

A Study on Performance Prediction in Early Design Stage of Apartment Housing Using Machine Learning

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Abstract—As the development of information and communication technology, the convergence of machine learning of the ICT area and design is attempted. In this way, it is possible to grasp the correlation between various design elements, which was difficult to grasp, and to reflect this in the design result. In architecture, there is an attempt to predict the performance, which is difficult to grasp in the past, by finding the correlation among multiple factors mainly through machine learning. In architectural design area, some attempts to predict the performance affected by various factors have been tried. With machine learning, it is possible to quickly predict performance. The aim of this study is to propose a model that predicts performance according to the block arrangement of apartment housing through machine learning and the design alternative which satisfies the performance such as the daylight hours in the most similar form to the alternative proposed by the designer. Through this study, a designer can proceed with the design considering various design alternatives and accurate performances quickly from the early design stage.

Keywords—Apartment housing, machine learning, multi-objective optimization, performance prediction.

I. INTRODUCTION

THE research on the use of machine learning in design is conducted along the development of information and communication technology. Researchers have been tried to automatically generate virtual worlds by learning real world architectural drawings using machine learning in the field of computer graphics [1]. Machine learning has been used to understand the consumer's preference according to design alternatives, which are difficult to grasp, the correlation and to quantify in the study on fashion design. With the convergence of machine learning and design, it is possible to solve the problems that are composed of multiple factors and difficult to grasp the correlation [2].

Research through machine learning was carried out to predict the structural performance of buildings to create the optimum design, and to predict the performance of building energy analysis or facility system to optimize operation in the field of architecture [3]. Research through machine learning also has been attempted in the field of architectural planning. Commonly used building performance simulation tools produce only static results after the design has been fixed. It is also difficult to explore a wide range of design alternatives and

their performance in a given design process with only limited time and resources [4]. Therefore, research works trying to solve these problems have been carried out. Such research works predicted various design alternatives (shapes) and their performance and tried to give feedback on performance to the designer in real-time [5].

Two typical problems usually occur when applying machine learning to architectural design. First, the amount of training data is important for machine learning [6]. However, there are few data to study because there is no structured data on the types of buildings and their performance [7]. In addition, the architectural planning should deal with many factors, and each factor has a non-linear ('non-linear' means that the result and cause or output and input are not necessarily proportional in this paper) relation with the performance of the building. Therefore, it is difficult to predict the various performances of the building by the existing machine learning algorithm [7]. To address these problems, previous research works suggested solutions as follows:

- 1) When the problem occurs because of the small amount of training data, a parametric model is used to automatically generate a large number of training data to solve the problem [8].
- 2) An Artificial Neural Network (ANN) was applied to predict nonlinear problems which consist of multiple factors more effectively [9].

Therefore, in order to solve the problems hard to predict due to the small amount of training data and the multiple factors that affect the building performance according to the arrangement of the buildings, making the training data with the parametric model and applying ANN were attempted in this research. The aim of this research is to help the designer search various design alternatives and their performance in real-time, through the performance prediction model based on building arrangement and shape.

The scope of this study was restricted to apartments among apartment houses. The building performance according to the shape and the arrangement of apartment housing is a typical non-linear problem. Daylight hour analysis was performed for each apartment housing type in this research. Daylight hours of apartment buildings are an important performance specified in the building code. Apartments can be classified into L-shaped type, two wing-shaped type, single wing-shaped type and flat type depending on the apartment plan [10]. The experiment was conducted on the L-shaped type in this study.

For this research, a number of training data for machine learning were generated. In this process, Rhinoceros-Grasshopper, a parametric modeling tool, Ladybug, an energy

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simulation tool capable of simulating building daylight hours, and RapidMiner Studio, a data mining tool, were used to establish a predictive model using machine learning.

This study was carried out according to the following process.

Step 1: Establishment of apartment building layout and construction of the parametric model.

Step 2: Execute performance analysis and generate training data for each design alternative created by parametric model.

Step 3: Construction of machine learning and performance prediction model.

Step 4: Performance evaluation of the building arrangement proposed by the designer.

Step 5: Performance prediction and optimal arrangement of morphological similar building arrangement.

Step 6: Applying the building arrangement and performance evaluation.

II. RELATED WORKS

The definition of machine learning refers to the process of predicting or optimizing output values by finding correlation of input variables [11]. Machine learning can save resources by enabling accurate and fast calculations or predictions of a wide range of data. Therefore, machine learning is used as a core technology in various convergence research fields [12].

A. Performance Prediction through Machine Learning

In general, most of the studies using machine learning are based on quantifiable data. In the field of industrial engineering, research has been attempted to predict the performance of a company through its financial status and patent index. Net profits and future profits were estimated by assigning quantified corporate financial components and patent indices such as firm's revenue, net profit, total assets, and shareholder's equity to the Restricted Boltzmann Machine (RBM), one of the deep learning algorithms [11]. In the field of electronics, research has been carried out to predict the output of sunlight using cloudiness, humidity, precipitation, atmospheric temperature, wind velocity and Support Vector Machine (SVM) algorithm [13]. Research is also being conducted on data that are difficult to quantify. For example, a study has been conducted to predict the logistics performance of a container using the Decision Tree after identifying the factors affecting the loading and unloading of containers on the ship, and quantifying the factors [14].

B. Performance Prediction through Machine Learning in Architecture

There are some research works to predict the performance of buildings by applying machine learning in the field of architecture. In the study on the structure of buildings, 237 training data were used to estimate the compressive strength of concrete according to the composition ratio of concrete components. And machine learning was performed using algorithms such as Linear Regression, SVM, and ANN [15]. However, it does not use enough training data and the average error rate exceeds 10%. Paterson used ANN to predict building

energy performance such as power consumption according to building type through factors such as building area and glazing ratio [7]. Because there was not enough data to cover all the shape and performance of the building needed for machine learning, the shape information of the building was entered manually based on the satellite images and the data was preprocessed to quantify the shape of the building. Asadi et al. used the Genetic Algorithm (GA) and ANN to predict the energy performance and cost of retrofitting existing buildings and set the type of insulation and the type of windows as factors affecting the building performance [16].

Through machine learning, it is possible to predict the performance according to the design alternatives by grasping the correlation between various design elements that were difficult to grasp previously [17]. The performance according to the arrangement and the shape of buildings is influenced by many factors and they are not in the proportional relation. In recent research, advanced machine learning algorithms such as ANN are applied. In order to apply the form of building to machine learning, it is necessary to quantify the shape of the building.

III. METHODOLOGY

This study was conducted according to the following process as shown in Fig. 1.

A. Constructing the Environment for Block Arrangement of Apartment Housing and the Parametric Model

To automatically generate the training data needed for learning, a parametric model that generates the apartment arrangement is constructed in the Rhinoceros-Grasshopper environment. The parametric model consists of fixed perimeter buildings and landforms and the shape and arrangement of variable apartment blocks. The parametric model was constructed in which L apartments were placed on the site with four blocks. The parameters that make up the model consist of items about the shape of each block of the apartment and items about the relationship between them. The parameters for each block type are Height, Angle, X-coordinate, and Y-coordinate. The parameters for the geometric relationship between blocks are the width, length, ratio of lateral to longitudinal length and area of polygon composed of reference points of each block.

B. Performance Analysis of Apartment Arrangement and Training Data Generation

Ladybug, an energy simulation tool, is used to analyze the daylight performance of each building shape and arrangement of apartment generated by the parametric model. Among the architectural energy performances, the simulation was conducted for the winter solstice (8:00 am to 4:00 pm) specified in the building code for each household the duration of daylight. An algorithm has been developed to verify whether all generations of four blocks meet daylight hour criteria using Grasshopper. Through this, the performance of the design alternatives created by parametric model is analyzed. To ensure the model's prediction probability, both the performance satisfying alternatives and the alternatives that do not satisfy the

performance must be sufficient.

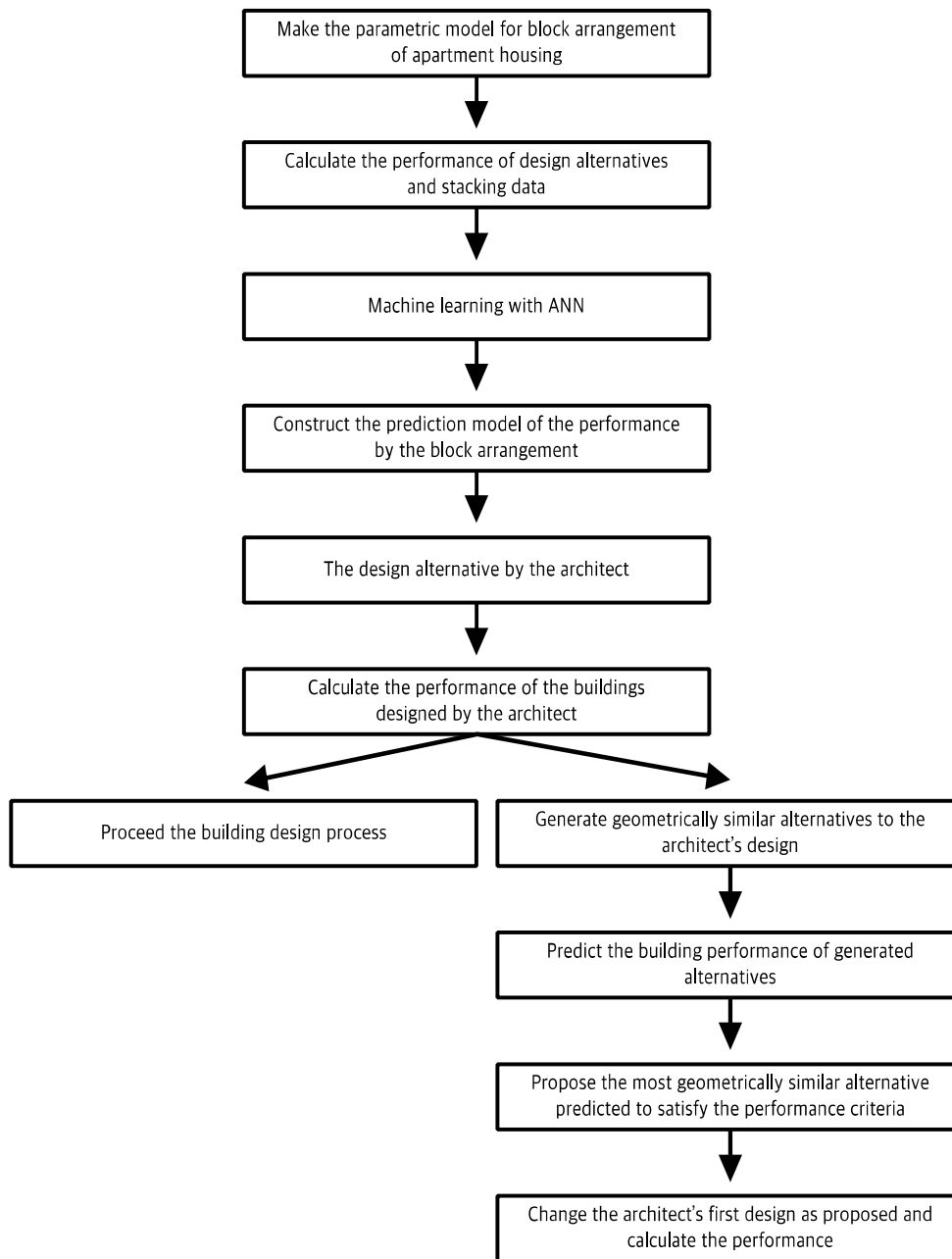


Fig. 1 Block Arrangement Process of Apartment Housing Using Machine Learning

Considering the number of all cases that can be generated by the parametric model, the amount to be computed is over 1.5×10^{11} iterations in this study. If design alternatives are generated randomly, it is difficult to ensure the reliability of the prediction model by generating few alternatives satisfying the performance. Therefore, the GA is used to collect design

alternatives satisfying the performance criteria.

The combination of the parameters and the performance when creating alternatives for the machine learning process is stored together with Excel data.

Training data consist of 20 independent variables and one dependent variable.

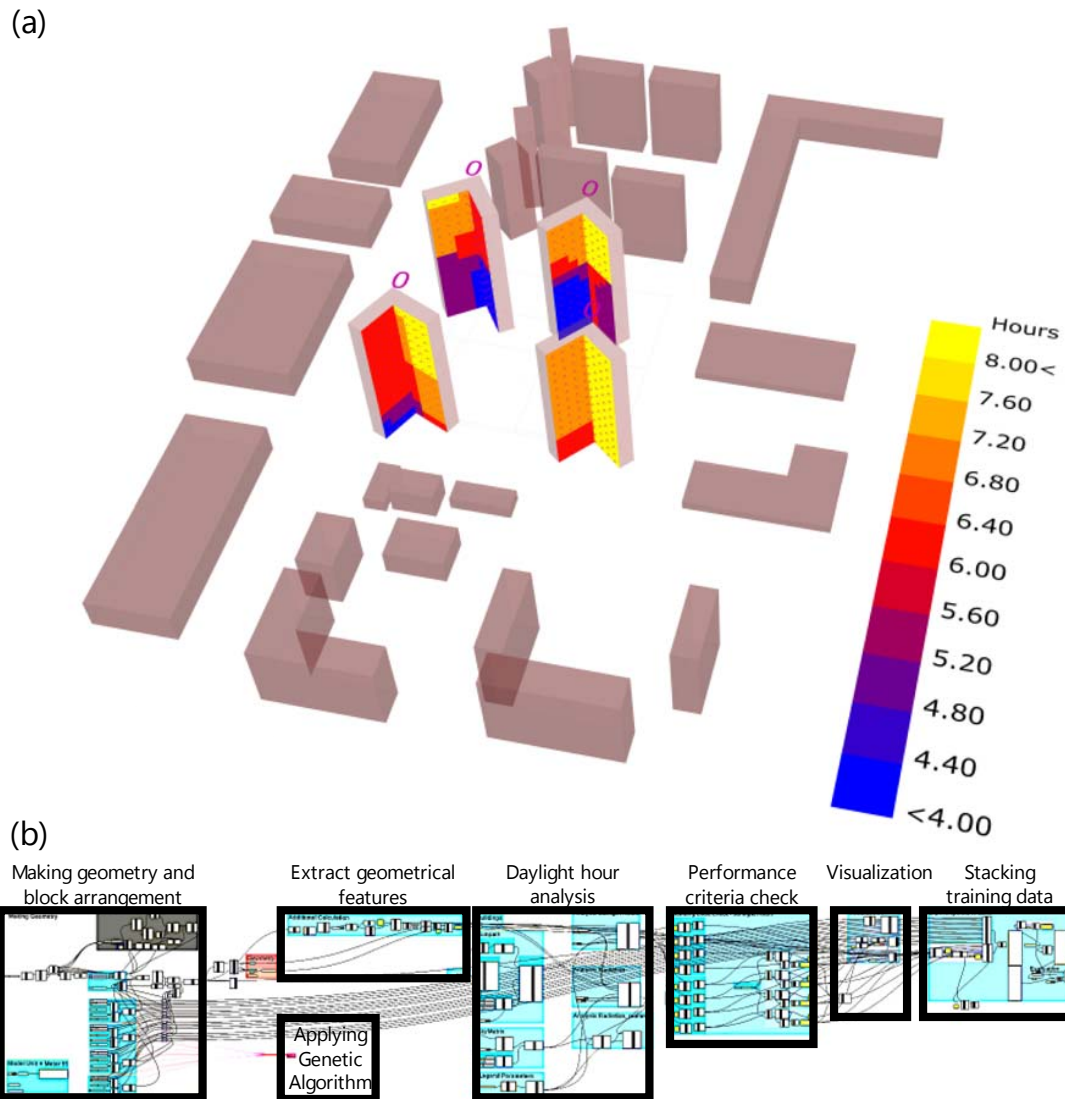


Fig. 2 (a) The result of daylight hour analysis and (b) the algorithm

C. Machine Learning Process and Performance Prediction Model

1) Artificial Neural Network

ANN is one of the machine learning algorithms and it is a modeling technique to find hidden patterns in data through iteration of learning process. ANN was created by mimicking the structure and progress of the neural network in the brain [18]. ANN consists basically of a combination of information processing units called neurons. One layer consists of multiple neurons, and the whole network consists of an input layer, a hidden layer, and an output layer. Each layer is connected through a weight, and the basic principle is to output the result calculated using the input value of the previous layer and the weight connected to the next layer using a transformation function, and finding the appropriate weight value between each layer is a machine learning process.

ANN can be divided into unsupervised learning which does

not have correct output value and supervised learning which has correct output value [7]. Supervised learning is applied to predict performance satisfaction expressed by boolean value in this research.

ANN is divided into Deep Neural Network (DNN), Convolutional Neural Network (CNN), Recurrent Neural Network (RNN) and Deep Boltzmann Machine (DBM) [19]. The performance prediction model was implemented using DNN in this research. DNN is composed of multi-layer and it is suitable to predict multi-factorial non-linear performance of building because it uses Back Propagation Algorithm to find optimal weight value.

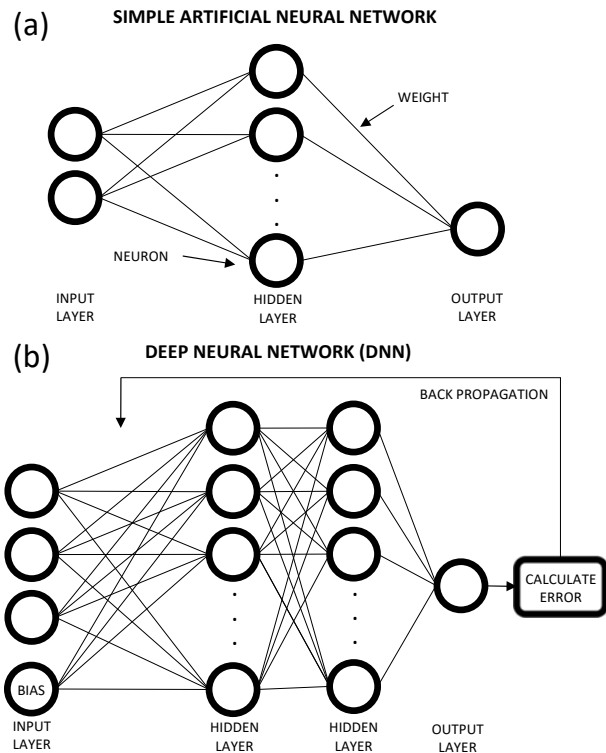


Fig. 3 (a) The basic structure of ANN and (b) the structure of DNN used in this research

2) Construction of Building Performance Prediction Model

The building performance prediction model was constructed through the following process (Fig. 4) in the RapidMiner Studio environment.

- Pre-processing of training data:** For machine learning decide the type of training data collected in previous stage. All training data collected in this study are classified into integer, real number and boolean data types. The data is also classified into two classes considering whether data is an attribute that affects performance or a label that indicates performance.
- Training data normalization:** All data are normalized to a real number between -1 and 1 for effective learning because the range of item values used for training data is different.
- K-Folds Cross Validation:** It is possible to derive the optimal machine learning model by constructing several ANN models and comparing the performance of them with the limited amount of training data through k-fold validation [20]. In this process, the entire training data is divided into k subsets called 'folds'. Then, in the process of applying the machine learning algorithm, k-1 folds are used in the learning process and 1 fold is used as the validation data, and this process is repeated a total of k times.
- DNN:** In this study, machine learning is performed using DNN, which is a type of ANN. The hidden layer that constitutes the optimal neural network and the activation function of the size and hidden layer of each layer were

determined through an iterative experiment process. In addition, setting values for machine learning such as epochs, epsilon, rho, and loss function have been decided. The Back Propagation Algorithm is used to find the optimal weight value in the learning process.

- Applying Model and Comparing Learning Performance:** Since this study has been performed through k-folds validation, a total of k DNN models are generated. After comparing the prediction performance of each DNN, the model with the highest performance is selected as the final model. This model is used to predict the performance of multiple design alternatives geometrically similar to the initial design alternative proposed by the designer.

D.Performance Prediction of Similar Design Alternatives, Proposal Optimal Alternative and Performance Analysis

TABLE I
PARAMETERS USED IN THE EQUATION FOR CALCULATING GEOMETRICALLY DIFFERENCE

Parameter	Description
H_0	Height of building designed by the architect (floor)
A_0	Angle of building designed by the architect from the South (degree)
X_0	X-coordinate of the building designed by the architect (m)
Y_0	Y-coordinate of the building designed by the architect (m)
H_n	Height of building generated in Step 2 (floor)
A_n	Angle of building generated in Step 2 from the South (degree)
X_n	X-coordinate of the building generated in Step 2 (m)
Y_n	Y-coordinate of the building generated in Step 2 (m)
d_k	Geometrical difference of building number k between a building designed by the architect and similar alternatives which are expected to satisfy the performance criteria
D	The whole geometrical difference of buildings between designer's alternative and similar alternatives which are expected to satisfy the performance criteria

If the performance of the design proposed by the designer does not satisfy the performance criteria, the following automated process is performed.

Step 1: Extract the geometric parameters covered in this study from the designer's design.

Step 2: Create the possible types and arrangements within the range using a parametric model that is constructed in advance after setting the extracted parameters to be variable within a certain range.

Step 3: The performance of the alternatives generated in step 2 is predicted through the performance prediction model.

Step 4: Calculate the geometrical difference between designer's alternative (black buildings in Fig. 5) and similar alternatives (white buildings in Fig. 5) which are expected to satisfy the performance criteria. The geometrical difference is calculated by the (1) and (2), and the calculation is performed on the entire alternatives which are expected to satisfy the performance.

$$d_k = (H_0 - H_n)^2 + (A_0 - A_n)^2 + (X_0 - X_n)^2 + (Y_0 - Y_n)^2 \quad (1)$$

$$D = d_1 + d_2 + d_3 + d_4 \quad (2)$$

Step 5: Arrangements in which the performance is expected to

be satisfied appear on the designer's work screen in order of decreasing D value. The designer looks at the proposed layout and changes the design. In this process, designer can verify that

the performance of the apartment housing that were expected to satisfy the performance agrees with the actual simulation results.

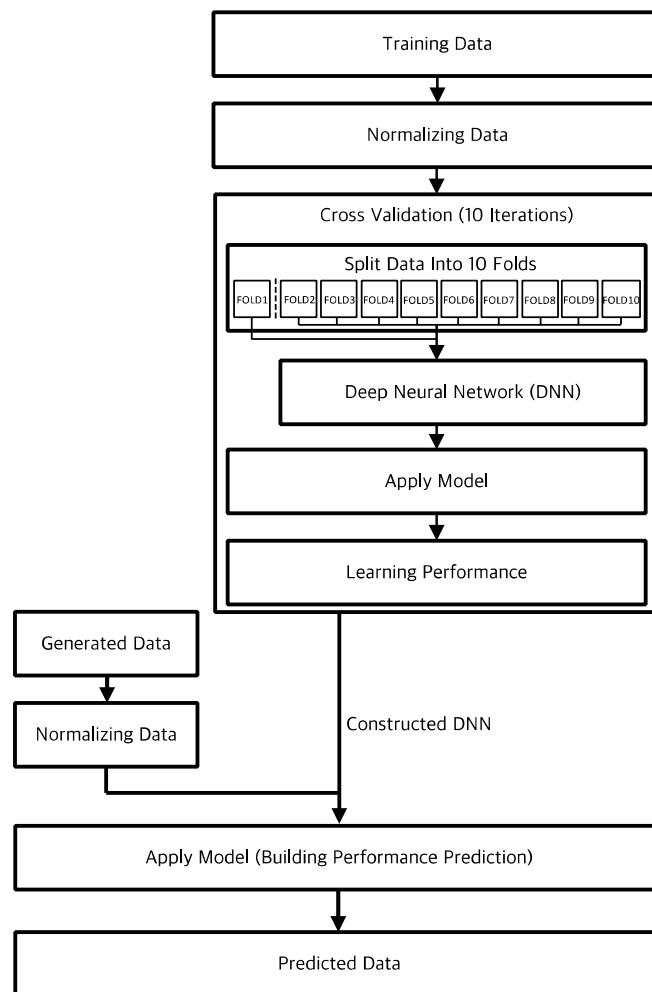


Fig. 4 The process of building performance prediction model construction and applying

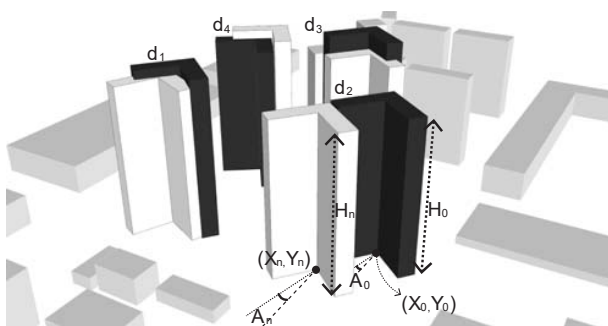


Fig. 5 The parameters for calculation geometrical difference between two alternatives

IV. FINDINGS

Four apartment blocks are arranged according to the experimental process designed in Chapter III in this chapter.

The designer has arranged four apartments with 'L' shape, analyzed the performance of the design, and assumed that the performance does not satisfy the daylight hour criteria.

A. Experiment Settings

- 1) Making Training Data: A parametric model is constructed to create training data. Four apartment buildings of 'L' shape were constructed in a square of 120 m x 120 m size and analyzed the daylight hour performance in each layout. The performance data is collected by returning false (0) for more than 4 hours, and true (1) if not more than 4 hours on the same day. The total training data size is 30,000.
- 2) Making Predicting Model: The entire training data is divided into 10 folds for k-fold validation. The hidden layer consisted of two layers and each layer consisted of 75 neurons. The activation function maxout is used to activate the hidden layer neurons. In constructing a single DNN, the epochs value was set to 51 to repeat the operation to find

the optimal weight value. The other set values are changed and determined repeatedly to ensure an optimal prediction probability.

- 3) Making Geometrically Similar Alternatives: If the apartment arrangement proposed by the designer does not satisfy the building performance criteria, 2,000 alternatives similar to the existing layout are generated using the parametric model. After extracting the required parameters from the parametric model constructed in the designer's layout, a new alternative generation process is carried out within the preset range. The set range is height - 4 m ~ 4 m, angle -10° ~ 10°, block reference point X, Y coordinate -10 m ~ 10 m. The performance of the alternatives generated from this process is predicted through the building performance prediction model.

TABLE II
THE VALUE OF PARAMETERS USED IN THIS RESEARCH

Phase	Parameters	Value
Making Training Data	Training Data Size	30,000
	Criteria of Minimum Daylight Hours	4
	Width of Site (m)	120
	Length of Site (m)	120
Making Predicting Model	Activation Function	Maxout
	Number of Hidden Layer	2
	Hidden Layer Sizes	75 / 75
	Epochs	51
	Epsilon	0.0000001
	Rho	0.9997
	Loss Function	Cross Entropy
	Number of Folds	10
Making Geometrically Similar Alternatives	Random Seed	1,997
	Number of Alternatives	2,000
	Range of Height (m)	-4 ~ 4
	Range of Angle (degree)	-10 ~ 10
	Range of X-Coordinate (m)	-10 ~ 10
	Range of Y-Coordinate (m)	-10 ~ 10

B. Experiment Results

The results of the prediction performance of the daylight hour prediction model based on the 30,000 training data generated by the experiment and the block arrangement of the apartment generated using DNN are as follows (Table III).

TABLE III
EVALUATION RESULT OF THE BUILDING PERFORMANCE PREDICTING MODEL

Predicting with Training Data			
	True (1)	True (0)	Class Precision
Predicted (1)	19330	234	98.80%
Predicted (0)	326	10110	96.88%
Class Recall	98.34%	97.74%	
Accuracy	98.13%		
Predicting with Geometrically Similar Alternatives			
	True (1)	True (0)	Class Precision
Predicted (1)	599	156	79.34%
Predicted (0)	374	871	69.96%
Class Recall	61.56%	84.81%	
Accuracy	73.50%		

The prediction accuracy is 98.13% when the performance prediction model is substituted with 30,000 training data used in the learning process. The prediction accuracy is 73.50% when the performance prediction model is substituted with 2,000 layouts similar to the designer's alternative. Among 2,000 alternatives similar to the initial design of the designer, 1,245 predicted to satisfy the performance criteria. Some 871 alternatives actually satisfy the performance criteria and the class precision is 69.96%. Among 2,000 similar alternatives, 1,027 alternatives satisfy the performance criteria and 871 alternatives are predicted to satisfy the performance. The class recall is 84.81%.

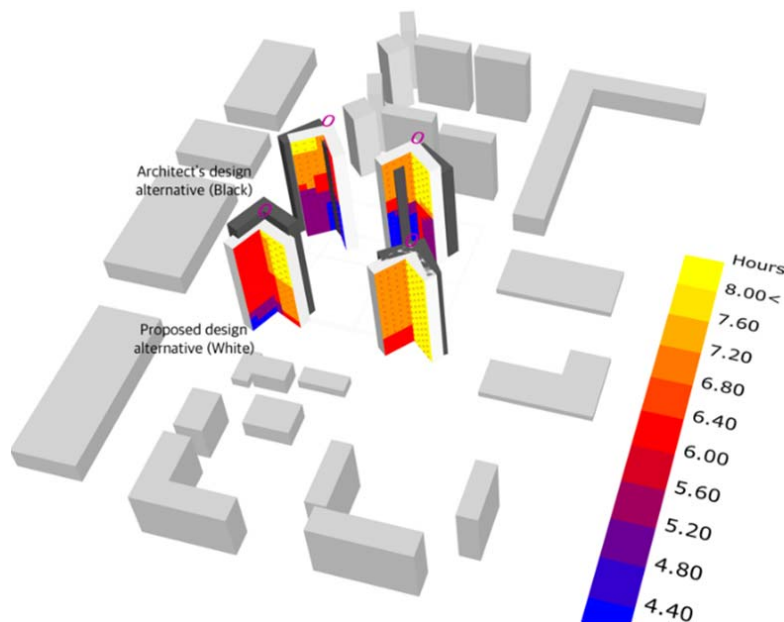


Fig. 6 Proposed design alternative and architect's design alternative

C. Analyzing Experiment Results

Experimental results show that prediction accuracy is reduced by 24.63% when new data is predicted compared to predicted performance based on training data. The reason for the lowered prediction probability is presumed as the following three reasons.

- 1) Overfitting of DNN: The prediction accuracy decreased by 20% or more in the process of predicting new data means that the constructed DNN is overfitted to the training data. The neural network is sensitive to training data, but it has low prediction performance for other data.
- 2) Value of epochs: The number of iterations of learning is an important factor influencing the prediction probability in the process of constructing a prediction model through machine learning [21]. In this study, since the optimization process of the epoch value that determines the repetition frequency of learning is not performed, the performance of the prediction model will be enhanced if the optimization process of the epoch value is performed.

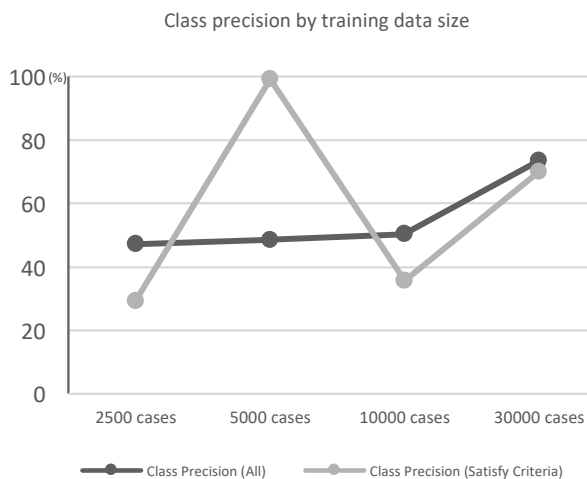


Fig. 7 Prediction performance according to the size of training data

- 3) Training data size: Overfitting occurs and the performance of the prediction model deteriorates when the amount of training data is small as the size of the neural network increases [22]. It was also confirmed, the tendency of the classification rate to decrease with decreasing amount of training data in this study. If more training data is used for machine learning, the performance of the prediction model would be improved.

V. CONCLUSION

In this research, a parametric model for generating the shape and the block arrangement of apartments was built and the building performance prediction model was constructed using machine learning. The aim of this research is to predict the performance on daylight hours for houses of four apartment buildings. A parametric model was constructed and used to generate enough training data to perform machine learning. DNN, which is a type of ANN, was used to construct a

predictive model for building performance affected by multiple factors. In case the design alternative proposed by the designer does not satisfy the performance, several similar alternatives are generated and the building performance prediction is performed. Among them, the most similar alternative predicted to satisfy the performance criteria is presented in the layout of the designer.

The building performance model was constructed with 30,000 cases of training data and the building performances of 2,000 cases of similar alternatives were predicted with the prediction model. The class precision of the prediction model was confirmed to be 73.5%.

This research found that the performance prediction model can be constructed by using the parametric model and ANN even if the building performance is difficult to predict due to a small amount of data or a large number of factors affecting the building performance. The research confirms that the designers can review various design alternatives and their performance in real time using the building performance prediction model according to the shape and the block arrangement of apartment housing. However, training data was generated in a limited environment so one performance prediction model is hard to cover the various building types. Furthermore, due to the overfitting of ANN, higher prediction performance could not be achieved. In addition, it is a limitation of this study that the prediction model cannot perform the prediction for the performance other than the daylight hours.

Therefore, it is necessary to construct training data for more types of buildings and various kinds of building performance, and to perform hyperparameter optimization which affects ANN in the machine learning process in order to avoid overfitting of ANN in future works.

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