics of mechanism's links. Moreover, Lagrange's equation in addition to suitable theory from structural mechanics can be considered as effective needed tools.

Many of recent research works dealing with elastic linkages considered the elastic links as a discrete or continuous system as in [25], [31]-[33].

The 2D-Euler's beam element selection can be considered the appropriate choice for system-oriented element matrices. Moreover, CR mechanism's dynamic behavior regarding the elastic effects can be considered as in [25].

Here, elastic linkage modeling via displacement or force method using finite element (FE) theory of lumped parameter approach or structural analysis can be considered as a suitable method for CR mechanism's elastic linkages. Besides, many useful assumptions can be considered as: mechanism's crank rotates with constant speed, the bearing's friction is very small which can be neglected with stable bearings, the elastic deformation is very small from equilibrium position of rigid-body. Hence, bearings can be represented as linear and torsional spring constants.

Elastic beam element motion can be described using Lagrange's equation as presented in [34], [35] as follows;

$$\frac{d}{dt}\left(\frac{\partial T_i}{\partial\{\dot{q}\}_i}\right)-\frac{\partial T_i}{\partial\{q\}_i}+\frac{\partial V_i}{\partial\{q\}_i}=\{Q\}_i+\{F\}_i \tag{14}$$

where $\{q\}_i$ represents the generalized nodal Degree of Freedom **(**DOF), T_i is the kinetic energy of the element, V_i is potential energy, $\{Q\}_i$ is the acting generalized forces, $\{F\}_i$ is the applied external forces.

Referring to (14), the equivalent mass matrix $[M]_e$ in addition to stiffness matrix $[K]_e$ can be written for individual elements in their local coordinates as presented in [25] as;

$$[M]_e=\begin{bmatrix} 2a & 0 & 0 & a & 0 & 0 \\ 0 & 156b & 22l_eb & 0 & 54b & -13l_eb \\ 0 & 22l_eb & 4l_e^2b & 0 & -13l_eb & -3l_e^2b \\ a & 0 & 0 & 2a & 0 & 0 \\ 0 & 54b & -13l_eb & 0 & 156b & -22l_eb \\ 0 & -13l_eb & -3l_e^2b & 0 & -22l_eb & 4l_e^2b \end{bmatrix}$$

$$[K]_e=\begin{bmatrix} EA/l_e & 0 & 0 & -EA/l_e & 0 & 0 \\ 0 & 12EI_e/l_e^3 & 6EI_e/l_e^3 & 0 & 12EI_e/l_e^3 & 6EI_e/l_e^2 \\ 0 & 6EI_e/l_e^3 & 4EI_e/l_e & 0 & -6EI_e/l_e^2 & 2EI_e/l_e^2 \\ -EA/l_e & 0 & 0 & EA/l_e & 0 & 0 \\ 0 & 12EI_e/l_e^3 & -6EI_e/l_e^2 & 0 & 12EI_e/l_e^3 & -6EI_e/l_e^2 \\ 0 & 6EI_e/l_e^2 & 2EI_e/l_e^2 & 0 & -6EI_e/l_e^2 & 4EI_e/l_e \end{bmatrix}$$

where: $a=(\rho A_e l_e)/6$ and $b=(\rho A_e l_e)/420$

Elements matrices $[M]_e$ and $[K]_e$ can be defined in the global coordinate system using transformation matrix as [25], [36].

Fig. 4 shows the finite element model of elastic mechanism. This figure describes the linkages structural deformations where system-oriented displacements are labeled to keep compatibility between elements at nodes.

At node (n), the notations $\boldsymbol{V_n}$ and $\boldsymbol{U_n}$ clearly describe nodal translations along orthogonal coordinate x and y-axis of a rigid-body position of linkages. Also at node (n), a rotational displacement (ϕ_n) describes the rotational deformations for two elements, (n and n+1), with respect to orientations of their rigid-body. In fact, forces and bearing's clearance control the movement, which can be denoted by spring stiffness.

Bearing's location of the model is shown in Fig. 5 for representing the stiffness effect of elastic mechanism on the stiffness matrix where the longitudinal and torsional equivalent stiffness of bearings are denoted by $\boldsymbol{K_l}$ and $\boldsymbol{K_t}$ respectively. These values ($\boldsymbol{K_l}$ and $\boldsymbol{K_t}$) can be added at suitable positions for arranging the oriented global stiffness matrix of the whole system.

Considering the system of free vibration, the equation of motion of modal system can be represented as follows;

$$[M]\{\ddot{q}\}+[K]\{q\}=0 \tag{15}$$

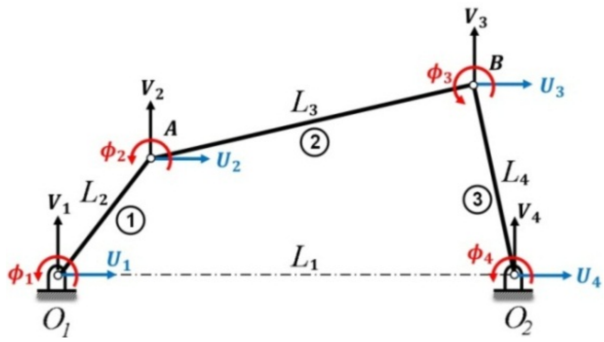

Fig. 4 Finite element model of elastic mechanism

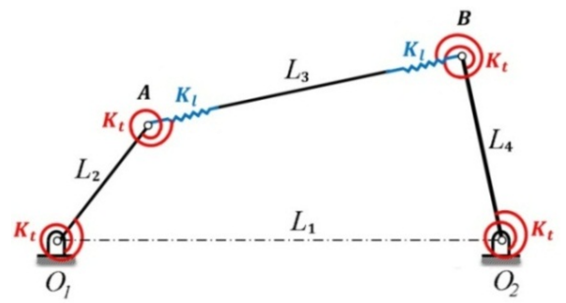

Fig. 5 Elastic mechanism model with flexible bearing stiffness

Moreover, the previous equation can be rewritten in new form regarding the system free vibration as;

$$|[K]-\lambda_i[M]|\{V\}=\{0\} \tag{16}$$

where $\{V\}$, λ_i are the eigenpairs. $[M]$ and $[K]$ are the mass and stiffness matrix of the composite mechanism. The last equation can be coded in computer program via software of (MATLAB7.1). The MATLAB software is capable of computing the eigen parameters for the elastic mechanism. Furthermore, the needed details of forming the previous matrices $[M]$ and $[K]$ are discussed in [37].

D.Experimental Modal of Crank Rocker Mechanism:

The prototype of composite linkages were manually fabricated using carbon fiber as strengthening mainstay in bidirectional fabric form in addition to polyester with catalyst as the matrix of the composite material. The laminated composite beam links of CR mechanisms have fiber volume fraction 50% at same five layers of carbon fiber as reinforcement. The composite's mechanical properties are calculated analytically using the mixture rule. The mechanical properties of fiber and polyester are listed in Table I.

TABLE I
MECHANICAL PROPERTIES OF COMPOSITE MATERIAL COMPONENTS

Material	Properties			
	Elasticity Modulus "E" [GPa]	Shear Modulus "G" [MPa]	Density "ρ" [g/cm^3]	Poisson´s ratio "υ"
Carbon Fiber	201±20	1526±643	1.73±0.01	0.21
Polyester	2.4±0.01	50±02	1.25±0.01	0.35

The prototype of CR mechanism's composite linkages is shown in Fig. 6.

The CR mechanism prototype was tested via frequency response measuring using appropriate dual channel analyzer and fast Fourier Transform (FFT) connected with computer as shown in Fig. 7. The corresponding fundamental frequencies of prototype are measured and recorded for various mechanisms at angle (μ=90°). The response frequencies are measured via (FFT) analyzer through the range (800:1600 Hz).

Fig. 6 Composite links of CR mechanism's prototype

Fig. 7 Experimental test using duel channel analyzer

III. RESULTS AND DISCUSSION

Four cases of CR mechanism of composite links were tested. The linkages proportions of these four cases are listed in Table II.

TABLE II
CR MECHANISM'S LINKAGE PROPORTIONS

Mechanism	R_2	R_3	R_4
Case-1	0.267	0.800	0.667
Case-2	0.417	1.000	0.792
Case-3	0.141	0.900	0.457
Case-4	0.209	0.914	0.457

The CR mechanism's linkages proportions of case-3 in addition to CR mechanism's linkages proportions of case-4 were selected considering the mini-max sense of optimum transmission angles means that the transmission angle's range of case-3 is $\mu=90\pm20^0$. While the transmission angle's range of case-4 is $\mu=90\pm30^0$. On the other hand, The CR mechanism's linkages proportions of case-1 in addition to CR mechanism's linkages proportions of case-2 were selected as general CR mechanisms where transmission angle's limits of case-1 are $\mu_{max}=119.18^0$ and $\mu_{min}=59.17^0$ in addition to $\mu_{max}=103.9^0$ and $\mu_{min}=35.7^0$ for case-2. The crank angular speed (ω_2) is equal to 3.27 rad/sec is used for calculated the needed driving torque. Driving torque can be calculated as in [38].

Fig. 8 illustrates the relation between crank angle (θ_2) and rocker angle (θ_4) for different cases of composite mechanisms. These relations reveal that the swinging angle's range of rocker equals to 66.9^0 in case-2, while it is equal to 33.9^0 in case-3. Moreover, the swinging angle's range of rocker is equal to 46.8^0 in case-1, while it is equal to 50.7^0 in case-4. Furthermore, regarding the third and fourth cases which have optimal transmission angles, the swinging angle's range of case-4 is higher than the range of case-3.

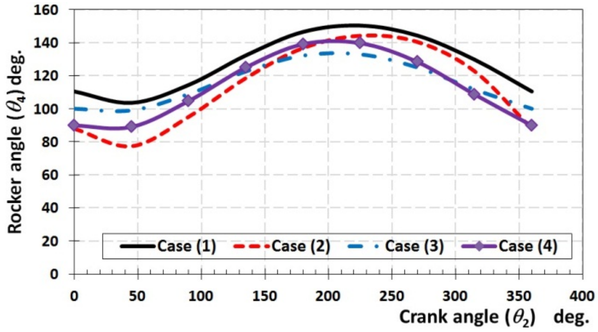

Fig. 8 Relation between crank angle (θ_2) and rocker angle (θ_4)

Fig. 9 demonstrates the relation between crank angle (θ_2) and the transmission angle (μ) for various cases of composite mechanisms. The curves indicate that both case-1 and case-4 have approximately the same trend, and their transmission angles range is approximately equal to 60^0. Also, case-1 and case-4 have the highest values of transmission angles, while case-2 has lowest value of transmission angles.

Fig. 10 illustrates the relation between crank angle (θ_2) and the driving torque for different cases of composite mechanisms. The relation's curves indicate that case-1 has the highest positive peak of the needed driving torque. Moreover, case-1 has the lowest negative peak of torque. On the other hand, case-4 is better than other cases because it has the lowest positive peak of needed driving torque. Here, the torque's positive peak of case-1 is equal to 44.29 N/m which is around four times of case-4. Also, the average torque of case-4 is around 29% of average torque in case-1. These observations are related to the values transmission angle where case-4 has the wide range of transmission angle $\mu=90\pm30^0$ with considering the mini-max sense. These observations are consistent with the results of needed driving torque of CR solar tracker mechanism as in [38].

The computed and measured fundamental response frequencies, regarding to four mode numbers at mechanisms transmission angle of value $\mu = 90^0$, are listed in Table III for different cases of composite mechanisms as follows;

TABLE III
FE AND MEASURED FREQUENCIES

Mode No.	case (2)		case (1)		case (4)		case (3)	
	FE	Ex	FE	Ex	FE	Ex	FE	Ex
1	30	27	36	33	73	67	85	79
2	74	71	89	85	181	172	213	203
3	215	207	259	249	526	505	618	594
4	517	507	622	610	1262	1237	1484	1455

The previous tabulated results of computed and measured fundamental frequencies reveal to a good conformity between the theoretical frequencies and measured ones. Mechanisms of case-3 and case-4, which are considering mini-max sense of optimal transmission angle, have the highest frequencies at four mode numbers. Also, the measured frequencies of case-3 are higher than the corresponding frequencies of case-4 as shown in Fig. 11 related to its small range of optimal angle (μ).

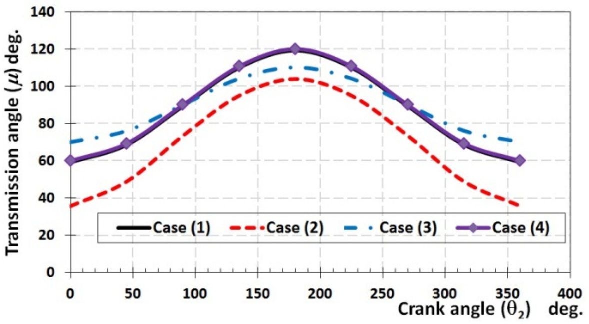

Fig. 9 Relation between crank angle (θ_2) and transmission angle (μ)

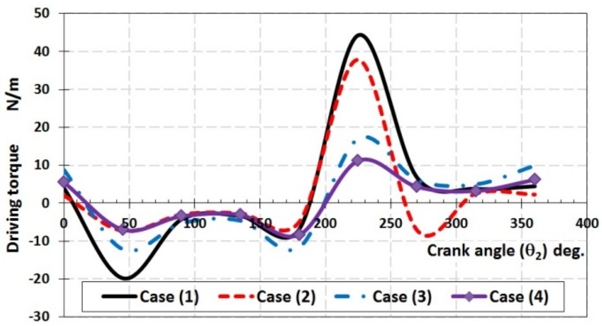

Fig. 10 Relation between crank angle (θ_2) and driving torque

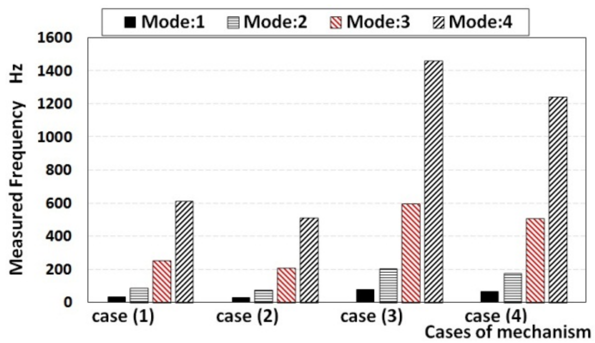

Fig. 11 Measured frequencies

IV. CONCLUSION

Most world countries are trying to do their best for significantly reducing the consumed energy of driving mechanical machines. According to this trend of reducing the energy consumption, this research work attempted to present a study for using light composite materials for manufacturing lightweight linkages of crank rocker mechanism in order to reduce the required driving torque besides treating the wear and corrosion problems.

Reducing driving power of crank rocker mechanism, as one of main important parts of mechanical machines, can be achieved via selecting its appropriate proportions of links lengths dealing with the optimum transmission angle. Moreover, mechanism's performance can be improved regarding composite links usage due to their stiffness and high damping.

The results reveal that mechanisms of optimal transmission angle ranges are better than the other cases because these mechanisms have the lowest positive peak of needed driving torque. Furthermore, the average torque of composite mechanisms of case-4 which has optimal transmission angles range is around 29% of average torque of general mechanisms of case-1. The theoretical and measured fundamental response frequencies of mechanisms are in a good conformity.

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