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A Quick Prediction for Shear Behaviour of RC Membrane Elements by Fixed-Angle Softened Truss Model with Tension-Stiffening

X. Wang, J. S. Kuang

***Abstract*—**The Fixed-angle Softened Truss Model with Tension-stiffening (FASTMT) has a superior performance in predicting the shear behaviour of reinforced concrete (RC) membrane elements, especially for the post-cracking behaviour. Nevertheless, massive computational work is inevitable due to the multiple transcendental equations involved in the stress-strain relationship. In this paper, an iterative root-finding technique is introduced to FASTMT for solving quickly the transcendental equations of the tension-stiffening effect of RC membrane elements. This fast FASTMT, which performs in MATLAB, uses the bisection method to calculate the tensile stress of the membranes. By adopting the simplification, the elapsed time of each loop is reduced significantly and the transcendental equations can be solved accurately. Owing to the high efficiency and good accuracy as compared with FASTMT, the fast FASTMT can be further applied in quick prediction of shear behaviour of complex large-scale RC structures.

***Keywords*—**Bisection method, fixed-angle softened truss model with tension-stiffening, iterative root-finding technique, reinforced concrete membrane.

I. Introduction

DURING the past several decades, remarkable progress has been achieved in theories of predicting shear strength of 2-D RC membrane elements under pure shear condition. A shear model is required to analytically predict the shear behaviour of RC elements under different loading conditions. A rational shear model has to rigorously satisfy Navier's three-principals of mechanics of material, including equilibrium equations, compatibility condition and constitutive models, to correctly predict a shear load and deformation history under different loading patterns [1].

A 2-D RC element subjected to membrane stresses is considered as a series of trusses, consisting of compressive concrete struts and tensile steel ties, following which a softened truss model was developed to employ a softened compression constitutive model for concrete to cope with the degraded concrete strength after cracking. Based on the softened truss model rules and rotating angle theory, the rotating-angle softened truss model (RASTM) [2] was proposed, where cracks are assumed to be perpendicular to the direction of principal tensile stress in concrete element. The constitutive model of concrete is based on actual nonlinear and softened behaviour of concrete under compression, and a sophisticated tensile stress-strain relationship is adopted. A smeared model for reinforcing bars covered by concrete was obtained from experiments. As the principal stress direction coincides with the crack direction, the contribution of concrete to shear strength is equal to zero.

To incorporate the contribution of concrete, which, together with contribution of steel, forms the shear strength of a membrane element, the fixed-angle softened truss model (FASTM) was proposed based on the softened truss model rules and fixed-angle theory [3], [4]. The principal direction deviates from the crack direction, creating a concrete shear stress on the cracked surface, which serves as the source of concrete contribution to the shear strength of the RC element.

The two softened truss models both adopt uniaxial constitutive relationship for concrete, which may overestimate the shear strengths of specimens. The softened membrane model (SMM) [5], which is similar to FASTM except for using a biaxial constitutive relationship, was developed and the two-dimensional softening effect was considered. Nevertheless, the improvement to biaxial constitutive relationship shows negligible influence on the prediction of peak shear strength by comparing FASTM with SMM. So FASTM has a superior performance with theoretical accuracy and calculation simplicity. However, the expression of concrete under uniaxial tension in these shear models could not correctly present the tensile stress-strain curve for concrete. It is shown in the experimental study [6] that the tensile stress of concrete vanishes rapidly after the initiation of first crack, which disagrees with the smooth and gentle ascending branch of the original tensile stress-strain relationship from FASTM. The FASTMT effect [7] was developed to enhance the accuracy of shear strength prediction by refining the tensile stress-strain curve of concrete with tension-stiffening effect. A comprehensive tension-stiffening model for post-cracking concrete was adopted including the influence of bond stress-slip relationship, orthogonal reinforcement and crack propagation. The predicted peak shear strength by FASTMT shows excellent agreement with the experimental results.

The modification of tension stiffening effect provides a more comprehensive theoretical background for FASTM and a more rational constitutive model for concrete in tension. However, complex transcendental equations are repeatedly solved in the calculation procedure. Massive computational effort is required for a full profile of shear stress-strain curve and consequently the solving procedure becomes extremely time-consuming. This paper aims to improve the calculation efficiency of shear

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prediction by FASTMT without sacrificing the accuracy. A fast FASTMT is proposed and an iterative root-finding method is adopted in the algorithm. The elapsed time of each iteration loop is reduced significantly thus further application in various large-scale RC structures is allowed.

II. Model Description

A. Basic Principles

The 2-D RC membrane element is subjected to in-plane shear stress τ_{lt}, and biaxial stresses, σ_l and σ_t, along $l-t$ coordinate, which is also the direction for longitudinal and transverse reinforcement. For reinforcement grid, ρ_l and ρ_t are the reinforcing ratios along $l-$ axes and $t-$ axes, and f_l as well as f_t are the average stresses in longitudinal and transverse steel, respectively. In fixed angle theory, the principal stresses of concrete stress are denoted as σ_1^c and σ_2^c and shear stress along crack surface is τ_{12}^c, and the cracks are assumed to be propagating along the $2-$ axis. The corresponding $1-2$ coordinate is defined as the principal direction of applied stresses, and it is obtained by rotating the $l-t$ coordinate of in-plane stresses by a fixed angle of α_1, as illustrated in Fig. 1.

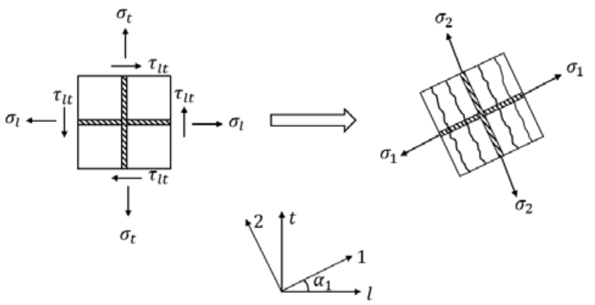

Fig. 1 Stress states and coordinates of fast FASTMT

B. Equilibrium Conditions

In fixed-angle theory, the concrete stress condition in $l-t$ coordinate is transformed into a $1-2$ coordinate, which is the principal coordinate of applied stresses. The angle between two coordinates is denoted as α_1 and its value remains fixed as the loading increases proportionally. Accordingly, the governing equations for equilibrium condition of fixed-angle softened truss model are:

$$\sigma_l = \sigma_1^c \cos^2\alpha_1 + \sigma_2^c \sin^2\alpha_1 - \tau_{12}^c 2\sin\alpha_1\cos\alpha_1 + \rho_l f_l \quad (1)$$

$$\sigma_t = \sigma_1^c \sin^2\alpha_1 + \sigma_2^c \cos^2\alpha_1 + \tau_{12}^c 2\sin\alpha_1\cos\alpha_1 + \rho_t f_t \quad (2)$$

$$\tau_{lt} = \left(\sigma_1^c - \sigma_2^c\right)\sin\alpha_1\cos\alpha_1 + \tau_{12}^c\left(\cos^2\alpha_1 - \sin^2\alpha_1\right) \quad (3)$$

C. Strain Compatibility

The compatibility conditions of fast FASTMT are obtained by assuming the cracks propagate along $2-$ axis and the principal stress direction coincides with the principal strain direction. By transforming a set of strains in $l-t$ coordinate into $1-2$ coordinate, the governing equations are:

$$\varepsilon_l = \varepsilon_1\cos^2\alpha_1 + \varepsilon_2\sin^2\alpha_1 - \frac{\gamma_{lt}}{2}2\sin\alpha_1\cos\alpha_1 \quad (4)$$

$$\varepsilon_t = \varepsilon_1\sin^2\alpha_1 + \varepsilon_2\cos^2\alpha_1 + \frac{\gamma_{lt}}{2}2\sin\alpha_1\cos\alpha_1 \quad (5)$$

$$\frac{\gamma_{lt}}{2} = \left(\varepsilon_1 - \varepsilon_2\right)\sin\alpha_1\cos\alpha_1 + \frac{\gamma_{12}}{2}\left(\cos^2\alpha_1 - \sin^2\alpha_1\right) \quad (6)$$

D. Constitutive Laws

The concrete under compression is treated as an orthotropic material with a softening coefficient ζ, which softens the strength capacity of cracked concrete and is developed based on experimental studies. The angle β represents the deviation between angle of principal direction of applied stresses and the angle of principal direction of concrete stresses. The governing equations for concrete under compression are:

$$\sigma_2^c = \zeta f_c^{'}\left[2\left(\frac{\varepsilon_2}{\zeta\varepsilon_0}\right) - \left(\frac{\varepsilon_2}{\zeta\varepsilon_0}\right)^2\right],\ \varepsilon_2/\zeta\varepsilon_0 \le 1 \quad (7a)$$

$$\sigma_2^c = \zeta f_c^{'}\left[1 - \left(\frac{\left(\varepsilon_2/\zeta\varepsilon_0\right) - 1}{\left(4/\zeta\right) - 1}\right)^2\right],\ \varepsilon_2/\zeta\varepsilon_0 > 1 \quad (7b)$$

$$\zeta = \left(\frac{5.8}{\sqrt{f_c^{'}}} \le 0.9\right)\left(\frac{1}{\sqrt{1+400\varepsilon_1}}\right)\left(1 - \frac{|\beta|}{24^\circ}\right) \quad (8)$$

$$\beta = \frac{1}{2}\tan^{-1}\left[\frac{\gamma_{12}}{\left(\varepsilon_1 - \varepsilon_2\right)}\right] \quad (9)$$

where ε_0 represents the strain level under maximum compressive stress and the value is taken as -0.00235.

For concrete in tension, a comprehensive tension-stiffening model [8]-[10] is employed. It assumes a linear bond stress-slip relationship between reinforcement and the surrounding concrete and includes the effect of orthogonal reinforcement. The stress-strain relationship can be divided into several stages and the governing equations are listed accordingly.

In linear stage, the concrete is within its elastic range and $\varepsilon_1 \le 0.00008$. The tensile stress-strain relationship gives:

$$\sigma_1^c = E_c\varepsilon_1 \quad (10a)$$

After the initiation of first crack, the maximum tensile stress in concrete starts to converge to tensile strength of concrete f_t' and the nonlinear stress-strain curve for $0.00008 \le \varepsilon_1 \le 0.001$ becomes:

$$\frac{\sigma_1^c}{f_t'} = \frac{1 - \dfrac{\tanh ka}{ka}}{1 - \operatorname{sech} ka} \tag{10b}$$

$$\frac{\varepsilon_1}{f_t'/E_c} = \frac{1 + \dfrac{\tanh ka}{\left(n_l'\rho_l \cos^4\theta + n_t'\rho_t \sin^4\theta\right)ka}}{1 - \operatorname{sech} ka} \tag{10c}$$

$$k^2 = \left(\frac{(p_l n_l + p_t n_t)E_b}{A_c}\right)\left(\frac{1}{E_{s,eq}} + \frac{1}{E_c}\right) \tag{10d}$$

$$E_{s,eq} = E_{sl}\rho_l \cos^4\alpha_1 + E_{st}\rho_t \sin^4\alpha_1 \tag{10e}$$

where a is average crack spacing, A_c is the unit area of a membrane element, p_l and p_t are the perimeters of a reinforcing bar, n_l and n_t are the numbers of bars in unit length, n_l' and n_t' are the modular ratios in the l and t directions, respectively.

When the maximum bond stress τ_b is reached, reinforcement starts to yield and the expression of the lower boundary point becomes when $\varepsilon_1 = \varepsilon_{ult,crack}$:

$$\frac{\sigma_1^c}{f_t'} = \frac{1}{2} \tag{10f}$$

$$\frac{\varepsilon_{ult,crack}}{f_t'/E_c} = \frac{\left(f_{yl}\rho_l \cos\theta + f_{yt}\rho_t \sin\theta\right) - \dfrac{\frac{p}{s}\tau_b a'}{2}}{E_{s,eq}} \frac{E_c}{\frac{p}{s}\tau_b a'} \tag{10g}$$

$$a' = \frac{f_t' A_c}{(p_l n_l + p_t n_t)\tau_b} \quad and \quad \frac{p}{s} = \frac{p_l}{s_l} + \frac{p_t}{s_t} \tag{10h}$$

where s_l and s_t are the reinforcement spacings in the l and t directions, respectively.

In the final stage of tension-stiffening, the termination point of tension is defined with the total failure of concrete element. The expression of the termination point of tension gives:

$$\sigma_1^c = 0 \tag{10i}$$

$$\frac{\varepsilon_1}{f_t'/E_c} = \frac{\left(f_{yl}\rho_l \cos\theta + f_{yt}\rho_t \sin\theta\right)}{E_{s,eq}} \frac{E_c}{\frac{p}{s}\tau_b a'} \tag{10j}$$

For the curve between the crack stabilisation point and the lower boundary point and the curve between the lower boundary point and the termination point, the normalised tensile stress-strain curve is plotted linearly.

It is observed in experiments [6] that a marked reduction occurs in the tensile capacity of concrete at higher deformations, and the normalised curve neglects such reduction. To account for this phenomenon and the accumulated damage to concrete by gradually increasing cracks, a variable tensile strength f_t [8] is introduced to replace the fixed tensile strength f_t':

$$f_t = f_t' \exp\left[-C\left(\varepsilon_1 - f_t'/E_c\right)\right] \tag{10k}$$

where C is a damage parameter and $C = 550$.

For concrete in shear, the relationship between shear stress and shear strain gives:

$$\tau_{12}^c = \frac{\sigma_1^c - \sigma_2^c}{2\left(\varepsilon_1 - \varepsilon_2\right)}\gamma_{12} \tag{11}$$

For reinforcement, a smeared model of steel is used and the stress-strain curve is obtained from experiments and only for embedded steel covered by concrete instead of bare steel bars. The notations f_l and f_t denote the reinforcement stresses along l and t directions, respectively. The governing equations for embedded mild steel bars are as:

$$f_l = E_{sl}\varepsilon_l,\ \varepsilon_l \le \varepsilon_{yl}' \tag{12a}$$

$$f_l = \left(0.91 - 2B_l\right) f_{yl} + \left(0.02 + 0.25B_l\right) E_{sl}\varepsilon_l,\ \varepsilon_l > \varepsilon_{yl}' \tag{12b}$$

where

$$\varepsilon_{yl}' = f_{yl}'/E_{sl} \tag{12c}$$

$$f_{yl}' = \left(0.93 - 2B_l\right) f_{yl} \tag{12d}$$

$$B_l = \frac{1}{\rho_l}\left(\frac{f_{cr}}{f_{yl}}\right)^{1.5} \tag{12e}$$

$$f_{cr} = 0.31\sqrt{f_c'\left(MPa\right)} \ and \ \rho_l \ge 0.15\% \tag{12f}$$

$$f_t = E_{st}\varepsilon_t,\ \varepsilon_t \le \varepsilon_{yt}' \tag{13a}$$

$$f_t = \left(0.91 - 2B_t\right) f_{yt} + \left(0.02 + 0.25B_t\right) E_{st}\varepsilon_t,\ \varepsilon_t > \varepsilon_{yt}' \tag{13b}$$

where

$$\varepsilon_{yt}' = f_{yt}'/E_{st} \tag{13c}$$

$$f_{yt}' = \left(0.93 - 2B_t\right) f_{yt} \tag{13d}$$

$$B_t = \frac{1}{\rho_t}\left(\frac{f_{cr}}{f_{yt}}\right)^{1.5} \tag{13e}$$

$$f_{cr} = 0.31\sqrt{f_c^{'}\left(MPa\right)} \ and \ \rho_t \geq 0.15\% \qquad (13f)$$

By subtracting and summing (1) and (2), two equilibrium equations are obtained to check the convergence of stress and strain solutions:

$$\rho_l f_l + \rho_t f_t = (\sigma_l + \sigma_t) - (\sigma_1^c + \sigma_2^c) \qquad (14)$$

$$\rho_l f_l - \rho_t f_t = (\sigma_l - \sigma_t) - (\sigma_1^c + \sigma_2^c)\cos 2\alpha_1 + 2\tau_{12}^c \sin 2\alpha_1 \qquad (15)$$

III. Analysis Method

A. Solution Algorithm

13 governing equations of fast FASTMT, (1)-(13f), contains 16 unknown variables, including 8 stresses (σ_l, σ_t, τ_{lt}, σ_1^c, σ_2^c, τ_{12}^c, f_l, f_t), 6 strains (ε_l, ε_t, γ_{lt}, ε_1, ε_2, γ_{12}), as well as the angle β and the softening coefficient ζ. If 3 additional unknown variables are provided, the remaining 13 parameters can be solved by the 13 governing equations and the required stresses and strains are obtained.

For a 2-D RC membrane specimen subjected to membrane stresses, the pure shear condition provides two of the three unknown variables, $\sigma_l = \sigma_t = 0$. The last unknown variable can be chosen by selecting a value for ε_2, due to the fact that ε_2 varies linearly with shear strain γ_{lt}. With all three unknown variables obtained, the set of equations can be solved by iterative procedures shown in Fig. 2:

1) Select a value for ε_2.
2) Assume the values for principal strains γ_{12} and ε_1.
3) Calculate the strains ε_l and ε_t in $l-t$ coordinate by (4) and (5).
4) Solve for concrete principal stresses σ_1^c, σ_2^c and τ_{12}^c by calculating the parameters including the angle β and softening coefficient ζ by (8) and (9).
5) Solve for reinforcement stresses f_l and f_t in $l-t$ coordinate by (12a)-(12f) and (13a)-(13f).
6) Check if the two convergence conditions (14) and (15) are satisfied. Otherwise, adjust the values of γ_{12} and ε_1 so that the convergence criteria can be satisfied.
7) Solve for shear strain γ_{lt} and shear stress τ_{lt} in $l-t$ coordinate by (3) and (6).
8) Change the value for ε_2 and repeat step 1 through 7. In this way, a set of γ_{lt} and τ_{lt} for various ε_2 can be obtained and the shear stress-strain curve can be plotted.

B. Fast FASTMT

The analyse method was performed off-line using a commercial software package MATLAB R2013b [11] and the key step is to find the values of γ_{12} and ε_1 that satisfy the equilibrium criteria. As shown in Fig. 3, this calculation procedure starts by enumerating all possible values of concrete principal strain denoted by $\varepsilon_1(i)$ and $\gamma_{12}(j)$. Following this, a $i \times j$ matrix P will be constructed and each grid $P(i,j)$ represents the square summation of the difference between the two sides of the two equilibrium equations, (14) and (15), with variables $\varepsilon_1(i)$ and $\gamma_{12}(j)$. The equilibrium conditions are satisfied if the value of square summation $P(i,j)$ approaches to zero with a certain tolerance. The minimum $P(i,j)$ will be located and the corresponding coordinate (i,j) will be returned, eventually one set of $\gamma_{12}(i,j)$ and $\varepsilon_1(i,j)$ which best satisfies the two equilibrium equations will be selected.

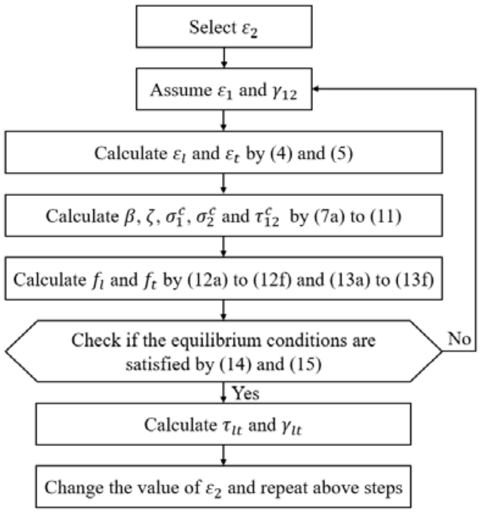

Fig. 2 Solution algorithm of fast FASTMT

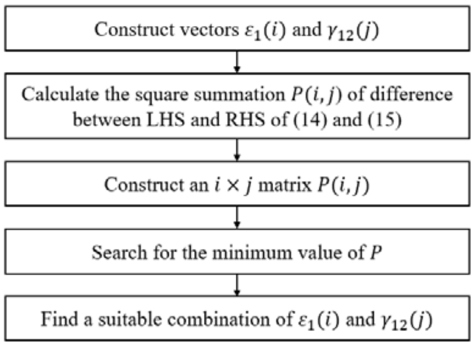

Fig. 3 Root-finding procedures of fast FASTMT

In the process of solving for the square summation value $P(i,j)$, multiple transcendental equations (10b) and (10c) in the tensile stress-strain relationship for post-cracking concrete will be involved repeatedly. In MATLAB, a built-in function solve is adopted and the program then returns the numerical solution of the stress-strain relationships. However, the elapsed time for each equation is non-negligible. As a result, it becomes significantly time-consuming since billions of transcendental calculations are required in order to obtain a full shear stress-strain curve.

An iterative root-finding method is then adopted instead of the built-in solve function in MATLAB. Due to the monotonicity and the continuity of post-cracking concrete tensile stress-strain equations, the bisection method is chosen to solve the multiple transcendental problems. This method repeatedly bisects a certain interval and selects the interval that contains a root, consequently the interval shrinks into half each time. The equations can be solved by the following procedures:

1) Select an initial interval for possible crack spacing a, and the upper bound of this initial interval should be relatively large to represent the uncracked stage.
2) Bisect the interval and select the section where the function values of the two end points have opposite sign.
3) Reset the initial interval and repeat step 1 and 2 until the interval length is within the tolerance.

IV. Results and Comparisons

The fast FASTMT is applied to a series of typical 2-D RC membrane elements tested by Pang and Hsu [12] as shown in Fig. 4 to validate the prediction accuracy. The elements are subjected to pure shear condition, where the inclination angle of crack is $\alpha_1 = 45^{\circ}$ and the initial stress value follows $\sigma_l = \sigma_t = 0$. The material properties are tabulated in Tables I and II.

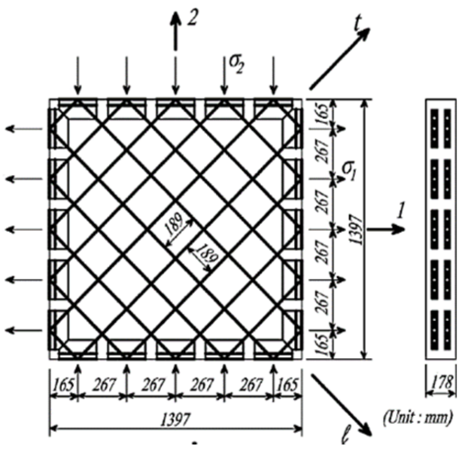

Fig. 4 Testing RC membrane element B2 [12]

TABLE I
REINFORCEMENT PROPERTIES

Longitudinal Reinforcement		Transverse Reinforcement	
ρ_l	1.789%	ρ_t	1.193%
f_{ly} (MPa)	446.5	f_{ty} (MPa)	462.6
E_{ls} (MPa)	200 000	E_{ts} (MPa)	192 400

TABLE II
CONCRETE PROPERTIES

f_c' (MPa)	44.1
ε_0	0.00235

The testing result for specimen B2 under pure shear is compared with the obtained values from fast FASTMT and also the original curve from FASTM in Fig. 5. The original FASTM overestimates shear strength of the 2-D RC membrane element under large shear strain and the deviation may be caused by the irrational decreasing curve of tensile stress without considering the effect of tension-stiffening. While the fast FASTMT, which includes the loss of bond stress under large slip distance and the gradual concrete tensile stress loss due to newly initiated cracks, exhibits a relatively conservative prediction for low strain level and accurate result for peak shear strength.

After adopting the bisection method, the fast FASTMT excels in the simplicity and efficiency in the root-finding process. For a single transcendental function employed by concrete tensile stress-strain relationship, the elapsed time for the built-in solve function in MATLAB is approximately 0.70 seconds, while by using bisection method the required time approaches to zero. For the full profile of tensile stress-strain curve with tension stiffening effect, the elapsed time for built-in solve function is approximately 2.80 seconds, while the bisection method requires only 0.06 seconds. To obtain the shear stress-strain curve of a RC panel under pure shear condition, the elapsed time of built-in solve function is more than 17 hours, while the bisection methods requires approximately 10 minutes.

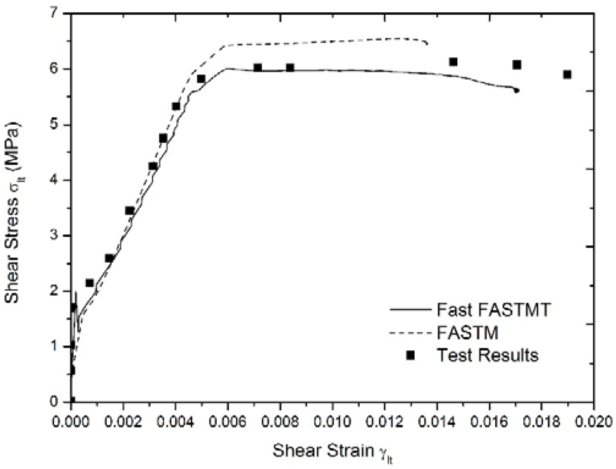

Fig. 5 Comparison of fast FASTMT, FASTM and experimental data

V. Conclusions

The fast FATMST aims for quickly predicting the shear strength of RC membrane element and it is based on the fixed-angle theory and softened truss model theory, which excels in both calculation accuracy and simplicity. The tensile stress-strain relationship in the fast FASTMT includes the comprehensive effect of tension-stiffening of RC after the first initiation of crack, thus the influences of bond stress-slip relationship, orthogonal reinforcement and crack propagation are incorporated. In the calculation procedure performed in MATLAB, an iterative root-finding method is adopted to solve the tensile stress-strain relationship for post-cracking concrete.

The testing result of a RC panel under pure shear condition is compared with the fast FASTMT and the original FASTM. Based on the comparison, the following conclusions can be drawn:

1) The calculated shear stress-strain curve of fast FASTMT shows a better agreement with the experimental data than the original FASTM. The overestimation of shear strength of FASTM is affected by the irrational decreasing rate of tensile stress under large strain level.
2) The total elapsed time of the fast algorithm is greatly shortened owing to the bisection root-finding method. Additionally, the fast FASTMT allows a smaller step when plotting the shear stress-strain curve, which inherently generates a more thorough and detailed shear behaviour of

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the full monotonic loading process.

3) The high efficiency as well as the accuracy of fast FASTMT enables a further application in quick shear behaviour prediction of complex large-scale RC structures.

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