Mathieu Stability of Offshore Buoyant Leg Storage and Regasification Platform

S. Chandrasekaran, P. A. Kiran

Abstract—Increasing demand for large-sized Floating, Storage and Regasification Units (FSRUs) for oil and gas industries led to the development of novel geometric form of Buoyant Leg Storage and Regasification Platform (BLSRP). BLSRP consists of a circular deck supported by six buoyant legs placed symmetrically with respect to wave direction. Circular deck is connected to buoyant legs using hinged joints, which restrain transfer of rotational response from the legs to deck and vice-versa. Buoyant legs are connected to seabed using taut moored system with high initial pretension, enabling rigid body motion in vertical plane. Encountered environmental loads induce dynamic tether tension variations, which in turn affect stability of the platform. The present study investigates Mathieu stability of BLSRP under the postulated tether pullout cases by inducing additional tension in the tethers. From the numerical studies carried out, it is seen that postulated tether pullout on any one of the buoyant legs does not result in Mathieu type instability even under excessive tether tension. This is due to the presence of hinged joints, which are capable of dissipating the unbalanced loads to other legs. However, under tether pullout of consecutive buoyant legs, Mathieu-type instability is observed.

Keywords—Offshore platforms, stability, postulated failure, dynamic tether tension.

I. INTRODUCTION

INCREASING demand for larger storage and regasification floating units to transport liquefied natural gas (LNG) results in exploration of offshore processing platforms in recent past; new geometrical form of BLSRP is proposed to meet increasing functional requirements [1], [2]. Circular deck used for storage and processing LNG is supported on six buoyant legs, which are symmetrically positioned with respect to CG of the deck. Hinged joints, placed between the deck and buoyant legs, isolate the deck partially. Buoyant leg structures show a few major advantages as easy installation, transportation, fabrication, and technical superiority [3]-[5]. Uniqueness of BLSRP is the partial isolation of the deck from buoyant legs, which reduces the deck response in rotational degrees-of-freedom making it safe for LNG processing [6]-[9]. The six buoyant legs, placed symmetrically, ensure greater stability and make the platform insensitive to wave directions. As the platform is positively buoyant, high initial pretension is imposed on tethers to ensure position restraint. Even though studies have been conducted on the dynamic analysis, literature shows lack of stability investigation in BLSRP, and hence, the present study aims at studying the stability

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condition of the same.

Mathieu's equation has been used in offshore industries to determine the stability of ships and spar platforms [10]-[12]. Mathieu equation is a special form of Hills equation, with only one harmonic mode. The canonical form of Mathieu equation is given by

$$\frac{d^2 f}{d\tau} + (\boldsymbol{\delta} - q\cos(2\boldsymbol{\tau}))f = 0 \tag{1}$$

where $\boldsymbol{\delta}$ and $\mathbf{q}$ are the Mathieu parameters which have to be derived for each problem. Studies on Mathieu stability analysis for taut moored compliant structures such as for TLP already exist [13], [14]. Researchers emphasized the significance of Mathieu stability analysis on deep water platforms, as tethers will be subjected to dynamic tension variations [15]. They also stated that modeling tethers as linear cable is fairly a decent approximation.

II. MATHIEU STABILITY EQUATION

Mathieu equation for the present case is derived from the idealized linear cable equation, and the Mathieu parameters are obtained as

$$\boldsymbol{\delta}_n = \frac{4}{M\omega^2}\left\{mg\frac{(I_2 + I_4)}{I_1}\right\} - (P + mgL)\frac{I_3}{I_1} \tag{2a}$$

$$q_n = \frac{2A}{M\omega^2}\frac{I_3}{I_1} \tag{2b}$$

where M is the total tether mass, ω is the wave frequency, m is the mass per unit length of tether, g is the acceleration due to gravity, P is the initial pretension, and A is the tension amplitude. And the corresponding integrals are given as

$$I_1 = \int_0^L X_n^{\,2}(x)dx \tag{3a}$$

$$I_2 = \int_0^L \frac{dX_n}{dx}\,dx \tag{3b}$$

$$I_3 = \int_0^L \frac{d^2 X_n}{dx^2}\,X_n\,dx \tag{3c}$$

$$I_4 = \int_0^L \frac{d^2 X_n}{dx^2}\,X_n\,(xdx) \tag{3d}$$

where X_n is

$$(X_n x) = J_0\left(2\beta_n\left[1 + \frac{mg(L-x)}{P}\right]^{1/2}\right) - \frac{J_0(2\beta n)}{Y_0(2\beta n)}\,Y_0\left(2\beta_n\left[1 + \frac{mg(L-x)}{P}\right]^{1/2}\right) \tag{4}$$

where β_n is obtained as the solution of:

$$J_0\left(2\beta_n\sqrt{1+\frac{mgL}{P}}\right)Y_0(2\beta_n) - Y_0\left(2\beta n\sqrt{1+\frac{mgL}{P}}\right)J_0(2\beta_n) = 0 \tag{5}$$

The solution to Mathieu equation is expressed as stability chart since the stability condition varies depending on the values of Mathieu parameters, and hence, a general solution cannot be obtained.

III. MATHEMATICAL MODELLING

The BLSRP is modelled using ANSYS AQWA software. BLSRP consists of six buoyant legs placed symmetrically at 60^0 to each other and is modelled as a line element since it qualifies for Morison region, and each BLS is a circular tubular structure with outer diameter 14.14 and thickness of 0.15 m. The deck is of 100 m diameter and it is connected to each buoyant leg using hinged joints ensuring the isolation of roll, pitch, and yaw motion. Each buoyant leg is inclined at 20^0 to the deck, and the BLS is moored using high pretension tethers and uses spread mooring system. Each leg consists of a group of four tethers, and a total of 24 tethers hold the BLSRP in its position. The static equilibrium between the buoyancy force, weight, and initial tether tension force is given by

$$F_b = W + 6\,T_0\cos(20) \tag{6}$$

Table I shows the geometric properties of the platform, and Fig. 1 shows the BLSRP model (case1) with buoyant leg numbering. For tether pullout study, two cases have been studied, where in case 2 tether connecting bls 1 to sea bed (tether 1) is removed, and for second study (case 3), one more tether is removed, i.e. tether 4 (tether connecting bls4 and sea bed). Fig. 2 shows the models with tethers removed.

TABLE I
GEOMETRIC PROPERTIES OF BLSRP

Description	BLSRP	Units
Water depth	1069.36	m
Total mass	641000	kN
Buoyant force	940880	kN
Diameter of buoyant leg	14.14	m
Plan dimension	100	m
Length of buoyant leg	132.48	m
Total tether force	319125.60	kN
Pretension in each leg	53187.61	kN
Tether length	964.81	m
No of tethers (six groups)	24	Four tethers in each group
AE/L	76830.67	kN/m
Unit weight of material	7850	kg/m^3
Unit weight of surrounding fluid	1025	kg/m^3

IV. RESULTS AND DISCUSSIONS

BLSRP was analyzed under regular Airy's wave of wave elevation 5 m and wave period 6.8 s for all three cases. The tension variation in tethers is plotted in Fig. 3, and the maximum tension amplitude is listed in Table II.
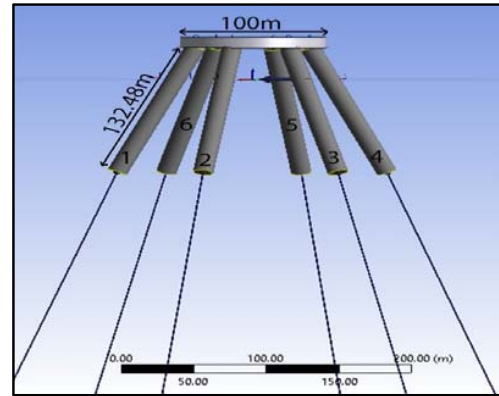

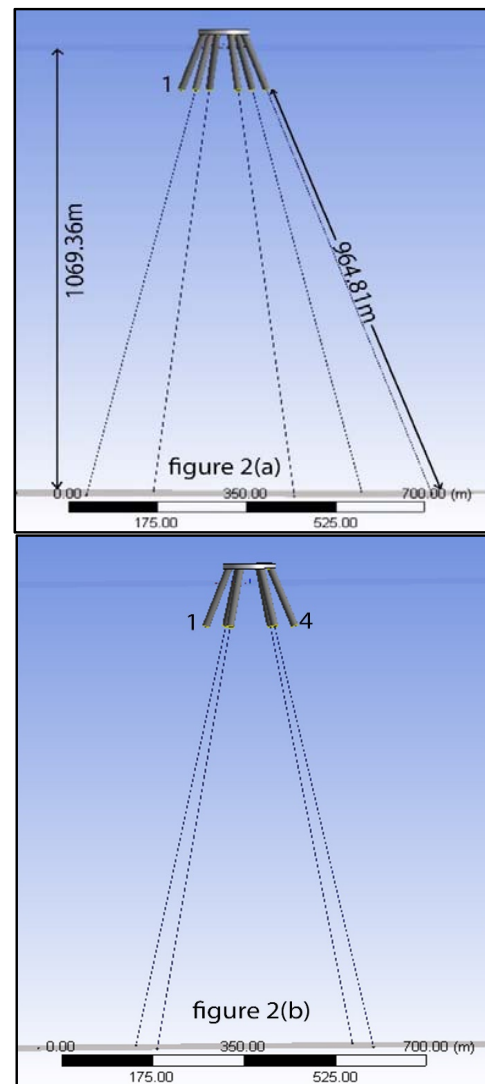Fig. 1 BLSRP model for case 1

Fig. 2 (a) The model for case 1 with tether 1 removed and (b) shows case 2 with tether 1 and tether 4 removed
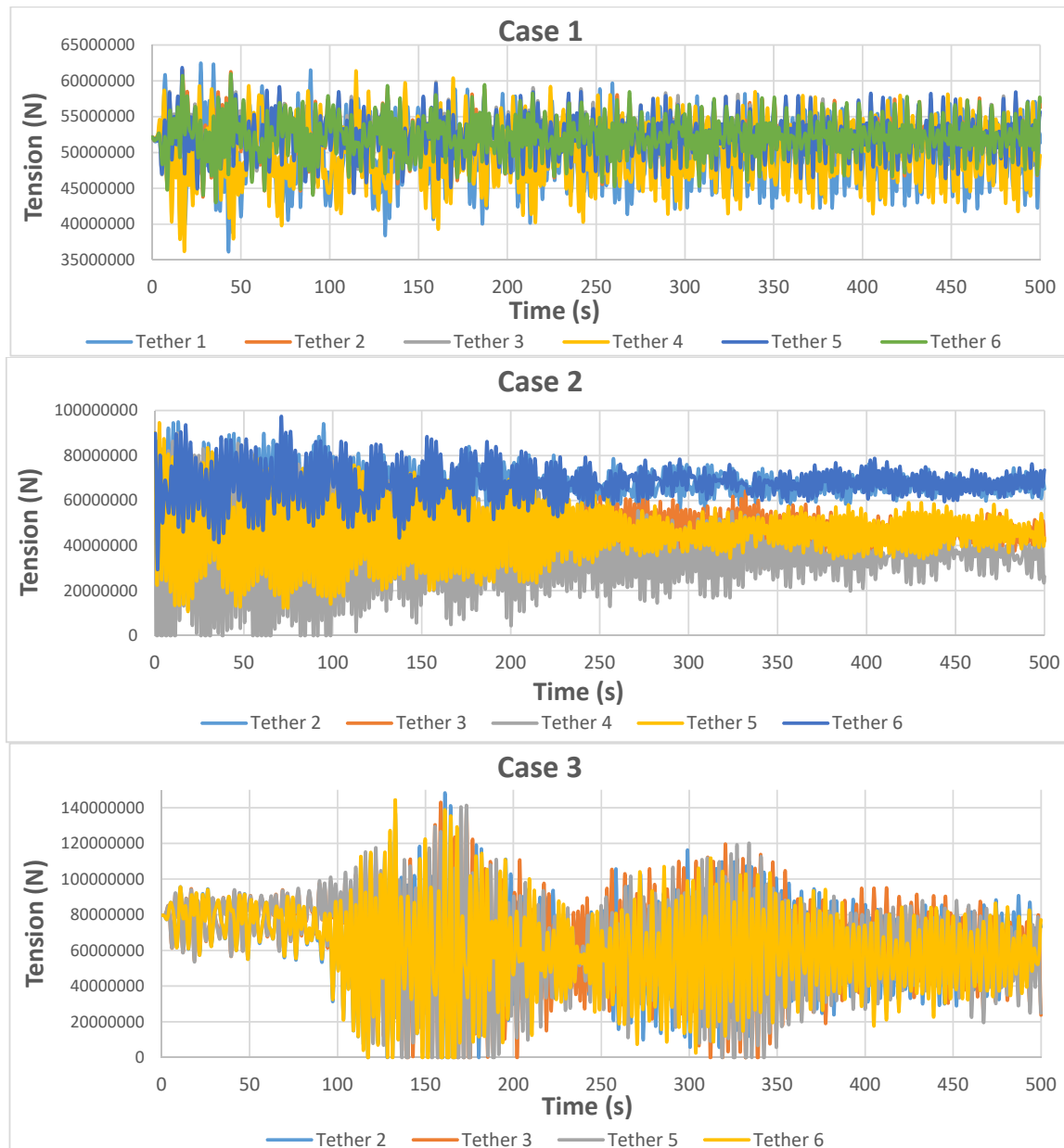
Fig. 3 Tension variation in tethers

TABLE II
MAXIMUM TENSION AMPLITUDE IN TETHERS FOR EACH CASE

Description	Tether 1 (N)	Tether 2 (N)	Tether 3 (N)	Tether 4 (N)	Tether 5 (N)	Tether 6 (N)	Maximum tension (N)
case 1	62485872	61529240	61430768	61397948	61848120	60971208	62485872
case 2	-	94936328	80994824	89918696	94570560	97407632	97407632
case 3	-	148282640	142983648	-	141300128	144282592	148282640

The tension variation in the initial case1 was noted to be 17.5% of the initial pretension, whereas for the case 2 the tension variation increased to 83%. While in case 2 where tether 1 is completely removed, it is seen that the maximum tension occurs in tether 6 and tether 2 which are adjacent to tether 1. For case 2 where tether 1 and tether 4 are removed, the tension variation in the remaining tethers is more or less the same. An increase of 49.9% in the initial tension is also noted at 0 sec, due to the distribution of tension force to compensate the loss of initial pretension in 2 tethers. Chaotic nature in the tether tension variation is noted for case 3, indicating possible failure or dynamic instability in the structure. With the dynamic tension variation in the tethers, stability analysis using Mathieu equation is performed and the

Mathieu parameters are obtained for each case and are plotted in stability chart in Fig. 4 and listed in Table III.
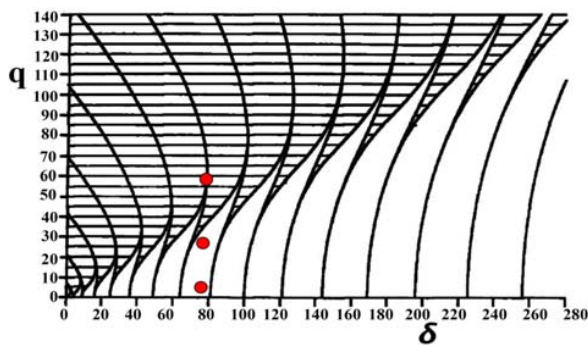

Fig. 4 Mathieu stability chart for BLSRP

TABLE III
MATHIEU PARAMETERS FOR BLSRP

Description	δ	q	Stability condition
case 1	75.07	5.9	STABLE
case 2	75.07	28.06	STABLE
case 3	75.07	60.34	UNSTABLE

Since the parameter δ depends on the tether stiffness and initial pretension, δ remains the same for all the cases. Significant increase in parameter q is noted as tether pullout occurs. The solution to Mathieu equation is expressed as stability chart and the parameters were plotted to obtain stability condition. The shaded region in the plot shows unstable region and the unshaded region shows stable region. Case 1 is well within the unshaded region confirming stable condition. For case 2 as tether 1 is removed, the parameters even though move vertically towards the unstable region the structure remains stable and Mathieu instability does not occur. Further removal of tether 4 as in case 3 leads to unstable region, the parameters fall under shaded region, and hence, platform is in unstable condition. The chaotic nature in the tension variation plots is hence justified. The structure undergoes Mathieu instability under case 3.

V. CONCLUSIONS

BLSRP is a promising new geometric form for oil and gas industries. The six legs placed symmetrically makes the platform insensitive to wave direction and gives better stability. A detailed numerical analysis was carried out for BLSRP using Mathieu equation of stability. Stability analysis shows good stability condition for the new geometry form. Even when single tether undergo pullout, the structure shows stable condition; however, significant increase in tethers were noted. When two buoyant legs are disconnected in case of tether pullout, the tension variation in the tethers are alarming and undergo a chaotic nature. Detailed investigation shows tethers undergo Mathieu instability. The present work aims to emphasize the significance of Mathieu stability analysis in offshore platforms.

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