Optical Fiber Data Throughput in a Quantum Communication System

Arash Kosari, Ali Araghi

***Abstract*—**A mathematical model for an optical-fiber communication channel is developed which results in an expression that calculates the throughput and loss of the corresponding link. The data are assumed to be transmitted by using of separate photons with different polarizations. The derived model also shows the dependency of data throughput with length of the channel and depolarization factor. It is observed that absorption of photons affects the throughput in a more intensive way in comparison with that of depolarization. Apart from that, the probability of depolarization and the absorption of radiated photons are obtained.

***Keywords*—**Absorption, data throughput, depolarization, optical fiber.

I. INTRODUCTION

INFORMATION technology has long been under development and now it is well penetrated all over the world. Under this circumstance, one of the main challenges about this technology that will arise is the way that the transmitted data are going to be protected. In recent years, quantum cryptographic communication channels have been used quite frequently to cope with the mentioned challenge. This technology provides absolute secrecy of the transmitted information thanks to its encoding technique which is mainly based on the polarization of photons [1].

To assess the data throughput of such systems, apart from the characteristics of the source and the receiver, the physical characteristics of the transmission medium[1] should also be taken into consideration. The reason is that optical fiber, in comparison with other types of transmission media, benefits from larger data throughput.

In the literature, a great proportion of studies is focused on symmetric cryptography [2], without considering probability of depolarization, or to polarize photons only in special protocol such as BB84 or the theoretical concepts of the subject in hand [3]-[5]. However, study on the data throughput in quantum cryptography has not been conducted in a notable approach.

The data throughput in optical fibers are affected by many factors which of them, two main ones are depolarization probability and absorption of transmitted photons. As up to now, no studies were conducted on the effect of the two pointed issues on the optical fiber throughput, and therefore, the aim of this paper is set to find out this effect. As an experimental unit, the commercially available optical fiber PANDA is used, which preserves optical polarization.

The rest of the paper is organized as follows; Section II is dedicated to mathematical model of the communication channel while in Section III, we are going through the obtained results based on the presented mathematical model, and finally, Section IV is for the conclusion.

II. MATHEMATICAL MODEL OF THE COMMUNICATION CHANNEL

At this section, we will get the expression for calculating of the optical fiber data throughput. To meet this aim, a mathematical model of the communication link should be developed in advance.

Further discussion will be based on the fact that information is transmitted by binary symbols ("0" and "1"). Let us set the probability of occurrence of symbols "0" and "1" at the channel input as $P_s(0)$ and $P_s(1)$, and the probability of occurrence of symbols "0" and "1" at its output – as $P_s'(0)$ and $P_s'(1)$ respectively.

Optical fiber absorbs some of the radiated photons passing through it, so when transmitting a symbol ("0" or "1") at the channel output the transmitted symbol may be omitted. Let us set such probability as $P_s'(-)$. Thus, character set of code words at the input does not coincide with the character set of code words at the output. The probability of receiving of symbols "0", "1" or of the absence of the symbol depends neither on the symbol at the channel input nor on the previously received symbols. Given these features, the considered communication channel is a discrete, binary, asymmetric uniform one, without memory and erasable [6].

It should be noted that optical fiber suffers from depolarization [7], so some of the transmitted photons will be usually depolarized. As a result, at the output of optical fiber (output of communication channel) symbol "0" and "1" may occur while at its input symbol "1" and "0" was transmitted respectively. Let us set such probability as $P(0/1)$ and $P(1/0)$ respectively.

According to [8], [9], rate of information transfer (RIT) of optical fiber C is defined as the amount of data I per average time duration of one bit (one symbol) τ_b transfer:

$$C = I / \tau_b = \left[H(B) - H(B/A) \right] / \tau_b, \qquad (1)$$

$H(B)$ is the entropy at the communication channel output, and $H(B/A)$ is the conditional entropy, which determines the

---

Arash Kosari is with the Iran Telecommunication Research Center, Iran, Islamic Republic Of (e-mail: arash222.ko@gmail.com).
[1] Where optical fiber is most commonly used.

"unreliability" of the communication channel or information loss under the influence of interference attack.

The entropy at the communication channel output will be written as [9]:

$$H(B) = -P_s'(0)\log_2 P_s'(0) - P_s'(1)\log_2 P_s'(1) - $$
$$-P_s'(-)\log_2 P_s'(-). \tag{2}$$

Entering into (2) probabilities $P_s'(0)$, $P_s'(1)$ and $P_s'(-)$ are equal respectively:

$$P_s'(0) = P_s(0)P(0/0) + P_s(1)P(0/1), \tag{3a}$$

$$P_s'(1) = P_s(0)P(1/0) + P_s(1)P(1/1), \tag{3b}$$

$$P_s'(-) = P_s(0)P(-/0) + P_s(1)P(-/1), \tag{3c}$$

$P(0/0)$ and $P(1/1)$ are the probabilities of occurrence of symbols "0" and "1" at the channel output while at the channel input there are symbols "0" and "1" respectively. $P(-/0)$ and $P(-/1)$ are the probability of the absence of the transmitted symbol at the channel output while at the channel input symbols "0" and "1" are transmitted.

Substituting (3) into (2), we get:

$$H(B) = -\left[P_s(0)P(0/0) + P_s(1)P(0/1)\right] \times \log_2\left[P_s(0)P(0/0) + P_s(1)P(0/1)\right] - $$
$$-\left[P_s(0)P(1/0) + P_s(1)P(1/1)\right] \times \log_2\left[P_s(0)P(1/0) + P_s(1)P(1/1)\right] - $$
$$-\left[P_s(0)P(-/0) + P_s(1)P(-/1)\right] \times \log_2\left[P_s(0)P(-/0) + P_s(1)P(-/1)\right]. \tag{4}$$

Conditional entropy $H(B/A)$ is [2]:

$$H(B/A) = -P_s(0) \times \left[P(0/0)\log_2 P(0/0) + P(1/0)\log_2 P(1/0) + \right.$$
$$\left. +P(-/0)\log_2 P(-/0)\right] - P_s(1)\left[P(0/1)\log_2 P(0/1) + \right.$$
$$\left. +P(1/1)\log_2 P(1/1) + P(-/1)\log_2 P(-/1)\right]. \tag{5}$$

By substituting (4) and (5) to (1), the RIT can be evaluated as;

$$C = -\left\{\left[P_s(0)P(0/0) + P_s(1)P(0/1)\right] \times \right.$$
$$\times \log_2\left[P_s(0)P(0/0) + P_s(1)P(0/1)\right] - $$
$$-\left[P_s(0)P(1/0) + P_s(1)P(1/1)\right] \times $$
$$\times \log_2\left[P_s(0)P(1/0) + P_s(1)P(1/1)\right] - $$
$$-\left[P_s(0)P(-/0) + P_s(1)P(-/1)\right] \times $$
$$\times \log_2\left[P_s(0)P(-/0) + P_s(1)P(-/1)\right] + $$
$$+P_s(0)\left[P(0/0)\log_2 P(0/0) + P(1/0)\log_2 P(1/0) + \right.$$
$$\left. +P(-/0)\log_2 P(-/0)\right] + P_s(1)\left[P(0/1)\log_2 P(0/1) + \right.$$
$$\left. +P(1/1)\log_2 P(1/1) + P(-/1)\log_2 P(-/1)\right]\right\}/\tau_b. \tag{6}$$

RIT reaches its maximum value (data throughput) C_{max}

when the entropy $H(B)$ is maximal. According to the properties of entropy, it is equal to the maximum when $P_s(0) = P_s(1) = 0.5$ [6], [9]. Then, the formula of the considered communication channel data throughput can be written in the following form:

$$C_{max} = $$
$$\left\{-\left[0.5\left[P(0/0) + P(0/1)\right]\right]\log_2\left[0.5\left(P(0/0) + P(0/1)\right)\right] - \right.$$
$$-\left[0.5\left[P(1/0) + P(1/1)\right]\right]\log_2\left[0.5\left[P(1/0) + P(1/1)\right]\right] - $$
$$-\left[0.5\left[P(-/0) + P(-/1)\right]\right]\log_2\left[0.5\left[P(-/0) + P(-/1)\right]\right] + $$
$$+0.5\left[P(0/0)\log_2 P(0/0) + P(1/0)\log_2 P(1/0) + \right.$$
$$\left. +P(-/0)\log_2 P(-/0)\right] + 0.5\left[P(0/1)\log_2 P(0/1) + \right.$$
$$\left. +P(1/1)\log_2 P(1/1) + P(-/1)\log_2 P(-/1)\right]\right\}/\tau_b. \tag{7}$$

Probabilities $P(0/0)$, $P(1/0)$, $P(-/0)$ and $P(0/1)$, $P(1/1)$, $P(-/1)$ are respectively equal to:

$$P(0/0) = \left(1 - P_{dep}\right)\left(1 - P_{abs}\right),$$
$$P(1/0) = P_{dep}\left(1 - P_{abs}\right), \tag{8a}$$
$$P(-/0) = P_{abs}$$

$$P(0/1) = P_{dep}\left(1 - P_{abs}\right),$$
$$P(1/1) = \left(1 - P_{dep}\right)\left(1 - P_{abs}\right), \tag{8b}$$
$$P(-/1) = P_{abs},$$

P_{dep} and P_{abs} are respectively the probabilities of depolarization and absorption of the radiated photon in the optical fiber.

According to [7], the probability of depolarization of the transmitted photon in the optical fiber is determined by the:

$$P_{dep} = \left(1 - e^{-2hl}\right)/2, \tag{9}$$

h is the coefficient of cross polarization channel of the optical fiber, and l is the optical fiber length.

As follows from the Bouguer's law [10], the probability of the absorption of photons of optical radiation in the optical fiber is equal to:

$$P_{abs} = 1 - e^{-\alpha l}, \tag{10}$$

where α is the absorption coefficient.

From (9) and (10), we can see that P_{dep} and P_{abs} can vary in the range of $0.0 \sim 0.5$ and $0.0 \sim 1.0$, respectively.

After substituting the corresponding expressions (9) and (10) to (7), the formula of the data throughput communication channel will be obtained as:

$$C_{\max} = \left\{ -e^{-\alpha l} \log_2\left(0.5 e^{-\alpha l}\right) + 0.5\left(1 + e^{-2hl}\right) \times \right.$$
$$\times e^{-\alpha l} \log_2\left[0.5\left(1 + e^{-2hl}\right)e^{-\alpha l}\right] + 0.5\left(1 - e^{-2hl}\right)e^{-\alpha l} \times \qquad (11)$$
$$\left. \times \log_2\left[0.5\left(1 - e^{-2hl}\right)e^{-\alpha l}\right] \right\} / \tau_b .$$

III. RESULTS AND DISCUSSION

Based on the obtained modeling, the dependencies of depolarization and probability of absorption of transmitted photon on the optical fiber's length are calculated as illustrated in Fig. 1.
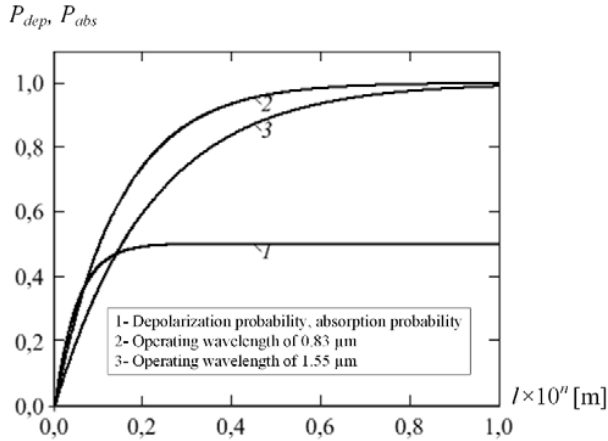
Fig. 1 The dependency of depolarization probability (curve 1, $n = 6$) and the probability of absorption of transmitted photons (curve 2 and 3, $n = 4$) on the optical fiber length. The cross polarization coefficient is $h = 10^{-5}\,\text{m}^{-1}$

As can be seen from Fig.1, depolarization probability of transmitted photons within the range l from 0 to 2.8×10^5 m increases, while within $l > 2.8\times10^5$ m takes its maximum value (theoretical limit), i.e. 0.5. Similar tendencies of change are possessed by the dependencies of probability of absorption of transmitted photons, where saturation of dependency $P_{abs}(l)$ up to the value of 1.0 for the operating wavelength of 0.83 μm is observed on a shorter optical fiber length (from 8.4×10^3m) than for the operating wavelength of 1.55 μm (from 10^4m). The reason for this is the fact that the attenuation in wavelengths of 0.83 μm and 1.55 μm is 3 dB/km and 2 dB/km, respectively. Moreover, the dependency of data throughput of optical fiber on its length for two operating wavelengths (0.83 μm and 1.55 μm) is obtained and depicted in Fig. 2. We used (11) for the calculation of the throughput $C_{\max}$. Since it is impractical to choose parts of optical fiber where P_{abs} and P_{dep} exceed 0.5, we analyzed pieces of part within the range of l from 0 to 1.5×10^3 m, where these dependencies do not exceed 0.5 (Fig. 1, curve 3). In [11]-[13], it is shown that the throughput of modern quantum cryptographic systems is mainly limited by the receiver modules. In [14], the authors manage to get the minimum value 100 ns of receiving dead time for such modules when using quantum counter based on avalanche photodetectors. According to active avalanche attenuation, τ_b is chosen to be 100 ns. With this τ_b, the maximum value of throughput can be at least 10^7 bit/s.

The dependency of throughput on fiber's length is shown in Fig. 2. As it is clear from this figure, while the fiber's length is increasing, the data throughput experiences a decreasing behavior. It is subject to the fact that with the increasing of optical fiber's length, the probability of depolarization or absorption of the radiated photons will be increased (see Fig. 1).

As a result, while fiber's length is increasing, the probability of errors in the data transmission $P(1/0)$, $P(-/0)$, $P(0/1)$, $P(-/1)$, and the conditional entropy $H(B/A)$ will be increased and the throughput will be reduced. This can be seen from the corresponding expressions (5), (7) and (8).
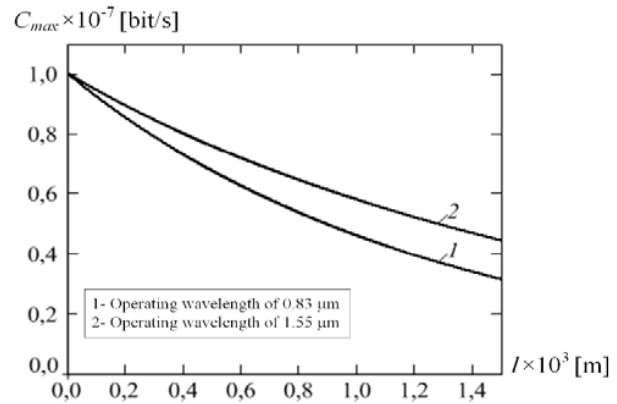

Fig. 2 Dependency of optical fiber throughput on its length. The coefficient of cross polarization coefficient is $h = 10^{-5}\,\text{m}^{-1}$

The probability of the depolarization is not depended on the operating wavelength. However, the absorption probability is depended on it as it is stated above. Therefore, the conditional entropy $H(B/A)$ is higher in case of 0.83 μm wavelength, and the throughput is lower when the transmitted wavelength is 1.55 μm.

Due to our calculations, with $l = 0.1\times10^3\,m$, $C_{\max}$ is 0.977, and with $l = 1.5\times10^3\,m$, $C_{\max}$ is equal to 0.708. Wherein $C_{\max}$ is not depended on τ_b.

Within the range l from $0 \sim 1.0\times10^2$ m, the achieved throughput for both 0.83 μm and 1.55 μm operating wavelength is almost the same with deviation of less than 2.3% (see Fig. 2).

As the receiver, the silicon avalanche photodetectors (based on germanium and gallium compounds, quantum dots) are used in our measurement which are not needed to be cooled to ultralow temperatures.

To further analyze the effect of fiber's length on throughput, two different cases are assumed. In case no. 1, photons transmission is done without considering the probability of depolarization and absorption, while in case no.

2, both mentioned probabilities are taken into account. According to these two cases, data throughput is calculated and the result is presented in Fig. 3. Based on the conducted studies, we obtained the same throughput for both 0.83 μm and 1.55 μm wavelengths. The range of the studied values for l and τ_b are set the same as ones used for the curves shown in Fig. 2.

As it is clear from Fig.3, the behavior of curves 1 and 2 are similar to curves 1 and 2 of Fig. 2, i.e. the longer the length, the more reduction in throughput in both case no. 1 and no. 2.

In case no. 1 with $l = 0.1 \times 10^3 m$, C_{max} is obtained as 0.989, and in case no. 2 with $l = 1.5 \times 10^3 m$, C_{max} is equal to 0.889. It is worth mentioning that C_{max} is not depended on τ_b.
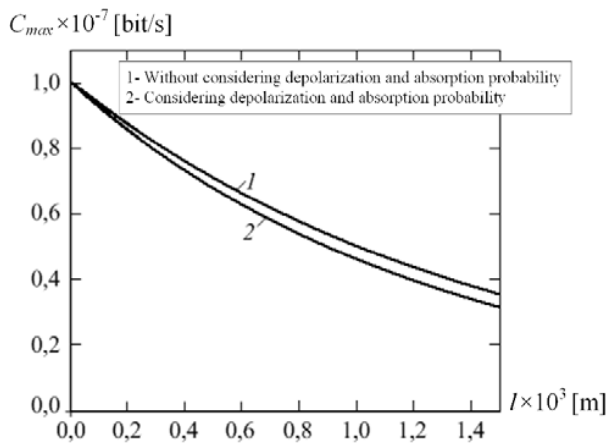

Fig. 3 Dependency of optical fiber throughput on its length, cross polarization coefficient $h = 10^{-5}\,m^{-1}$, operating wavelength is 0.83 μm

IV. CONCLUSION

A mathematical model for an optical fiber communication channel is developed, in which data are transmitted using separate photons of different polarization, and an expression for the corresponding data throughput is obtained, which takes into account the probability of depolarization and the absorption of the transmitted photons. It is observed that the effect of photons absorption on the throughput is more intensive compared to the depolarization effect. It is also revealed that in short pieces of optic fiber's length, there will be no considerable difference between use of 0.83 μm and 1.55 μm wavelengths.

REFERENCES

[1] S. I. Killeen, *"Quantum cryptography: Ideas and Practice"*. Minsk., 2007.
[2] G. Mogos, *"Cubic Quantum Security"*, International Conference on Computational Science and Computational Intelligence (CSCI), 2014.
[3] W. Liu, C. Zhu, L. Zhang, C. Pei, *"Performance Analysis of Polarization Coding BB84 Quantum Key Distribution System Under non-Markovian Channel"*, International Conference on Computer, Information and Telecommunication Systems (CITS), 2017.
[4] K. Lim, H. Ko, C. Suh, J. Kevin Rhee, *"Security Analysis of Quantum Key Distribution on Passive Optical Networks"*, Optics Express, vol. 25, no. 10, pp. 11894- 11909, 2017.
[5] M. Lopes, N. Sarwade, *"Cryptography From Quantum Mechanical Viewpoint"*, International Journal on Cryptography and Information Security (IJCIS), vol. 4, no. 2, pp. 13- 25, 2014.
[6] L. L. Kluev *"The theory of electrical connection"* Minsk, 2008.
[7] A. L. Dmitriev, *"Optical data transmission system"*. St. Petersburg., 2007.
[8] I. P. Panfilov, *"The theory of electrical connection"*. Moscow, 1991.
[9] T. V. Alekseev, *"Theory of telecommunications."* Moscow, 1991.
[10] F. C. Strong, *"Theoretical Basics of Bouguer-Beer Law of Radiation Absorption"*, ASC, pp. 338-342, 1952.
[11] A. O. Zenevich, A. M. Timofeev, *"Data Throughput of an Optical Communication Channel for the Transmission of a Message by Individual Photons"*, 2011. No. 7 (61). pp. 5-9.
[12] I. R. Gulakov, A. O. Zenevich, A. M. Timofeev, *"Instruments and Methods of Measurement"*, no. 1 (4). pp. 104-109, 2012
[13] S. A. Zenevich S. K. Komarov, A. M. Timofeev, *"Telecommunication"* no.10, pp. 14-16, 2010.
[14] A. O. Zenevich, E. V. Novikov, *"The investigation of zoned sensitivity of avalanche photodetectors operating in a single photon count mode"*, no. 1 (13), pp. 27-31, 2006.