

# Prediction of Product Size Distribution of a Vertical Stirred Mill Based on Breakage Kinetics

C. R. Danielle, S. Erik, T. Patrick, M. Hugh

**Abstract**—In the last decade there has been an increase in demand for fine grinding due to the depletion of coarse-grained orebodies and an increase of processing fine disseminated minerals and complex orebodies. These ores have provided new challenges in concentrator design because fine and ultra-fine grinding is required to achieve acceptable recovery rates. Therefore, the correct design of a grinding circuit is important for minimizing unit costs and increasing product quality. The use of ball mills for grinding in fine size ranges is inefficient and, therefore, vertical stirred grinding mills are becoming increasingly popular in the mineral processing industry due to its already known high energy efficiency. This work presents a hypothesis of a methodology to predict the product size distribution of a vertical stirred mill using a Bond ball mill. The Population Balance Model (PBM) was used to empirically analyze the performance of a vertical mill and a Bond ball mill. The breakage parameters obtained for both grinding mills are compared to determine the possibility of predicting the product size distribution of a vertical mill based on the results obtained from the Bond ball mill. The biggest advantage of this methodology is that most of the minerals processing laboratories already have a Bond ball mill to perform the tests suggested in this study. Preliminary results show the possibility of predicting the performance of a laboratory vertical stirred mill using a Bond ball mill.

**Keywords**—Bond ball mill, population balance model, product size distribution, vertical stirred mill.

## I. INTRODUCTION

STIRRED milling technology has been firmly established in the last 20 years as superior to ball mills for fine and regrinding operations due to its superior energy efficiency [1]. Stirred mills are now commonly used in many sectors of the mining industry, though they have been used in other industries for many years [2]. This technology has proven to be more energy efficient with greater opportunities for future optimization in both fine and coarse grinding.

The first stirred vertical mill was developed in Japan by the Japan Tower Mill Company Ltd which was later renamed to Kubota Tower Mill Corporation, KTM. The Japanese Tower Mill was the first vertical grinding mill to be used in the mining industry [3]. The Tower Mill<sup>®</sup> is now produced by Nippon-Eirich. The Vertimill<sup>™</sup> is a modified and improved version of the Tower Mill<sup>®</sup> and it was developed by Metso, Inc. The Vertimill<sup>™</sup> and the Tower Mill<sup>®</sup> have similar design

configurations. Both technologies are gravity-induced mills that use high density grinding media as the charge. The Tower Mill<sup>®</sup> and the Vertimill<sup>™</sup> are typically operated in a closed circuit, where the non-comminuted product material returns to the mill to enhance energy efficiency. The rotating and lifting action generated by the helical agitator is responsible for the movement of the grinding media and the grinding mechanism within the mill [3]. Fig. 1 illustrates the Tower Mill<sup>®</sup> and the Vertimill<sup>™</sup>.

The efficient operation of grinding mills requires that parameters such as feed rate, feed size distribution, solids concentrate, slurry density, grinding media size distribution, and grinding power should be constantly monitored and adjusted for better grinding results. The smaller grinding media size used in stirred mills increase the contact probability between the media and the particles, and, therefore, the number of stress events inside the mill also increases. Collisional energy in stirred mills is also not lost by high-intensity impacts between the grinding media and the equipment internal walls. Thus, stirred mills have been preferred for fine, ultra-fine and regrinding operations.

Metso, the manufacturer of the Vertimill<sup>™</sup>, has been using batch tubular ball mills to scale-up vertical mills successfully. In theory, a laboratory size vertical stirred mill could be used to scale-up industrial units. However, the vertical stirred mill geometry requires that the balls must be proportionally scaled-down. This also leads to a reduction in the feed size that can be tested in a small scale vertical mill to preserve the ratio between balls size and feed size. Therefore, the reduced feed size may not reflect an industrial size operation [4]. Batch ball mill tests can predict vertical stirred mill power using an efficiency factor to correct the higher energy efficiency of vertical stirred mills. This scale-up procedure can be used together with the *selection function* to consider other relevant operational parameters that can affect particles breakage.

In order to simplify the methodology in this study, a Bond ball mill is used to collect the breakage parameters that will be compared to the same values obtained in a laboratory scale vertical stirred mill testing the same material. The breakage kinetics of both mills is compared to define a relationship that will allow the prediction of product size distribution of a laboratory vertical stirred mill from a Bond ball mill. It is important to notice that the ratio between the efficiencies of both mills is independent of the ore type. All processes and operating conditions were measured under controlled conditions to check the accuracy of the results obtained.

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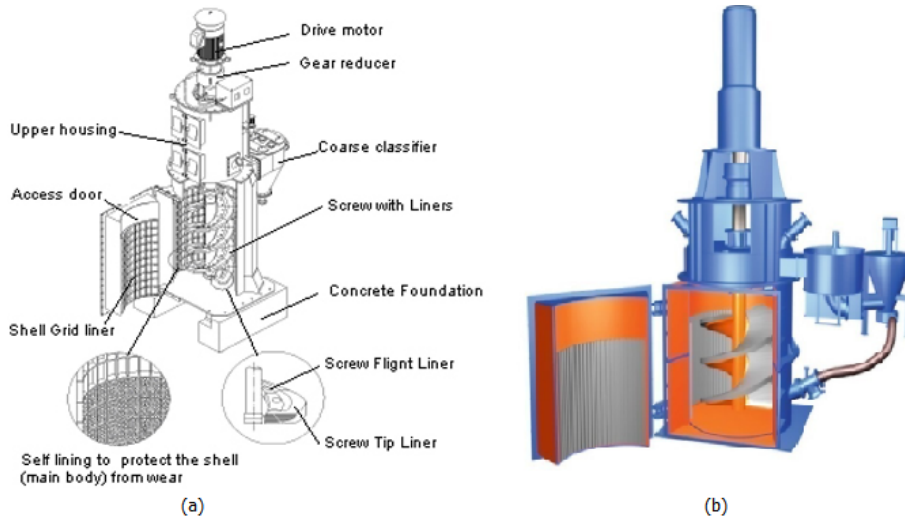


Fig. 1 (a) Tower Mill® (Eirich courtesy) and (b) Vertimill™ (Metso courtesy)

The objective of the work is to predict the performance of a laboratory vertical stirred mill from parameters obtained in a batch Bond ball mill using small quantities of material that will be later considered to develop a scale-up methodology.

## II. POPULATION BALANCE MODEL

PBMs have found increased usage in the design, optimization, and control of grinding circuits due to its ability to predict complete size distribution of the product [5]. In these models, the breakage behavior of each particle of a given size class can be calculated.

The Population Balance Equation is a mathematical description of the evolution of the particle size distribution when submitted to grinding processes over time in a batch operation. Equation (1) illustrates the size-mass balance model [6].

$$\frac{dm_i(t)}{dt} = -S_i m_i(t) + \sum_{j=1}^{i-1} b_{ij} S_j m_j(t), i = 1, 2, \dots, n \quad (1)$$

where  $m_i(t)$  is the mass fraction of particles contained in size interval  $i$  after grinding time  $t$ ;  $b_{ij}$  represents the size distribution of particles in the intervals  $i$  after a breakage event of particles in size interval  $j$ ;  $S_i$  represents the selection function or the specific rate of breakage of particles in size interval  $i$ .

### A. Breakage Function

The term  $b_{i,j}$  represents the breakage function from a feed particle in size  $j$  to a product size distribution in intervals  $i$ . A simple method to calculate the cumulative breakage function, denominated  $B$ , is to do it experimentally by taking a sample of material in one size fraction, grinding it for a pre-determined time, and then determining the product size distribution by sieve analysis. The use of the *BII* method developed by Austin et al. [6] can be employed. In this approach, to compensate for re-breakage of primary fragments, the product of the breakage and selection functions,

$S_j B_{ij}$ , is considered approximately constant [7]. It is important to assure that no more than 30% of the initial mass is broken to avoid errors related to material re-selected for breakage when using the *BII* method. Based on these conditions and according to the *BII* method, the breakage distribution function can be calculated using (2).

$$B_{i,1} = \frac{\log\left(\frac{1-P_i(0)}{1-P_i(t)}\right)}{\log\left(\frac{1-P_2(0)}{1-P_2(t)}\right)} \quad (2)$$

where  $P_i(t)$  is the cumulative mass fraction less than size interval  $i$  at time  $t$ ; and  $B_{i,1}$  is the cumulative mass fraction of particles passing the top size interval  $i$  from breakage of particles of size  $j=1$ .

The term  $B_{i,j}$  represents the cumulative weight fraction of the material broken from size  $j$  which falls into size intervals below the upper size of size interval  $i$ . Equation (3) illustrates the non-cumulative form of the breakage function.

$$B_{i,j} = B_{i,j} - B_{i+1,j} \quad (3)$$

### B. Selection Function

The selection function represents the breakage rate of particles in the size interval  $i$ . The rate of disappearance of material in the size 1 is described by (4).

$$\frac{-dm_i(t)M}{dt} = \alpha m_1(t)M \quad (4)$$

Considering that the total mass,  $M$ , inside the mill is constant:

$$\frac{dm_i(t)}{dt} = -S_i m_i(t) \quad (5)$$

The term  $S_j$  is assumed to be constant, and therefore does not vary with time. The integration of (5) results in:

$$m_1(t) = m_1(0)e^{(-S_1 t)} \quad (6)$$

where  $S_i$  ( $\text{min}^{-1}$ ) is the selection function for size interval  $i$  and it is considered equipment, operating conditions, and material dependent.

### III. EXPERIMENTAL

#### A. Samples

Commercial granite for civil construction and pavement is used in the laboratory tests. The as-received aggregate of minus 19 mm of diameter was reduced in size using a jaw crusher and a roll crusher in two stages. The final product was classified based on its size. The single size intervals used in this work are described in Table I.

TABLE I  
SINGLE SIZE INTERVALS

Interval	Size ( $\mu\text{m}$ )
1	-841 +595
2	-595 +420
3	-420 +297
4	-297 +210
5	-210 +149

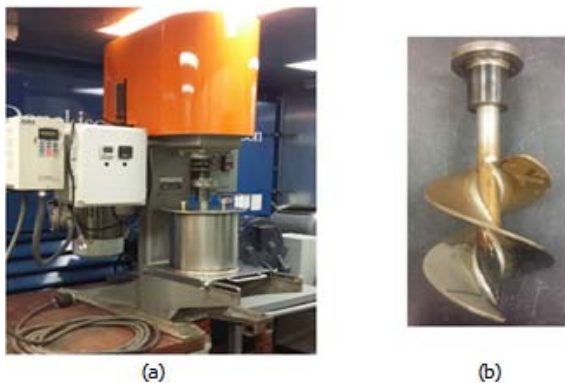


Fig. 2 (a) Laboratory vertical stirred mill, and (b) Agitator

#### B. Bond Ball Mill

A Bond ball mill was used to collect the breakage parameters to be used in the PBM (1). The *BII* method developed by Austin, Klimpel, and Luckie [6] is used to determine the breakage function values for each single size fraction described in Table I. The total material weight for each test is 1890 grams. The same standard operating conditions for the Bond Work Index test were used to collect the fine breakage parameters [8]. Grinding time was defined to be 20 seconds and the operation environment was kept dry. The selection function was calculated using six grinding times: 20, 55, 130, 195, 285, and 405 seconds. The product size distribution is measured and recorded after each grinding time.

#### C. Laboratory Vertical Stirred Mill

A small size vertical stirred mill was manufactured for this study. The geometry of the laboratory unit represents 1/20 of

the industrial size Vertimill™ VTM-1500. A variable speed and a torque meter were also installed. Fig. 2 illustrates the laboratory vertical mill used in this work.

Table II summarizes the operating conditions used in the tests performed in the vertical stirred mill. The breakage function was calculated after 30 seconds of grinding time. The selection function was determined using four grinding times: 30, 85, 155, and 275 seconds. The product size distribution is also measured and recorded after each grinding time.

TABLE II  
OPERATING CONDITIONS USED IN THE VERTICAL STIRRED MILL TESTS

Grinding media size	mm	9.5
Agitator velocity	rpm	130
Grinding media density	kg/m <sup>3</sup>	7850
Grinding media total mass	kg	20.2
Ore density	kg/m <sup>3</sup>	2700
Ore total mass	kg	1.615
Solids percent in mass	%	35

### IV. BREAKAGE PARAMETERS

The breakage and selection functions were determined in laboratory physical tests as a function of grinding time. The methodology described by Austin et al. [6] was used. Fig. 3 illustrates the breakage function determined for the Bond and vertical stirred mills tested.

The breakage function represents the product size distribution in intervals  $i$  after the breakage of particles from interval  $j$ . This parameter is calculated based on the relationship between the masses of product particles in each single size interval  $i$  and the original particle in size  $j$ . The breakage process in a ball mill is a combination of attrition between the ore particles, and impact between the balls and the ore. The final product size distribution is directly dependent on the breakage environment promoted by the equipment and the ore characteristics. The research center Julius Kruttschnitt (JKMRC) considers the breakage function to be material specific and dependent on the breakage energy promoted by the equipment [9]. If the total breakage energy promoted by the mill is less than the necessary to fracture the ore particles, the material will not break and internal fractures may be developed. If the breakage energy is higher, the excess may be used for secondary breakage processes on the product of the primary breakage event.

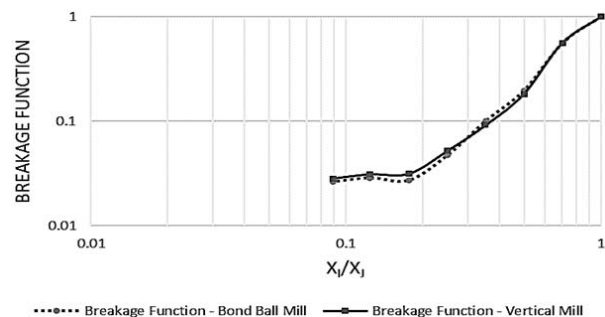


Fig. 3 Breakage function determined for the Bond ball mill and vertical stirred mill tested

The Bond ball mill and vertical stirred mill breakage functions show similarity for the size interval 1 analyzed in Fig. 3. The differences in the results obtained may be related to the breakage mode promoted by the two grinding mills. Vertical stirred mills promote a higher number of low-energy collisions and a lower number of high-energy collisions when compared to a ball mill [10].

Fig. 4 illustrates the selection functions determined for the Bond ball mill and vertical stirred mill. It is possible to see that the rate of breakage of finer particles is higher in the vertical mill. The size of the grinding media is the variable that most influences the grinding performance [11]. Therefore, the higher rate of breakage calculated in the vertical mill may be associated to the size of balls selected for the test. Other operating variables are currently being tested to confirm this hypothesis considering a laboratory size vertical stirred mill.

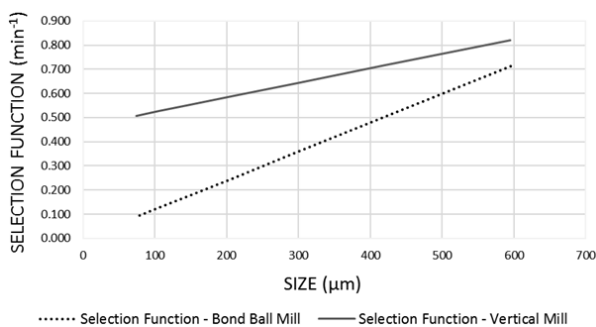


Fig. 4 Selection function determined for the Bond Ball Mill and Vertical Stirred Mill tested

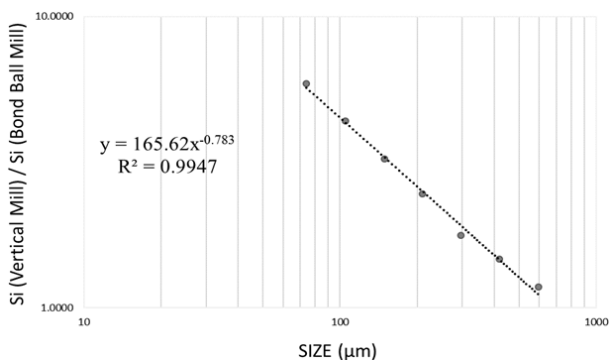


Fig. 5 Relationship between the selection function obtained from the vertical mill and Bond ball mill tested

The higher rate of breakage of the vertical stirred mill tested is in accordance with the fact that this equipment has a higher energy efficiency than ball mills. The result obtained suggests that the energetic efficiency in the vertical mill compared to a Bond ball mill increases with a reduction in feed particle size

for the material tested. Fig. 5 illustrates the relationship between the selection functions obtained for the Bond and vertical mills tested.

The power mathematical relationship between the two selection functions determined for the grinding mills tested indicate that the values increase at a specific rate. The R-square value of 0.9947 suggests that an almost perfect relationship exists. Equation (7), obtained from Fig. 5, can be used to calculate the selection function of the vertical mill tested based on the selection function results obtained from a Bond ball mill.

$$Si_{VM} = Si_{BM} 165.62p^{-0.783} \quad (7)$$

where  $Si_{VM}$  is the selection function for the vertical stirred mill used in this work;  $Si_{BM}$  is the selection function obtained using a Bond ball mill; and  $p$  is the feed size in  $\mu\text{m}$ .

## V. PREDICTION OF PRODUCT SIZE DISTRIBUTION

Figs. 6 and 7 illustrate the measured and predicted product size distribution obtained using a Bond ball mill and a laboratory vertical stirred mill for the size interval 1, respectively. The breakage parameters determined using both grinding mills show good data accuracy when predicting the product size distribution as a function of grinding time.

The PBM predicted very good fits for all tests. It is important to point out that the breakage parameters were obtained in the laboratory tests and no retrofitting or parameters adjustments were used. The product size distribution was measured after each selected grinding time eight, to compare with the values predicted by the model using PBM. There are no scale-up factors or efficiency factors being used to correct for the higher rate of breakage in the vertical stirred mill tested.

## VI. CONCLUSION

The performance of a vertical stirred mill can be described using the PBM described by Austin et al. [6] for ball mills. This work shows the possibility of calculating the selection function values for the vertical stirred mill tested using a Bond ball mill. The similarity obtained between the breakage functions for the Bond and vertical mills suggests that the total collisional energy between the balls inside both units should be similar, although it is known that vertical stirred mills produce a higher number of low-energy collisions than a ball mill. Continuation of this work includes the study using different sizes of balls and different mill velocities to identify correlations between the breakage parameters and to develop a scale-up function using a Bond ball mill.

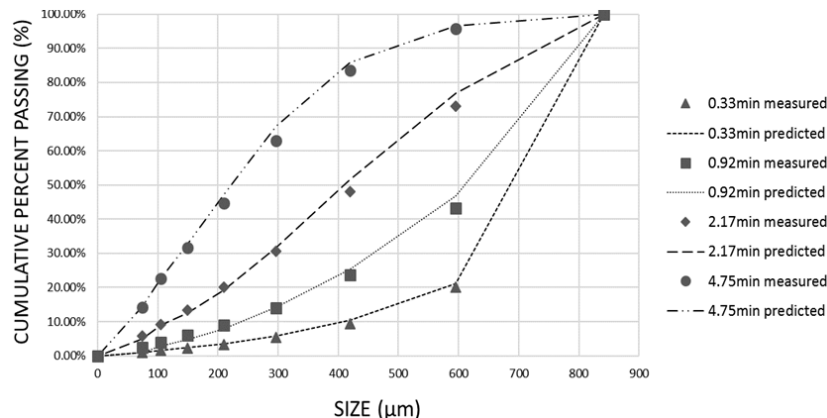


Fig. 6 Measured (symbols) and predicted (lines) product size distribution of a Bond ball mill

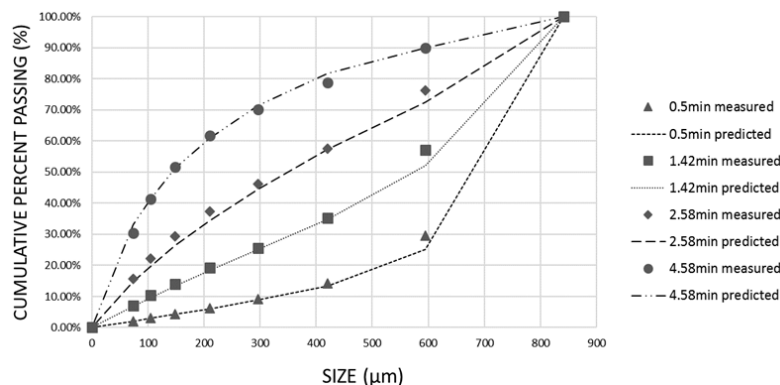


Fig. 7 Measured (symbols) and predicted (lines) product size distribution of a vertical stirred mill

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## REFERENCES

- [1] A. Jankovic, W. Valery Junior and D. La Rosa, "Fine Grinding in the Australian Mining Industry," 11 July 2006. (Online). Available: [http://www.metso.com/mini-ngandconstruction/mct\\_service.nsf/WebWID/WTB-120\\_106-22576-A45AE/\\$File/043.pdf](http://www.metso.com/mini-ngandconstruction/mct_service.nsf/WebWID/WTB-120_106-22576-A45AE/$File/043.pdf).
- [2] N. Stehr and J. Schwedes, "Investigation of the Grinding Behavior of a Stirred Ball Mill," German Chemical Engineering, vol. 6, pp. 337-342, 1983.
- [3] D. Stief, W. Lawruk and L. Wilson, "Tower Mill and its Application to Fine Grinding," Minerals and Metallurgy Processing, vol. 4, pp. 45-50, 1987.
- [4] D. B. Mazzinghy, C. L. Schneider, V. K. Alves and R. Galéry, "Vertical Agitated Media Mill scale-up and simulation," Minerals Engineering 73, pp. 69-76, 2015.
- [5] T. Napier-Munn, S. Morrel, R. Morrison and T. Kojovic, "Mineral Comminution Circuits: Their Operation and Optimisation," JKMR Monograph Series in Mining and Mineral Processing 2, 1996.
- [6] L. Austin, R. Klimpel and P. Luckie, Process Engineering of Size Reduction: Ball Milling, New York: SME, 1984.
- [7] A. Gupta and D. Yan, Mineral Processing Design and Operation: An Introduction, Amsterdam: Elsevier, 2006.
- [8] F. C. Bond, "The Third Theory of Comminution," AIME Transactions, vol. 193, p. 484, 1952.
- [9] S. Morrel, U. Sterns and K. Weller, "The Application of Population Balance Models to Very Fine Grinding in Tower Mills," Proceeding of XIX International Mineral Processing Congress, 1993.
- [10] R. Morrison, P. Cleary and M. Sinnott, "Using DEM to Compare the Energy Efficiency of Pilot Scale Ball and Tower Mills," Minerals Engineering, pp. 665-672, 2009.
- [11] A. Jankovic, Mathematical Modelling of Stirred Mills, Brisbane: University of Queensland, JKMR, 1999.