

A Method of Drilling a Ground Using a Robotic Arm

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Abstract—Underground tunnel face bolting and pipe umbrella reinforcement are one of the most challenging tasks in construction whether industrial or not, and infrastructures such as roads or pipelines. It is one of the first sectors of economic activity in the world. Through a variety of soil and rock, a cyclic Conventional Tunneling Method (CTM) remains the best one for projects with highly variable ground conditions or shapes. CTM is the only alternative for the renovation of existing tunnels and creating emergency exit. During the drilling process, a wide variety of non-desired vibrations may arise, and a method using a robot arm is proposed. The main kinds of drilling through vibration here is the bit-bouncing phenomenon (resonant axial vibration). Hence, assisting the task by a robot arm may play an important role on drilling performances and security. We propose to control the axial-vibration phenomenon along the drillstring at a practical resonant frequency, and embed a Resonant Sonic Drilling Head (RSDH) as a robot end effector for drilling. Many questionable industry drilling criteria and stability are discussed in this paper.

Keywords—Drilling, PDE control, robotic arm, resonant vibration.

I. INTRODUCTION

THE increase of the population in cities is becoming remarkable and the relatively recent rapid urbanization phenomenon poses serious traffic problems due to number of vehicles, consequently air pollution. In this regard, it becomes relevant to exploit the underground space to circumvent these problems. In tunnel construction, the two most adopted methods are the tunneling machine and the CTM. Namely, the second method has more advantages over to its adaptation with different types of project configurations and also different types of soils. Nevertheless, the execution of this process conventional drilling in highly urbanized areas can result in settlement and considerable land movements. To remedy this, the establishment of a reinforcing proves to be an interesting alternative in order to control the soil movements in the front upstream. However, this technique requires direct exposure of the workers to an enormous danger. Hence, the proposition of the *Newtun* project implies many academic institutions and industrial companies in the domain

For tunnel reinforcing, a CTM uses a drilling machine by means of a head embedded on a slide link of length between 3 and 24 m. The idea was directed towards the study of the drilling head mounted on a robot arm, and we get a *virtual slide link*, ambitiously this solution was declared for tunnel reinforcing in the future [7]. Note that axial vibrations are generated by a resonant sonic drilling head, and a wave is transmitted along the drillstring from the top boundary to the

tip boundary where a bouncing bit is fixed. These vibrations have a major impact on the performance of the drilling and can affect the stem penetration rate and direction of drilling. Unlike rotary drilling where the vibrations are those of torsion which require neutralization to avoid the phenomenon of stick-slip [8]-[11], axial vibrations are necessary to fracture the rock.

In order to ensure a drilling operation that meets the ground nature and loads, a vibration reference model is generated under an input force of which the amplitude depends on the needed vibration frequency. It is called resonance frequency. At this frequency, the bit's amplitude of vibration reaches its maximum value, and the generated mechanical energy remains stable at a significant needed value for the drilling operation.

In the literature, several methods have been proposed to control vibrations produced along the drillstring. The most popular control techniques are listed below:

- *Feedback control*: Presented by Halsey et al. [12], this method measures the torque at the surface of the drillstring and uses it as a state feedback. Therefore, the waves will be attenuated at the surface instead of being returned to the drillstring. The main disadvantage of this strategy is that it requires precise torque measurement, which remains difficult to obtain in practice because the measurements are made during drilling.
- *Soft torque rotary system (STRS)*: Presented by Sananikone [13], it is an improvement of the first method. It avoids the torque measurement task at surface by calculating it through the motor current.
- *PID control*: Introduced by Pavone and Desplans [14], it is a simple strategy to avoid the phenomenon of stick-slip. The gains of the PID control are obtained by an appropriate stability analysis. The disadvantage of this technique is that the vibrations are not sufficiently attenuated, and conditions of optimality are difficult to obtain.
- *Vibration absorber*: Presented by Jansen [15], it contributes to the first method improvement. But the feedback control uses the electrical variables (current and voltage) instead of the mechanical parameters. H_∞ control was proposed by Serrarens et al. [16]. The H_∞ control has robust qualities, and ensures stability in presence of modeling errors. However, in order to obtain a good performance, a very precise model is required. Another disadvantage of this method is that the saturation constraints are not well handled.
- *Drilling Oscillation KILLer (D-OSKIL)*: This method is presented by Canudasde-wit et al. [17]. It uses the Weight On the Bit (WOB) as control variable. An optimal

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compromise between the WOB and the penetration must be found. To adjust the control law, the implementation requires the repetitive addition and removal of the stabilizer sections.

- *Active Vibration Damper (AVD)*: Introduced by Cobern and Wassell [18], the basic idea of this method is to increase the viscous friction at the tip boundary (bottom of the hole). The damping coefficient is modified through the injected fluid which allows to manipulate the viscosity properties. This contributes to the stick-slip vibration attenuation. However, drilling optimality conditions require additional control variables.
- *Modeling error compensation*: Presented by Puebla and Alvarez-Ramirez [19], it consists of performing a feedback control at the Bottom Hole Assembly level (BHA). It is a robust method compared to the unknown parameters of the drillstring and the friction term.

Despite the development of many methods to control vibration, today, such phenomena still affect drilling performance. This is mainly due to lack of understanding the system dynamics. In fact, most of the proposed techniques were based on simplified models with localized parameters (EDO) that do not respect the distributed nature of the system. As a result, new methods have been developed using parameter models (EDPs). Lyapunov techniques were used by Challamel [20], Saldivar et al. [21] [22], and Alli et al. [23] to ensure asymptotic, exponential and practical stabilities. In [24]-[26] the methods of flatness and backstepping have been applied to construct control laws for tracking. Since oil extraction machines engage a rotary head, the torsion vibrations are the most important and their decrease becomes paramount to prevent deterioration of the bit. Thus, certain methods mentioned above have been used mainly to attenuate torsional vibrations in the oil field. Then, a more in-depth analysis of the drilling process of the attenuation of coupled torsional and axial vibrations. In the case of the Newtun project (described in the following section), drilling is sonic. Consequently, axial vibrations are the most apparent. Unlike petroleum drilling, it is not expected to reduce these vibrations because they are indispensable for drilling, but they must be mastered to optimize drilling.

The remaining of this paper is as follows: Section II describes the Newtun project and its impact on the conventional drilling automation. The importance of the proposed robot-arm in positioning the sonic head is shown. In Section III, we describe the drillstring axial vibrations through an EDP model where the boundary dynamics are introduced. The well posedness of this description is given. An energy based control is detailed in Section IV, where we prove the asymptotic stability in regard a defined reference vibration model. Finally, simulations and comments are achieved in Section V.

II. NEWTUN PROJECT

Since the middle of the 20th century, the world has experienced a very rapid acceleration of urbanization, which is reflected in the increase of the population in the cities.

This urban population increases the number of very large cities. There were, in 1950, more than 10 million inhabitants in New York and London. The high densification of urban agglomerations poses many problems. leads to the necessity of the exploitation of the underground. The development of underground structures (car parks, road tunnels, railways etc.) makes it possible to limit the congestion and to contribute strongly to the distribution of flows. These structures in urban areas meet often problems. The construction of tunnels is particularly difficult to which may cause damage to the surrounding structures. This threat makes underground work difficult to ensure and integrate into management plans. The development of Tunnel Boring Machine (TBM) was the first response from the construction industry to the concern of building owners. In 1953, Robbins built the first truly successful tunneling machine, of a cylindrical shield carrying a rotating head provided with peaks and rollers for to cut the rock (for details see [1]). Today, tunnel boring machines have become sophisticated industrial machinery and which can be considered as a factory underground. This method of digging offers a very high level of performance (up to 20 ml of tunnel per day) and environmental safety. On the other hand, high level of investment, very important preparatory work are usable only for long tunnels of constant section. In short, tunnel boring machines provide a safe and effective solution, but not sufficiently flexible to cover all needs where the use of a tunnel boring machine does not meet the requirements of the use the Conventional Method (CTM), also known Austrian method which can be defined by a cyclical process in 3 main steps:

- Digging: With explosives, or with the mechanical shovel or with the aid of a puncturing.
- Marinating: Loading and evacuation of cuttings.
- Support: A temporary support, installed at the time of digging (metal hangers, bolts, shotcrete ..., followed by a waterproofing membrane, then a definitive support (projected concrete, prefabricated concrete ...).

The conventional method lends itself to all configurations of projects and to a variety of soil types or rocks. Supporting and carrying out the work can be adapted in successive digging steps, and adjusted more finely to the depending on the soil actually encountered. This method is therefore a process agile and flexible and remains the only alternative for renovation of existing tunnels or to create the emergency access required by the new security standards. The digging of the tunnel is carried out by perforating the soil or rock starting with the cutting face. For this, nestable drill rods are used (1). These rods can be assembled and connected one by one during drilling to drill string with a drilling bit. The advance of the digging is done by section while, for security, the configuration of drilling is subject to geological firing plan.

A. Technical Solution Improvement

In highly urbanized areas, the use of the conventional method in fact causes large settlements and ground movement, with consequences human and economic benefits. Thus, the reinforcing in the working front presents a solution for the

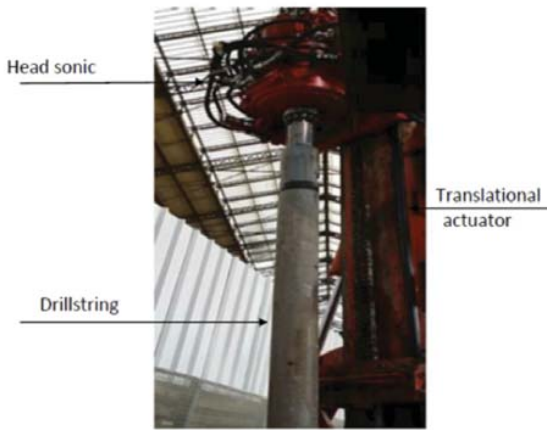


Fig. 1 A conventionnel drilling machine

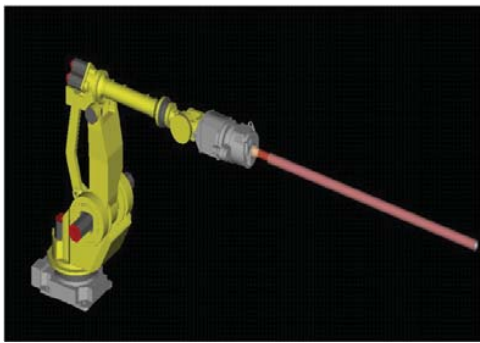


Fig. 2 Drilling robot carrying a RSH

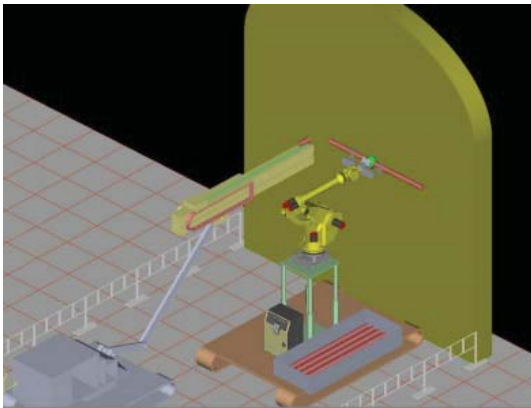


Fig. 3 Drilling scenario by a robot-arm

realization of a tunnel in grounds difficult and unpredictable situations where settlement should be limited. The importance of deployment of this type of reinforcement varies according to the nature of the soil and the excavation method.

There are different types of reinforcement, like front bolting, umbrella arch, and vault. For example, the bolting technique at the front consists of sealed bolts continuous drilling installation in sub-horizontal boreholes. This method increases the resistance of the front massif. Also, this leads to significant improvement of site safety. The bolts used in this type of

detenting are generally made of fiberglass, their high tensile strength is about 200 to 1000 MPa. At the same time their low shear strength (180 MPa), permits their easy destruction by earth-moving machinery. However, in the case of in situ stripping tests, it is necessary to use steel tubular self-drilling bolts. The continuous sealing of bolts is made with cement grout or with resin (for more details see [1]).

The main task is to robotize the drilling, more precisely the robotization of reinforcing techniques in tunneling (bolting to the forehead, umbrella arch). Given the enormous gap between the public works community and industry where there is the use of robots, a dialogue and many exchanges took place between Soletanche Bachy and our IBISC laboratory team to determine the combining existing drilling machines and robotics. These machines perform the drilling process by means of a head mounted on a slide length between 3 and 24 m. Therefore, the collective reflections were directed towards the possibility study of a drilling head on a robot in order to be able to get a *virtual slide link*. Given that these machines require semi-manual feeding of rods from a rack. We began by considering the integration of robots for this task of handling for loading and unloading the stems during drilling and thus preventing major human hazards to which the workers are exposed (see Figs. 2, 3). Consequently, the main challenge at first is to control axial vibrations under a resonant frequency. This permits not only to preserve the robot-arm structure but also to maintain the bit's amplitude at a high level in an unknown environment.

III. MODELING: WELL POSEDNESS OF THE PDE PROBLEM

The axial vibration characteristics for modeling are based on the structure given by Fig. 4. It introduces the axial displacement of the drillstring section, and the boundary conditions. The following notations are used $x \in [0, L]$:

- $u(x, t)$ axial displacement, with $x = 0$ at the top (drilling head), and $x = L$ at the tip (drilling bit).
- $u_t(x, t)$ the time derivative of $u(x, t)$.
- $u_x(x, t)$ the space derivative of $u(x, t)$.
- $H(t)$ drilling force with sine-form, function of the drillstring natural frequencies and the Sonic Drill Head geometric parameters. It is a control input.
- $F(u_t(L, t))$ is the Coulomb friction at the tip, depends on the ground characteristics.
- E is the elasticity modulus, and ρ the drillstring density.
- A is the drillstring cross section, and M the drill bit mass.
- α is the viscous friction at the top, and β is the viscous friction between the drillstring and the ground

From Fig. 4, the resulting axial vibration model is as (more details about these vibrations can be found in [2], [3] and [4]):

$$u_{tt}(x, t) = \frac{E}{\rho} u_{xx}(x, t) - \frac{\beta}{\rho A} u_t(x, t), \quad t > 0, 0 < x < L(1)$$

with the boundary conditions:

$$\begin{aligned} u_x(0, t) &= \frac{\alpha}{\rho A} u_t(0, t) - \frac{1}{AE} H(t) \\ u_x(L, t) &= -\frac{M}{AE} u_{tt}(L, t) - \frac{1}{AE} F(u_t(L, t)) \end{aligned} \quad (2)$$

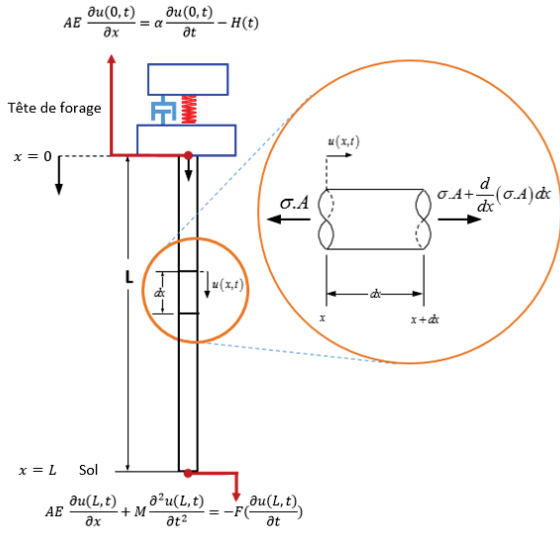


Fig. 4 Axial vibrations structure

A. Bit-Ground Interaction: Friction Models

The description of the interaction between the drilling bit and the ground is a crucial aspect for vibrations modeling. In fact, it is well known that the oscillation mechanism results from the friction force produced at the lower end. Several approaches of modeling can be found in the specialized literature (see [8]-[10], and the references therein). A model of friction makes it possible to obtain an overview of the vibratory phenomena thus characterizing the dynamic behavior of the drilling bit and making it the development of appropriate control strategies.

The friction force $F(u_t(L, t))$ introduced in [10] is given by the following nonlinear form:

$$F(u_t(L, t)) = c_b u_t(L, t) + c_a \mu_{outil}(u_t(L, t)) \operatorname{sgn}(u_t(L, t)) \quad (3)$$

The term $c_b u_t(L, t)$ denotes the viscous friction at the tip boundary, while the product $c_a \mu_{outil}(u_t(L, t)) \operatorname{sgn}(u_t(L, t))$ is the dry friction, with

$$\mu_{outil}(u_t(L, t)) = \mu_{cb} + (\mu_{sb} - \mu_{cb}) e^{-\gamma_b |u_t(L, t)|} \quad (4)$$

and $\mu_{sb}, \mu_{cb} \in (0, 1)$ are the static and Coulomb frictions, respectively, $0 < \gamma_b < 1$.

B. Well-Posedness of PDE Problem

Let $u(x, t)$ be the solution. One defines the vector $Q(t) = (u(\cdot, t), u_t(\cdot, t), u_t(L, t))^T$, leading to the following compact writing:

$$\dot{Q} = PQ + F(Q), \quad Q_0 \in X = K_1([0, L]) \times L_2([0, L]) \times \mathbb{R}$$

with

$$K_1([0, L]) = \{u \in H^1([0, L]) / u(L, t) = 0 \text{ and } \langle u, v \rangle_{K^1} = \int_0^L u_x v_x dx\}$$

with

$$P = \begin{pmatrix} 0 & 1 & 0 \\ \frac{E}{\rho} \frac{\partial^2}{\partial x^2} & \frac{-\beta}{\rho A} & 0 \\ \frac{AE}{M} < \delta'_L(x), \cdot > & 0 & 0 \end{pmatrix}$$

such that $\langle \delta'_L(x), u(x, t) \rangle = -u_x(L, t)$. From the Lumer-Philips theorem, we may prove that the operator P generates a C_0 semigroup with contraction $\{e^{Pt}\}_{t>0} \in X$.

IV. TRACKING OF REFERENCE VIBRATIONS

A first objective is to define the control input $H(t)$ and prove the stability of the system. For this purpose, the calculation of the energy accumulated during this task and the study of its variation time will guide us on the design of the control law. Indeed, the use of energy has been widely addressed in the problem of stabilization, controllability and observability of PDE [5], [6]. At this step, we define a reference vibration model that should the system (drillstring, drill bit, percussion force) be followed (the superscript r denotes the reference):

$$u_{tt}^r(x, t) = \frac{E}{\rho} u_{xx}^r(x, t) - \frac{\beta}{\rho A} u_t^r(x, t), \quad t > 0, \quad 0 < x < L \quad (5)$$

with the reference boundary conditions:

$$\begin{aligned} u_t^r(0, t) &= \frac{AE}{\alpha} u_x^r(0, t) - \frac{1}{\alpha} H^r(t) \\ u_t^r(L, t) &= -\frac{1}{M} F^r(u_t^r(L, t)) \end{aligned}$$

Let us define the errors $e(t) = u(\cdot, t) - u^r(\cdot, t)$, $e_x = u_x - u_x^r$, $H_e(t) = H(t) - H^r(t)$, and $F_e(t) = F(u_t(L, t)) - F^r(u_t(L, t))$. The PDE model of errors with the boundaries is as:

$$\begin{aligned} e_{tt}(x, t) &= \frac{E}{\rho} e_{xx}(x, t) - \frac{\beta}{\rho A} e_t(x, t), \quad t > 0, \quad 0 < x < L \\ e_t(0, t) &= \frac{AE}{\alpha} e_x(0, t) - \frac{1}{\alpha} H_e(t) \\ e_{tt}(L, t) &= -\frac{1}{M} F_e(t) \end{aligned} \quad (6)$$

The main objective is to prove that system of errors is asymptotically stable with the appropriate determination of the residual control input $H_e(t)$, consequently $H(t)$. Let us examine the following proposition.

Proposition 1: The asymptotic convergence of the drill pipe axial vibrations is asserted under the following control input at the top boundary

$$H(t) = (AE - \frac{\alpha E}{\rho}) e_x(0, t) + \alpha e(0, t) + H^r(t) \quad (7)$$

□

Proof: Let us define the system's energy obtained from the above system's of errors:

$$E_e(t) = \frac{1}{2} \int_0^L \frac{E}{\rho} e_x^2(x, t) dx + \frac{1}{2} \int_0^L e_t^2(x, t) dx + \frac{1}{2} e^2(0, t) \quad (8)$$

The time derivative is given by:

$$\begin{aligned} \frac{dE_e(t)}{dt} &= \frac{E}{\rho} \int_0^L e_x(x,t) e_{xt}(x,t) dx \\ &+ \int_0^L e_t(x,t) e_{tt}(x,t) dx + e(0,t) e_t(0,t) \end{aligned}$$

From (6), we can write

$$\begin{aligned} \frac{dE_e(t)}{dt} &= \frac{E}{\rho} \int_0^L e_x(x,t) e_{xt}(x,t) dx \\ &+ \frac{E}{\rho} \int_0^L e_t(x,t) e_{xt}(x,t) dx \\ &- \frac{\beta}{\rho A} \int_0^L e_t^2(x,t) dx + e(0,t) e_t(0,t) \end{aligned}$$

After an integration by parts, we obtain:

$$\begin{aligned} \frac{dE_e(t)}{dt} &= \frac{E}{\rho} \int_0^L e_x(x,t) e_{xt}(x,t) dx + \frac{E}{\rho} e_t(L,t) e_x(L,t) \\ &- \frac{E}{\rho} e_t(0,t) e_x(0,t) - \frac{E}{\rho} \int_0^L e_x(x,t) e_{xt}(x,t) dx \\ &- \frac{\beta}{\rho A} \int_0^L e_t^2(x,t) dx + e(0,t) e_t(0,t) \end{aligned}$$

Using the tip boundary in (6), one obtains:

$$\frac{dE_e(t)}{dt} = e_t(0,t) \left[-\frac{E}{\rho} e_x(0,t) + e(0,t) \right] - \frac{\beta}{\rho A} \int_0^L e_t^2(x,t) dx$$

Let $V(t) = e_t(0,t)$, then from the system equations

$$V(t) = \frac{AE}{\alpha} e_x(0,t) - \frac{1}{\alpha} H_e(t) \quad (9)$$

Consequently,

$$\frac{dE_e(t)}{dt} = V(t) \left[-\frac{E}{\rho} e_x(0,t) + e(0,t) \right] - \frac{\beta}{\rho A} \int_0^L e_t^2(x,t) dx$$

If we take

$$V(t) = -\left[-\frac{E}{\rho} e_x(0,t) + e(0,t) \right] \quad (10)$$

Also, from (9), and the defined $V(t)$ by (10), the residual control input $H_e(t)$ is as:

$$H_e(t) = (AE - \frac{\alpha E}{\rho}) e_x(0,t) + \alpha e(0,t) \quad (11)$$

with $H_e(t) = H(t) - H^r(t)$. $H^r(t)$ is a known function as it was defined from the references. So, we may define easily the control variable $H(t)$, as it is given by the proposition. Hence, it is straightforward that $\frac{dE_e(t)}{dt} \leq 0$. At this stage, we proved only stability of the drillstring including the drill bit. It remains to prove the system's asymptotic stability. The LaSalle's invariance principle is used here to prove that the only equilibrium is 0 when the energy $E_e(t)$ decreases. From $E_e(t) = 0$, we have

$$e_x(0,t) = 0, \quad e_t(0,t) = 0, \quad e(0,t) = 0$$

From $e_x(0,t) = 0$, we understand that $e(x,t) = \Phi(t)$ (only function of time). Further, from $e_t(0,t) = 0$, $\Phi(t) = cst$ as $\frac{d\Phi(t)}{dt} = 0$. From $e(0,t) = 0 \quad \forall t$, we obtain $\Phi(0) = 0$,

consequently, $\forall t \quad \Phi(t) = 0$. This means that $E_e(t) = 0$ equivalent to $e(x,t) = 0 \quad \forall x, t$. On the other hand, from $\frac{dE_e(t)}{dt} = 0$, we get

$$e(0,t) = \frac{E}{\rho} e_x(0,t), \quad e_t(x,t) = 0$$

As $e_t(x,t) = 0 \quad \forall x$, then $e(x,t) = \Psi(x)$. This also implies that $e_{tt}(x,t) = 0$. From (6), we prove that $e_{xx}(x,t) = 0$. Consequently, $\Psi'(x) = cte$. Let $\Psi(x) = \varrho x + \xi$. with $\Psi(0) = \xi$ and $\Psi(L) = \varrho L$. Now, from using $e(0,t) = \frac{E}{\rho} e_x(0,t)$, this relation can be defined $\Psi(0) = \frac{E}{\rho} \Psi'(0)$ or $\xi = \frac{E}{\rho} \varrho$. We have also $e_x(L,t) = 0$, then $\Psi'(L) = 0$. This leads to $\varrho = 0$ and $\xi = 0$. As a result $\frac{dE_e(t)}{dt} = 0 \Leftrightarrow e(x,t) = 0 \quad \forall x, t$. We conclude that the proposed control law $H(t)$ conducts to the asymptotic convergence of the system's states to the equilibrium. ■

V. SIMULATION RESULTS

In order to validate our theoretical investigation and the proposed control scheme, the vibrations reference behavior and the system's (drillstring, drill bit, applied force) model are solved numerically. The PDE represents a damped wave equation with dynamics in the boundary. A drillstring natural frequency currently used in practice as resonant frequency is $\omega = 120\text{Hz}$. While the remaining system's variables are shown in Table I,

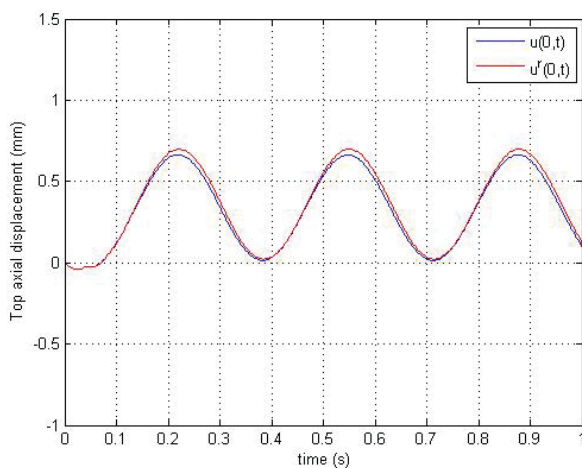
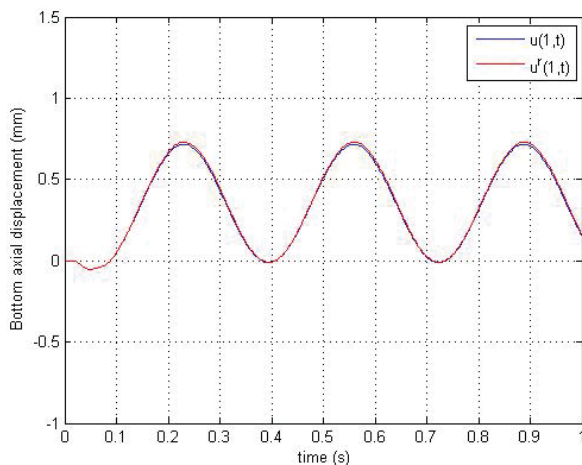
TABLE I
THE SYSTEM PARAMETERS

E	210 GPa	A	0.0146 m ²
ρ	7850 kg/m ³	L	3 m
M	150 kg	α	0.02 kg.s ⁻¹
β	0.02 Kg.m.s ⁻¹	δ	0.03

Figs. 5 and 6 show the real and reference trajectory at the top boundary and the tip boundary, respectively. Under the obtained control-input, an asymptotic convergence of the drillstring and the drill bit is theoretically demonstrated and shown by simulations. Note that a coulomb-friction parameters defining a known ground environment are adopted. Also, we consider that the measurement of $u(L,t)$ is possible or can be estimated.

VI. CONCLUSION

The operation of reinforcing in tunnels is carried out manually/semi-autonomous by means of a drilling machine where the security, the time of realization, and reliability are questionable. A robotic arm in reinforcing was proposed during the Newtun project. However, embarking a head, like the resonant sonic drill head, is not straightforward as it generates axial vibrations along the drillstring. The axial vibrations must be rigorously studied before to be embedded to the robot arm. First, the axial vibration control problem proved that is well posed, and an energy-based control analysis was defined. An asymptotic convergence in the Lyapunov sense towards the reference trajectory was obtained and tested in simulations. The stability results guarantee the use of a robot

Fig. 5 The real and reference trajectories at the top $x = 0$.Fig. 6 The real and reference trajectories at the tip $x = L$.

arm, but the solution should take into account another factor due injected fluid dynamic dynamics. The fluid, under high pressure, used to evacuate the cut rock impacts the axial vibration. It will be our investigation in the future to complete the method of drilling a ground using a robotic arm.

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