Design and Analysis of a Piezoelectric-Based AC Current Measuring Sensor

Easa Ali Abbasi, Akbar Allahverdizadeh, Reza Jahangiri, Behnam Dadashzadeh

Abstract—Electrical current measurement is a suitable method for the performance determination of electrical devices. There are two contact and noncontact methods in this measuring process. Contact method has some disadvantages like having direct connection with wire which may endamage the system. Thus, in this paper, a bimorph piezoelectric cantilever beam which has a permanent magnet on its free end is used to measure electrical current in a noncontact way. In mathematical modeling, based on Galerkin method, the governing equation of the cantilever beam is solved, and the equation presenting the relation between applied force and beam's output voltage is presented. Magnetic force resulting from current carrying wire is considered as the external excitation force of the system. The results are compared with other references in order to demonstrate the accuracy of the mathematical model. Finally, the effects of geometric parameters on the output voltage and natural frequency are presented.

Keywords—Cantilever beam, electrical current measurement, forced excitation, piezoelectric.

I. INTRODUCTION

NOWADAYS, electrical energy as an essential energy in human life, is highly considered and it has been done a lot to optimize its production and consumption. Since that its production and transmission costs very much, and disturbing disorders will happen in daily life without it, energy saving in many cases is necessary and requires the use of several methods. It can be said that, in today's life, any person in any situation is obligated to use electrical devices and tools. Therefore, by assuming this high usage, every tool should have minimum consumption and maximum efficiency among available types. Accordingly, electrical devices and tools must be controlled from two sides; at first by quality control organizations and then by consumers [1], [2]. Before providing a device to market, quality control organizations check the quality of the device and present its efficiency. Also, consumers check the consumption rate of the device in different times, so they can have an evaluation of its performance and also they can limit its usage in peak hours [3]. Due to this, some tools should be available to measure the

Easa AliAbbasi was M.Sc. Student at the School of Engineering-Emerging Technologies, University of Tabriz, Tabriz, Iran (e-mail: easa_aliabbasi@yahoo.com).

Akbar Allahverdizadeh is with the School of Engineering-Emerging Technologies, University of Tabriz, Tabriz, Iran (phone: +98-41-3339-3877; fax: +98-41-3329-4626; e-mail: allahverdizadeh@tabrizu.ac.ir).

Reza Jahangiri, is with the Department of Mechanical Engineering, Islamic Azad University, Salmas Branch, Salmas, Iran (e-mail: r_jahanghiri@tabrizu.ac.ir).

Behnam Dadashzadeh is with the School of Engineering-Emerging, Iran (e-mail: b.dadashzadeh@tabrizu.ac.ir).

parameters of devices so that these values could be given accurately to consumers in an easy way. One of the important parameters of an electrical device is its electrical current which can be measured by specific sensors.

Generally, electrical current measurement has two methods: contact and noncontact. The sensor must be in touch with measurement site in contact method. Although it is easy, in some cases, there is not a possibility to attach the sensor on devices or the surface is covered and cannot be removed. In addition, in many situations, installation of contact electrical current sensors requires a suspension of system. In environments with up-time needs, the suspension of system may provide a lot of problems. Military bases are an example of that. Because in such environments electrical requirements change with missions and in every mission special devices add or drop to the system, always an electrician should be in alert to interrupt the system and to change the location of sensor. It is time consuming, and by time, it would harm the sensor or system [1]. Hence, noncontact methods are used which has the ability of solving these problems. Among all noncontact measurement methods, piezoelectric based sensors are reliable ones [4].

Piezoelectricity is the ability of some crystals like tourmaline, quartz, and topaz that produce voltage in mechanical stress. This is called the direct piezoelectric effect. Inverse piezoelectric effect happens when electrical potential subjects to the material and causes a shape change and mechanical stress [5], [6].

The first research about piezoelectric sensors was accomplished by Leland et al. [7] which was for commercial and household uses. The sensor was composed of a piezoelectric cantilever beam and a permanent magnet attached to the free end of the beam. The system had no need to external resources and was electrically isolated from the wire. Over time, the results got better and also the exact place of permanent magnet presented in two modes: in single wire and two wires which were 45 and 90 degrees, respectively [8]. Then, two different configurations of sensors in four different wires studied. Electrodes were covered all over the length of the first sensor's conductor and half of the second sensor's conductor in which, the resonant frequency for the first one was 1.23 kHz and for the second one was 960 Hz. The second sensor, which had lower resonant frequency, resulted in heavier and bigger permanent magnet because of the variety in micro-magnet's manufacturing process. Unlike other sensors, this sensor could measure the current of two wires without surrounding it completely [9]. Isagawa et al. [10] presented the design of a noncontact MEMS-scale DC-current meter in both

single and two wires. It was non-drive, noncontact, and could measure single wire and two wire's current. This piezoelectric sensor could be improved by placing permanent magnet on top and bottom of the beam's free end. Thus, a current control sensor with this property was presented by Xu et al. [11]. It could be enabled wirelessly and also could be retrofitted to sub-meter existing buildings in order to have individual cost monitoring. A new method based on piezoelectric and magnetostrictive materials was presented by He et al. [12]. The structure of the system was composed of a magnetostrictive layer, piezoelectric layer, and a proof mass. One side of the beam was clamped and the piezoelectric layer was on the magnetostrictive layer. Magnetostrictive material was magnetized longitudinally and piezoelectric material used to polarize along the beam's thickness. The proof mass was also used to make the cantilever beam vibrate in crossing current's resonant frequency. When the sensor was near a current carrying conductor, because of magnetostriction, the cantilever beam used to bend and the piezoelectric material could generate a voltage proportional to crossing current. The designed system in this paper had no need to external power source and was electrically isolated from electrical current, too. The sensor was working in a range from 1 A_{rms} to 10 A_{rms} and resonant frequency of 50 Hz in which this range was a little low and could be improved.

In this paper, a piezoelectric cantilever beam will be designed to measure noncontact electrical current. It is obvious that every current carrying wire has a magnetic field around it and because of that, it generates a magnetic force in which, by the means of electromagnetic equations, the wire's current could be evaluated by measuring this force. According to this, mathematical model for a piezoelectric cantilever beam that has a permanent magnet attached to its free end will be presented. Coupling between external magnetic field and permanent magnet's magnetic field creates a relationship between the conductor and the beam which causes vibration in beam and polarization in piezoelectric layer. So, according to direct piezoelectric effect, output voltage will be generated in clamped end of the beam.

II. MAGNETIC FIELD AROUND A CURRENT CARRYING WIRE

Crossing current from a conductor causes a magnetic field and also a magnetic force around it. Lorentz law could be used for measuring these magnetic forces. This law expresses that the magnetic force of a magnetic field, by considering the right hand law, is perpendicular to both the wire and magnetic field. It is proved that the relation between magnetic force, F and electrical current I is [13]:

$$\vec{F} = \vec{I}L \times \vec{B} \qquad (1)$$

which L is the wire's length and B is the magnetic field. The magnetic field of a wire could be calculated from Amperes law that r is wire radius and $\mu_0 = 4\pi \times 10^{-7} \text{T.m/A}$:

$$B = \frac{\mu_0 I}{2\pi r} \qquad (2)$$

III. BIMORPH PIEZOELECTRIC CANTILEVER BEAM'S MATHEMATICAL MODEL

The method for calculating forces from current carrying wires was presented in previous section. Here, an equation must be driven to provide the output voltage of piezoelectric cantilever beam which is proportional to applied force. Therefore, a bimorph piezoelectric cantilever beam like Fig. 1 is considered. In such beams, piezoelectric layers could be connected in series or parallel, in which, in this paper, only parallel mode will be studied, and also, modeling is based on Euler-Bernoulli theory.

A. Derivation of the Beam's Governing Equation

The following equation could be written for an undamped cantilever beam [14]:

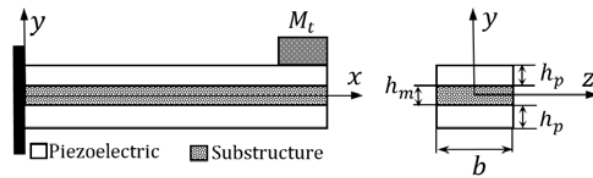$$YI\frac{\partial^4 w(x,t)}{\partial x^4} + m\frac{\partial^2 w(x,t)}{\partial^2 t} = 0 \qquad (3)$$

Fig. 1 Bimorph piezoelectric cantilever beam

where YI, m, and w are the bending stiffness, mass per unit length, and transverse displacement of the beam. For the undamped beam, two types of damping mechanisms are included: viscous air damping and strain-rate damping, respectively. The damped equation of motion is written in (4) where I is the representation of moment of inertia. Viscous air damping coefficient is c_s, and strain-rate damping coefficient is c_a [15].

$$YI\frac{\partial^4 w(x,t)}{\partial x^4} + c_s I\frac{\partial^5 w(x,t)}{\partial x^4 \partial t} + c_a\frac{\partial w(x,t)}{\partial t} + m\frac{\partial^2 w(x,t)}{\partial t^2} = 0 \qquad (4)$$

As mentioned before, wire's magnetic field and the beam must be in a relation for measuring electrical current. So, a permanent magnet is placed on the free end of the cantilever beam as the proof mass. By considering the effect of this proof mass to the end of the beam in which a magnetic force will be added to the beam, the governing equation of the beam would be written as following:

$$-\frac{\partial^2 M(x,t)}{\partial x^2} + c_s I\frac{\partial^5 w(x,t)}{\partial x^4 \partial t} + c_a\frac{\partial w(x,t)}{\partial t} + m\frac{\partial^2 w(x,t)}{\partial t^2} = f(t) \qquad (5)$$

M_t presents the proof mass and $M(x,t)$ presents the bending moment which can be evaluated as:

$$M(x,t) = b\left(\int_{-h_p-\frac{h_m}{2}}^{-\frac{h_m}{2}} T_p z dz + \int_{-\frac{h_m}{2}}^{\frac{h_m}{2}} T_m z dz + \int_{h_m/2}^{h_p+h_m/2} T_p z dz\right) \qquad (6)$$

In this equation b, h, and T are width of the beam, height of the beam, and stress component in x-direction, respectively. In addition, m and p are used to indicate the piezoelectric and

non-piezoelectric layers. Thus, axial stress components for every layer could be as following:

$$T_m = Y_m S_m \tag{7}$$

$$T_p = c_{11}^E S_p - e_{31} E_3 \tag{8}$$

where Y and c^E present the elastic modulus of piezoelectric layer and non-piezoelectric layer, and also, S, e, and E are the representations of axial strain components in each layer, effective piezoelectric stress constant, and electric field, respectively. The axial strain can be evaluated easily from (9).

$$S(x,z,t) = -z \frac{\partial^2 w(x,t)}{\partial x^2} \tag{9}$$

According to (6), the bending moment is related to backward coupling term coefficient, ϑ by:

$$M(x,t) = -YI \frac{\partial^2 w(x,t)}{\partial x^2} + \vartheta v(t)[H(x) - H(x-L)] \tag{10}$$

H, v, and L are Heaviside function, voltage, and length of the beam, respectively. Bending stiffness and backward coupling term coefficient are as following:

$$YI = \frac{2b}{3}\left\{ Y_m \frac{h_m^3}{8} + c_{11}^E \left[\left(h_p + \frac{h_m}{2} \right)^3 - \frac{h_m^3}{8} \right] \right\} \tag{11}$$

$$\vartheta = \frac{e_{31}b}{h_p}\left[\left(h_p + \frac{h_m}{2} \right)^2 - \frac{h_m^2}{4} \right] \tag{12}$$

In addition, mass per unit length is calculated accordingly:

$$m = b\left(2\rho_p h_p + \rho_m h_m \right) \tag{13}$$

Substituting (10) into (5), gives the final governing differential equation of the beam.

$$YI \frac{\partial^4 w(x,t)}{\partial x^4} + c_s I \frac{\partial^5 w(x,t)}{\partial x^4 \partial t} + c_a \frac{\partial w(x,t)}{\partial t} + m \frac{\partial^2 w(x,t)}{\partial t^2} - \vartheta v(t)\left[\frac{d\delta(x)}{dx} - \frac{d\delta(x-L)}{dx} \right] = f(t) \tag{14}$$

B. Solving the Governing Equation of the Cantilever Beam

Galerkin method is used to solve the equation of motion. Equation (15) demonstrates the transverse deflection in time t and position x. In this equation, $\phi_i(x)$ is the mass-normalized eigenfunction of the ith vibration mode, and $\eta_i(t)$ is the modal mechanical coordinate expression.

$$w(x,t) = \sum_{i=1}^{\infty} \phi_i(x)\eta_i(t) \tag{15}$$

Proof mass must be considered in evaluation mass-normalized eigenfunction because it has high effect on the response of the beam. So, the eigenfunction will be:

$$\phi_i(x) = A_i \left[\cos\frac{\lambda_i}{L}x - \cosh\frac{\lambda_i}{L}x + \varsigma_i\left(\sin\frac{\lambda_i}{L}x - \sinh\frac{\lambda_i}{L}x \right) \right] \tag{16}$$

in which:

$$\varsigma_i = \frac{\sin\lambda_i - \sinh\lambda_i + \lambda_i\frac{M_t}{mL}(\cos\lambda_i - \cosh\lambda_i)}{\cos\lambda_i + \cosh\lambda_i - \lambda_i\frac{M_t}{mL}(\sin\lambda_i - \sinh\lambda_i)} \tag{17}$$

In (16), modal amplitude constant is represented by A_i which could be calculated by normalizing the eigenfunctions according to (18) and (19). Also, I_t is the mass moment of inertia of the tip mass in $x = L$ and δ_{is} is Kronecker delta.

$$\int_0^L \phi_s(x)m\phi_i(x)dx + \phi_s(L)M_t\phi_i(L) + \left[\frac{d\phi_s(x)}{dx} I_t \frac{d\phi_i(x)}{dx} \right]_{x=L} = \delta_{is} \tag{18}$$

$$\int_0^L \phi_s(x)YI \frac{d^4\phi_i(x)}{dx^4}dx - \left[\phi_s(x)YI \frac{d^3\phi_i(x)}{dx^3} \right]_{x=L} + \left[\frac{d\phi_s(x)}{dx}YI \frac{d^2\phi_i(x)}{dx^2} \right]_{x=L} = \omega_i^2 \delta_{is} \tag{19}$$

ω_i is undamped natural frequency of the ith vibration mode which is:

$$\omega_i = \lambda_i^2 \sqrt{\frac{YI}{mL^4}} \tag{20}$$

The eigenvalues of the system, λ_i, can be obtained from (21).

$$1 + \cos\lambda\cosh\lambda + \lambda\frac{M_t}{mL}(\cos\lambda\sinh\lambda - \sin\lambda\cosh\lambda) - \frac{\lambda^3 I_t}{mL^3}(\cosh\lambda\sin\lambda + \sinh\lambda\cos\lambda) + \frac{\lambda^4 M_t I_t}{m^2 L^4}(1 - \cos\lambda\cosh\lambda) = 0 \tag{21}$$

1) Deriving the Equation of the Electric Circuit of the Piezoceramic Cantilever Beam under Mechanical Deflection

In this section, at first the electrostatic behavior of a thin layer will be discussed in order to derive the governing equation of piezoelectric. Axial strain is considered only and because of that, electric displacement is written as following in which ε_{33}^S is the permittivity component at constant strain.

$$D_3 = e_{31}S_p + \varepsilon_{33}^S E_3 \tag{22}$$

Showing output resistance with R_L, Gauss's law will be written as (23) where n represents the unit outward normal, D shows the vector of electric displacement components and the integration is done over the electrode area A.

$$\frac{d}{dt}\left(\int_A D.ndA \right) = \frac{v(t)}{R_L} \tag{23}$$

Considering (9) and knowing the relation between electric field and voltage in parallel connection, $E_3 = -\frac{v(t)}{h_p}$, (22) will be substituted into (23) and the governing equation of a piezoelectric layer's output voltage which is under strain will be as following:

$$\frac{\varepsilon_{33}^S bL}{h_p}\frac{dv(t)}{dt} + \frac{v(t)}{R_L} + e_{31}h_{pc}b\int_0^L \frac{\partial^3 w(x,t)}{\partial x^2 \partial t}dx = 0 \tag{24}$$

where h_{pc} is the distance between neutral axis and the center of each piezoelectric layer and is calculated as:

$$h_{pc} = \frac{h_p + h_m}{2} \tag{25}$$

Substituting (15) into (24) provides:

$$\frac{\varepsilon_{33}^S bL}{h_p} \frac{dv(t)}{dt} + \frac{v(t)}{R_L} + \sum_{i=1}^{\infty} \kappa_i \frac{d\eta_i(t)}{dt} = 0 \tag{26}$$

where K_i is the modal coupling term and is written:

$$\kappa_i = -e_{31} h_{pc} b \int_0^L \frac{d^2\phi(x)}{dx^2} dx = -e_{31} h_{pc} b \frac{d\phi(x)}{dx}\Big|_{x=L} \tag{27}$$

The schematic of electric circuit representing piezoelectric layers is shown in Fig. 2 (A). Using Kirchhoff's current law, gives (28) which by equating it with (26), the internal capacitance and the dependent current source terms will be obtained in (29) and (30).

$$C_p \frac{dv(t)}{dt} + \frac{v(t)}{R_L} - i_p(t) = 0 \tag{28}$$

$$C_p = \frac{\varepsilon_{33}^S bL}{h_p} \tag{29}$$

$$i_p(t) = -\sum_{i=1}^{\infty} \kappa_i \frac{d\eta_i(t)}{dt} \tag{30}$$

By substituting (15) into (14), the mechanical equation of motion in modal coordinates is:

$$\frac{d^2\eta_i(t)}{dt^2} + 2\zeta_i\omega_i \frac{d\eta_i(t)}{dt} + \omega_i^2 \eta_i(t) - \chi_i v(t) = f_i(t) \tag{31}$$

where the electromechanical coupling term is as following.

$$\chi_i = \vartheta \frac{d\phi_i(x)}{dx}\Big|_{x=L} \tag{32}$$

Knowing that the applied force from a current carrying wire's magnetic field varies with time, the modal mechanical forcing function will be as (33) where F_i is the modal force amplitude.

$$f_i(t) = F_i e^{j\omega t} \tag{33}$$

Fig. 2 (B) presents the schematic of each piezoelectric layer with two separate dependent current sources in parallel connection. According to Fig. 4 and Kirchhoff's current law, one can derive the electrical circuit equation as following:

$$C_p \frac{dv(t)}{dt} + \frac{v(t)}{2R_L} - i_p(t) = 0 \tag{34}$$

By substituting the harmonic solutions for $\eta_i(t)$ and $v(t)$ and by solving the governing equation for cross-sectional movements of the beam, the steady-state voltage response could be written as:

$$v(t) = \frac{\sum_{i=1}^{\infty} \frac{-j\omega\kappa_i F_i}{\omega_i^2 - \omega^2 + j2\zeta_i\omega_i\omega}}{\frac{1}{2R_L} + j\omega C_p + \sum_{i=1}^{\infty} \frac{-j\omega\kappa_i\chi_i}{\omega_i^2 - \omega^2 + j2\zeta_i\omega_i\omega}} e^{j\omega t} \tag{35}$$
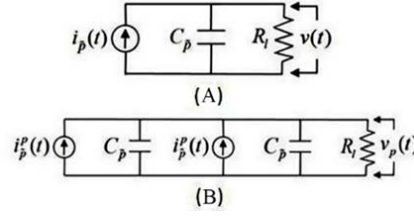

Fig. 2 Schematic of the equivalent electrical circuit, (A) with one dependent current source, (B) with two dependent current sources [16]

In order to have simplicity in problem analysis, one can reduce the output voltage of bimorph cantilever beam to single-mode as:

$$v(t) = \frac{j2\omega R_L \kappa_i F_i}{(1 + j2\omega R_L C_P)(\omega_i^2 - \omega^2 + j2\zeta_i\omega_i\omega) + j2\omega R_L \kappa_i \chi_i} e^{j\omega t} \tag{36}$$

IV. RESONANT FREQUENCY OF THE PIEZOELECTRIC CANTILEVER BEAM

In resonance, the system vibrates with maximum amplitude. Although it is possible to vibrate the system in other frequencies, more energy and more steady input may be needed in respect with resonant frequency. By increasing system's output amplitude and decreasing damping, the system will be able to reach resonant frequency. After selecting the material of the beam, the resonant frequency is dependent on dimensions and proof mass. The following equation is the frequency relation for a piezoelectric cantilever beam with a proof mass which is used to estimate the amount of proof mass in resonant frequency [17]-[19].

$$f_i = \frac{v_i^2}{2\pi} \sqrt{\frac{0.236 D_p b}{\left(L - \frac{l_m}{2}\right)^3 [m_e + \Delta m]}} \tag{37}$$

In (37), f_i, l_m, v_i, Δm, and D_p are ith resonant frequency, length of the proof mass, ith eigenvalue ($v_1 = 1.375$ V), mass of the proof mas, and bending modulus, respectively. Also m_e is mass per unit length in the middle of proof mass which equals:

$$m_e = 0.236 mb \left(L - \frac{l_m}{2}\right) + mb \frac{l_m}{2} \tag{38}$$

Eventually, bending modulus of a bimorph piezoelectric cantilever beam could be written as:

$$D_p = \frac{2Y_m c_{11}^E h_p^3}{3} + c_{11}^E h_m h_p^2 + \frac{c_{11}^E h_m^2 h_p}{2} + \frac{h_m^3 Y_m}{12} \tag{39}$$

V. VALIDATION OF MATHEMATICAL MODEL

In 2009, a research was presented in piezoelectric energy harvesting [16]. In that paper, a bimorph piezoelectric cantilever beam with base excitation was used. The result of our research is compared with reference [16] and is shown in Fig. 3. It is obvious that the results have good compatibility.
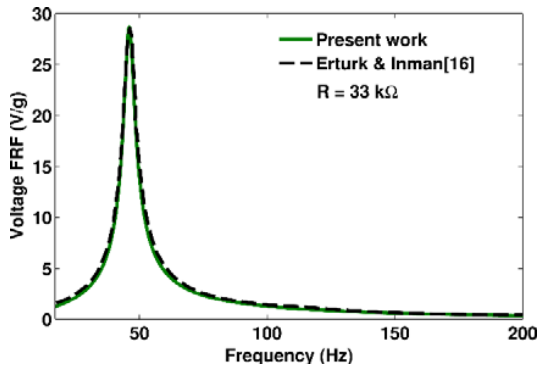
Fig. 3 Validation of present work with reference

VI. RESULTS

In simulations, the piezoelectric material is PZT-5A and the middle layer is from steel. PZT is selected because according to [20], PZT composites present piezoelectric properties greatly. Also, it was concluded from [21] that PZT is the best piezoelectric material for low noise. In addition to that, A-type PZTs have more output voltage with respect to H-types [22].

The length, the width, and the height of the cantilever beam are 20 mm, 14 mm, and 0.016 mm, respectively. These dimensions were selected because the magnetic field around a current carrying wire is very weak and the dimensions should be in low range in order to sense the weak magnetic field. Besides, the length, the width and the height of the cantilever beam must be selected in a way to adjust the frequency on desired value. The parameters of Table I are used in simulations.

TABLE I
GEOMETRIC AND MATERIAL PARAMETERS

Parameter	Piezoelectric	Steel
Layer's length (mm)	20	20
Layer's width (mm)	14	14
Layer's height (mm)	0.06	0.04
Mass of permanent magnet (kg)	1.82	
Density (kg.m^{-3})	7750	7850
Elastic modulus (GPa)	62	200
Dielectric constant (pm.V^{-1})	-171	-

As mentioned before, by using electromagnetic equations, the applied force from current carrying wire would be calculated. For an AC current with 15 A amplitude, the amplitude of applied force will be 0.00045 N. The frequency response diagram of the beam for 26 $k\Omega$ output resistance is presented in Fig. 4. It is obvious that in 47 Hz, output voltage has the maximum amount. In measuring electrical current by piezoelectric cantilever beam, the main point is the determination of output voltage. By applying a force equivalent to 15 A current, output voltage is plotted in time domain and shown in Fig. 5. It shows that after some milliseconds, the output voltage reaches its maximum amount and AC voltage appears in beam's output. Also, in both Figs. 4 and 5, maximum output voltage is observed, which is 0.35 V. While studying the beam's responses, many items must be

considered because changing beam's parameters could have large effects on both frequency response and output voltage response. One of these important parameters is the length of the beam. In Fig. 6, natural frequency for different lengths is presented. It is shown that by increasing the length, natural frequency decreases.
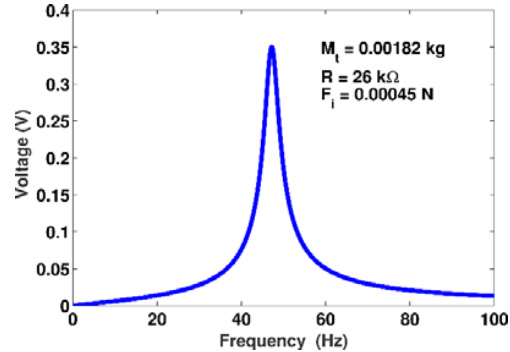

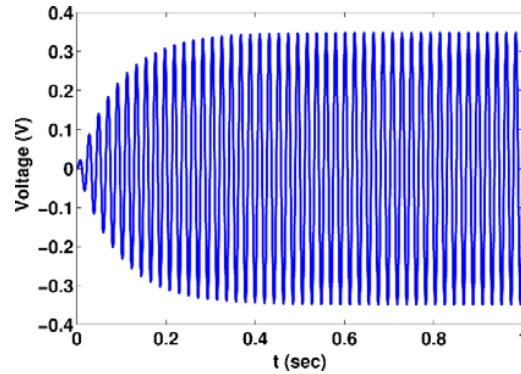Fig. 4 Frequency response of the cantilever beam
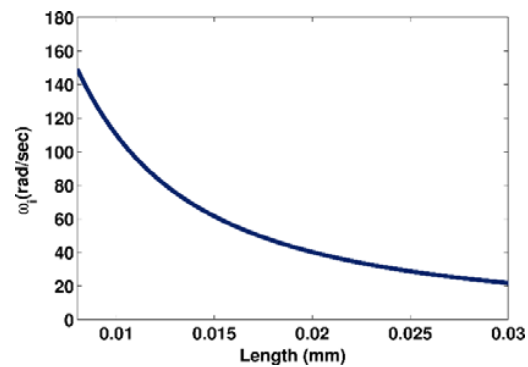

Fig. 5 Output voltage of the sandwich beam

Fig. 6 Natural frequency of the beam vs. different beam lengths

In addition to frequency, output voltage changes by length too. In Fig. 7, frequency response of the beam is plotted for five different lengths. It can be seen that, resonant frequency is decreased and output voltage amplitude is increased.
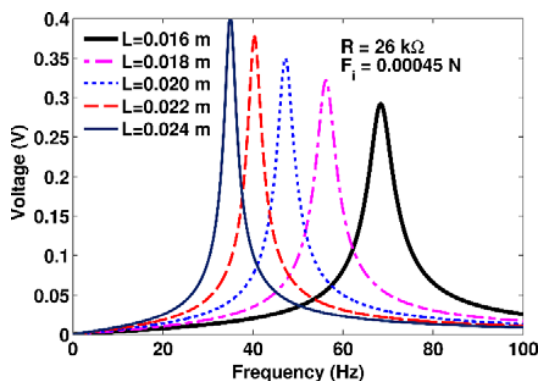
Fig. 7 Frequency response of the beam for different lengths

Width is another important parameter of the beam. This parameter, like length, has effects on beam's response, too. The changes of natural frequency by changing the width is studied and presented in Fig. 8, whereas other parameters are fixed.
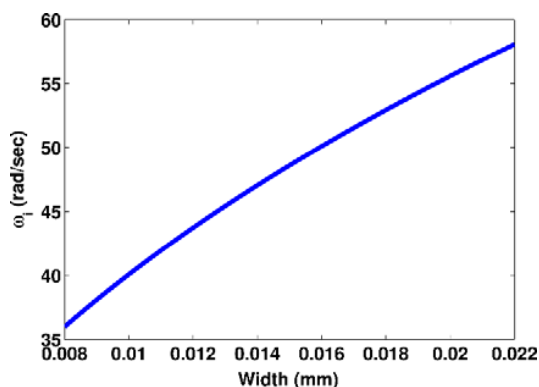

Fig. 8 Natural frequency changes vs. different widths

It is observed that by increasing the width, the natural frequency increases, too.

VII. Conclusion

Piezoelectric cantilever beams are being used in measurement processes widely. In order to measure a wire's electrical current in noncontact way, mathematical model for the bimorph cantilever beam is presented. Numerical results of the model studied and its validation in order to check the correctness are also presented. Afterwards, the variation of both output voltage and natural frequency by changing beam's geometric parameters are expressed.

By designing these cantilever beams accurately, they could be used in noncontact electrical current measurement processes. After determining the amount of output voltage, the amount of electrical current crossing the conductor will be determined, too. In simulations, the output voltage of 0.35 V with respect to the current of 15 A is obtained.

References

[1] J. S. Donnal and S. B. Leeb, "Noncontact Power Meter," *Sensors Journal, IEEE,* vol. 15, pp. 1161-1169, 2015.

[2] K. Rajitha and M. Manasa, "Home Electrical Parameters Measurements Using Power Sensor Tag," *Journal research in electrical electronics and communications,* vol. 02.

[3] E. J. Moniz, "Engaging Electricity Demand," presented at the MIT Study on the Future of the Electric Grid, Cambridge, MA, USA, Dec. 2011.

[4] S. S. Rao and M. Sunar, "Piezoelectricity and its use in disturbance sensing and control of flexible structures: a survey," *Applied mechanics reviews,* vol. 47, pp. 113-123, 1994.

[5] A. Vázquez Carazo and R. Bosch i Tous, "Novel piezoelectric transducers for high voltage measurements," Doctoral, d'Enginyeria Elèctrica, Universitat Politècnica de Catalunya, Barcelona, 2000.

[6] J. Yang, *An introduction to the theory of piezoelectricity* vol. 9: Springer Science & Business Media, 2004.

[7] E. Leland, P. Wright, and R. White, "Design of a MEMS passive, proximity-based AC electric current sensor for residential and commercial loads," in *Procedings of PowerMEMS*, Freiburg Germany, 2007, pp. 77-80.

[8] E. S. Leland, P. Wright, and R. M. White, "A MEMS AC current sensor for residential and commercial electricity end-use monitoring," *Journal of Micromechanics and Microengineering,* vol. 19, p. 094018, 2009.

[9] E. S. Leland, C. T. Sherman, P. Minor, R. M. White, and P. K. Wright, "A new MEMS sensor for AC electric current," in *Sensors*, Kona, HI, 2010, pp. 1177-1182.

[10] K. Isagawa, D. F. Wang, T. Kobayashi, T. Itoh, and R. Maeda, "Development of a MEMS DC electric current sensor applicable to two-wire electrical appliance cord," in *International Conference on Nano/Micro Engineered and Molecular Systems (NEMS)*, 2011, pp. 932-935.

[11] Q. Xu, M. Seidel, I. Paprotny, R. M. White, and P. K. Wright, "Integrated centralized electric current monitoring system using wirelessly enabled non-intrusive ac current sensors," in *IEEE Sensors*, Limerick, Ireland, 2011, pp. 1998-2001.

[12] W. He, P. Li, Y. Wen, and C. Lu, "A self-powered high sensitive sensor for AC electric current," in *Sensors*, 2011, pp. 1863-1865.

[13] D. K. Cheng, *Field and wave electromagnetics*: Pearson Education India, 1989.

[14] S. S. Rao, *Vibration of continuous systems*: John Wiley & Sons, 2007.

[15] A. Erturk and D. J. Inman, *Piezoelectric energy harvesting*: John Wiley & Sons, 2011.

[16] A. Erturk and D. J. Inman, "An experimentally validated bimorph cantilever model for piezoelectric energy harvesting from base excitations," *Smart materials and structures,* vol. 18, p. 025009, 2009.

[17] J. W. Yi, W. Y. Shih, and W.-H. Shih, "Effect of length, width, and mode on the mass detection sensitivity of piezoelectric unimorph cantilevers," *Journal of applied physics,* vol. 91, pp. 1680-1686, 2002.

[18] D. Shen, J.-H. Park, J. H. Noh, S.-Y. Choe, S.-H. Kim, H. C. Wikle*, et al.*, "Micromachined PZT cantilever based on SOI structure for low frequency vibration energy harvesting," *Sensors and actuators A: physical,* vol. 154, pp. 103-108, 2009.

[19] X. Li, W. Y. Shih, I. A. Aksay, and W. H. Shih, "Electromechanical Behavior of PZT-Brass Unimorphs," *Journal of the American Ceramic Society,* vol. 82, pp. 1733-1740, 1999.

[20] Z. De-Qing, W. Da-Wei, Y. Jie, Z. Quan-Liang, W. Zhi-Ying, and C. Mao-Sheng, "Structural and electrical properties of PZT/PVDF piezoelectric nanocomposites prepared by cold-press and hot-press routes," *Chinese Physics Letters,* vol. 25, p. 4410, 2008.

[21] L. Capineri, L. Masotti, V. Ferrari, D. Marioli, A. Taroni, and M. Mazzoni, "Comparisons between PZT and PVDF thick films technologies in the design of low-cost pyroelectric sensors," *Review of Scientific Instruments,* vol. 75, pp. 4906-4910, 2004.

[22] M. J. Ramsay and W. W. Clark, "Piezoelectric energy harvesting for bio-MEMS applications," in *SPIE's 8th Annual International Symposium on Smart Structures and Materials*, 2001, pp. 429-438.