International Journal of Information, Control and Computer Sciences
ISSN: 2517-9942
Vol:11, No:9, 2017

On Fault Diagnosis of Asynchronous Sequential Machines with Parallel Composition

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***Abstract*—**Fault diagnosis of composite asynchronous sequential machines with parallel composition is addressed in this paper. An adversarial input can infiltrate one of two submachines comprising the composite asynchronous machine, causing an unauthorized state transition. The objective is to characterize the condition under which the controller can diagnose any fault occurrence. Two control configurations, state feedback and output feedback, are considered in this paper. In the case of output feedback, the exact estimation of the state is impossible since the current state is inaccessible and the output feedback is given as the form of burst. A simple example is provided to demonstrate the proposed methodology.

***Keywords*—**Asynchronous sequential machines, parallel composition, fault diagnosis.

I. Introduction

As a novel automatic control scheme for event-driven systems, corrective control has been successfully used to solve various control problems of asynchronous sequential machines. The efficiency of corrective control is remarkable especially in fault diagnosis and fault tolerant control for asynchronous sequential machines; refer to [1]–[3] for theoretical development of this topic, and to [4], [5] for experimental verification on FPGA-based asynchronous digital systems.

In this paper, we address the problem of fault diagnosis for a composite asynchronous sequential machine made of parallel composition of two single input/state asynchronous sequential machines. Parallel composition is widely used in manufacturing systems [6] and modeling and control of discrete event systems [7]. The main objective is to diagnose any unauthorized state transition occurring to a single submachine comprising the composite machine. Two control configurations, state feedback and output feedback, are considered separately in fault diagnosis. When state feedback is available, the controller knows the state at which the fault occurs as well as the state reached by the machine as the result of the fault. On the other hand, the output feedback makes it impossible for the controller to derive the current state. In particular, we assume that the output feedback value is transmitted as the form of burst, a quick succession of output characters [8]. Since exact identification of the current state is impossible, we derive the change of state uncertainty throughout the unauthorized transition. Fault detectability is also examined to investigate whether the end of an authorized transition can be determined by the controller. Note that the construction of a fault tolerant controller is not discussed in this paper.

Recent study on control of composite asynchronous sequential machines can be found in [9] where fault tolerant control for composite asynchronous sequential machines with cascade connection is addressed, and in [10] where model matching of switched asynchronous sequential machines is addressed. Note that the present study differs from both [9], [10] since they do not use the modeling formalism of parallel composition.

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (No. 2015R1D1A1A01056764) and by the Ministry of Science, ICT and future Planning (No. 2015R1A2A1A15054026).

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II. Modeling

A major part of this content is borrowed from our prior work (e.g, [9], [11]). A composite asynchronous sequential machine $\Sigma = \Sigma_1||\Sigma_2$ consists of parallel composition of two input/state asynchronous sequential machines Σ_1 and Σ_2 with

$$\begin{aligned}\Sigma_1 &= (A, X, x_0, f_1)\\ \Sigma_2 &= (A, Y, y_0, f_2)\end{aligned} \tag{1}$$

where X and Y are the state set of Σ_1 and Σ_2, respectively, $x_0 \in X$ and $y_0 \in Y$ are the initial states, and $f_1 : X \times A \to X$ and $f_2 : Y \times A \to Y$ are the state transition functions partially defined on $X \times A$ and $Y \times A$. The input set A is further divided into $A = A_n \dot{\cup} A_d$ where A_n and A_d are the set of normal and adversarial inputs, respectively.

Σ_1 (and Σ_2) is operated with the feature of asynchrony. A valid state–input pair $(x, v') \in X \times A$ of Σ_1 is a stable combination if $f_1(x, v') = x$; otherwise, it is a transient combination. Owing to the absence of a synchronizing clock, Σ_1 stays at a stable combination (x, v') indefinitely. If the input v' changes to another value v for which (x, v) is a transient combination, Σ_1 engages in a series of transient transitions

$$\begin{aligned}f_1(x, v) &= x_1\\ f_1(x_1, v) &= x_2\\ &\vdots\end{aligned} \tag{2}$$

where v remains fixed. If no infinite cycles exist, Σ_1 reaches the *next stable state* x_k such that $x_k = f_1(x_k, v)$ at the end of the chain with k transient transitions $x, x_1, \ldots, x_{k-1}$. Since the transition speed of asynchronous sequential machines is instantaneous (ideally zero), the meaningful behavior of asynchronous sequential machines can be represented only by

stable states. To this end, we introduce the stable recursion function s as follows [1]:

$$s_1 : X \times A \to X$$
$$s_1(x, v) = x' \tag{3}$$

where x' is the next stable state of a valid state–input combination (x, v). A sequence of transient transitions from a stable state to the corresponding next stable state, as represented by s_1, is called *stable transition*. The domain of s_1 can be expanded to $X \times A^+$ in a natural way as follows, where A^+ is the set of all nonempty strings of characters in A.

$$s_1(x, v_1v_2 \cdots v_k) = s_1(s_1(x, v_1), v_2 \cdots v_k),$$
$$v_1v_2 \cdots v_k \in A^+. \tag{4}$$

Note that the aforementioned definitions and properties are equally applied to Σ_2.

Σ is described as an input/output asynchronous sequential machine

$$\Sigma = \Sigma_1 || \Sigma_2 = (A, Z, X \times Y, (x_0, y_0), s, h) \tag{5}$$

where Z is the output set, $X \times Y$ are the state set with the initial state (x_0, y_0), $s : X \times Y \times A \to X \times Y$ and $h : X \times Y \to Z$ are the stable recursion function and output function, respectively. To prohibit unpredictable outcomes caused by the absence of a synchronizing clock, Σ_c is assumed to comply with the principle of fundamental mode operations [12] whereby a variable must change its value when both C and Σ are in stable states, and no two or more variables may be changed simultaneously. Under the assumption of fundamental mode operations, we naturally assume that once the input $u \in A$ changes, one of Σ_1 and Σ_2 takes a stable transition in the first, and only after the end of the transition does the second asynchronous sequential machine initiate its stable transition. Which asynchronous sequential machine among Σ_1 and Σ_2 takes the first transition is nondeterministic. Regardless of the order, however, the next stable states reached by Σ_1 and Σ_2 are always deterministic. Thus we represent Σ as a stable-state machine described only by s as

$$s(x, y, u) := \begin{cases} (s_1(x, u), s_2(y, u)) & s_1(x, u)! \text{ and } s_2(y, u)! \\ (s_1(x, u), y) & s_1(x, u)! \text{ and } s_2(y, u)¡ \\ (x, s_2(y, u)) & s_1(x, u)¡ \text{ and } s_2(y, u)! \\ \text{undefined} & \text{otherwise} \end{cases} \tag{6}$$

where $s_1(x, u)!$ and $s_1(x, u)¡$ mean that $s_1(x, u)$ is defined and undefined, respectively.

The output of Σ is given as the form of *burst* [8], a quick succession of output characters. For a stable transition $s(x, y, u) = (x', y')$, assume that Σ_1 and Σ_2 traverse a series of transient states $x_1, \ldots, x_k$ and $y_1, \ldots, y_l$, respectively. According to the foregoing discussion, either Σ_1 or Σ_2 may conduct its stable transition in the first, followed by that of the other asynchronous sequential machine. Hence the trajectory of state pairs is one of the following two strings.

$$\text{(i)} \quad (x, y)(x_1, y) \cdots (x_k, y)(x', y)(x', y_1) \cdots (x', y_l)(x', y')$$
$$\text{(ii)} \quad (x, y)(x, y_1) \cdots (x, y_l)(x, y')(x_1, y') \cdots (x_k, y')(x', y') \tag{7}$$

where (i) is the outcome with the assumption that Σ_1 takes the first transition and (ii) is the outcome with the reverse order. To address the nondeterministic feature, we assign two output bursts $b_1, b_2 \in Z^+$ for each stable transition $s(x, y, u) = (x', y')$ by defining a mapping

$$B : X \times Y \times A \to P(Z^+) \tag{8}$$

as ($P(Z^+)$ is the power set of Z^+)

$$B(x, y, u) := \{b_1, b_2\}$$
$$b_1 := \beta(h(x, y)h(x_1, y) \cdots h(x_k, y)h(x', y)h(x', y_1) \cdots h(x', y')) \tag{9}$$
$$b_2 := \beta(h(x, y)h(x, y_1) \cdots h(x, y_l)h(x, y')h(x_1, y') \cdots h(x', y'))$$

where $\beta(\cdot)$ replaces each segment of repeating characters by a single one, e.g., $\beta(z_1z_1z_2z_2) = z_1z_2$. For later usage, denote by $b_1^f, b_2^f \in Z$ the last character of b_1 and b_2. By definition,

$$b_1^f = b_2^f = h(x', y'). \tag{10}$$

Also, denote by $b_1^{-1}, b_2^{-1} \in Z^+$ the string obtained by removing b_1^f and b_2^f from b_1 and b_2, respectively. Then,

$$b_1^{-1} := \beta(h(x, y)h(x_1, y) \cdots h(x', y)h(x', y_1) \cdots h(x', y_l))$$
$$b_2^{-1} := \beta(h(x, y)h(x, y_1) \cdots h(x, y')h(x_1, y') \cdots h(x_k, y')). \tag{11}$$

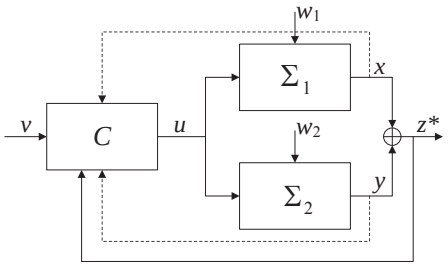

Fig. 1 Corrective control system for a composite asynchronous sequential machine $\Sigma = \Sigma_1 || \Sigma_2$

Fig. 1 illustrates the corrective control system for a composite asynchronous sequential machine Σ. C is the corrective controller, $v \in A_n$ is the externa input, $u \in A_n$ is the control input generated by C, x and y are the state of Σ_1 and Σ_2, z is the output of Σ, and $w_1, w_2 \in A_d$ are the adversarial inputs occurring to Σ_1 and Σ_2. w_1 are w_2 override the control input $u \in A_n$ and cause the corresponding asynchronous sequential machine to undergo an unauthorized state transition. For instance, when Σ_1 has been staying at a stable state x when w_1 occurs such that $s_1(x, w_1)!$, Σ_1 undergoes the unauthorized transition from x to $s_1(x, w_1)!$. The next operation of Σ would be incorrect if Σ is not counteracted from this fault immediately.

In this paper, we consider two control configuration separately — (i) state feedback and (ii) output feedback where the feedback value is given as output burst. In Fig. 1, the route of state feedback is marked in dashed lines to highlight our setting. In the case of state feedback, both x and y are transmitted to C. Hence the formulation of C is written as

$$C = (A_n \times X \times Y, A_n, \Xi, \xi_0, \phi, \eta) \text{ with } (x, y) \tag{12}$$

where $A_n \times X \times Y$ is the input set (v, x, and y), A_n is the output set serving as the control input u, Ξ is the state set, $\xi_0 \in \Xi$ is the initial state, $\phi : \Xi \times X \times Y \times A_n \to \Xi$ is the recursion function, and $\eta : \Xi \to Z$ is the output function. In the case of output feedback, on the other hand, output burst termed z^* in Fig. 1 is relayed to C as the output feedback. Hence the form of C is

$$C = (A_n \times Z^+, A_n, \Xi, \xi_0, \phi, \eta) \text{ with } z^* \tag{13}$$

The objective of fault diagnosis by C also depends on the control configuration as follows.

(i) In the case of state feedback, C must identify the original state of Σ at which the unauthorized state transition initiates and the deviated state reached by Σ as the result of the fault.

(ii) As mentioned earlier, the exact observation of the state is impossible in the case of output feedback. Instead, we must specify a state set, one element of which Σ stays at the moment of the fault occurrence, and another state set representing all the possible states that can be reached by Σ as the result of the fault.

III. Fault Diagnosis

A. State Feedback

Provided that state feedback x and y are available in the architecture of Fig. 1, let us discuss fault diagnosis on w_1 and w_1. First, assume that Σ has been staying at a stable state $(\bar{x}, \bar{y})$ when w_1 occurs, enforcing Σ_1 to reach $s_1(\bar{x}, w_1) = x'$. C can diagnose the occurrence of w_1 by observing that the state feedback of Σ_1 changes to x' while the external input v remains fixed. Since only one variable can change at a time under the principle of fundamental mode operations, w_2 never happens at the moment w_1 happens. Thus the next state Σ reaches by w_1 is $(x', \bar{y})$.

The foregoing discussion is equally applied to an occurrence of w_2. If w_2 occurs to Σ_2 such that $s_2(\bar{y}, w_2) = y'$, C can diagnose this fault by observing that the state feedback changes from $(\bar{x}, \bar{y})$ to $(\bar{x}, y')$ while the external input v remains fixed.

In summary, when full state feedback is available to C, one can diagnose any fault event as follows in line with the change of state feedback.

(i) $(\bar{x}, \bar{y}) \to (x', \bar{y})$: w_1 occurs to Σ_1 such that $s_1(\bar{x}, w_1) = x'$.
(ii) $(\bar{x}, \bar{y}) \to (\bar{x}, y')$: w_2 occurs to Σ_2 such that $s_2(\bar{y}, w_2) = y'$.

B. Output Feedback

Since the exact identification of the current state of Σ is impossible in the control configuration of output feedback, we introduce the notion of state uncertainty in this paper. Let $\chi \subset X \times Y$ be uncertainty about the state of Σ. χ implies that the current state of Σ is unknown but an element of χ. Suppose that Σ stays at a stable state with the external input $v \in A_n$, the output $z \in Z$, and the output burst $b \in Z^+$. The latter means that z is the last character of b, i.e., $z = b^f$. Although direct access to the current state is impossible, the information on v, z, and b allows us to estimate all the possible states where Σ may stay, that is, the *state uncertainty* χ. The more information we have access to, the more we can reduce the size of the state uncertainty χ. First, we express the state uncertainty solely in terms of the output. Define $E_1(z) \subset X \times Y$ as

$$E_1(z) := \{(x, y) \in X \times Y | h(x, y) = z\}. \tag{14}$$

Using v, we can reduce the state uncertainty further since the current state makes a stable combination with v as well as it provides the output z. Let $E_2(v, z)$ denote the set of such states:

$$E_2(v, z) := \{(x, y) \in E_1(z) | s(x, y, v) = (x, y)\}. \tag{15}$$

If we know the previous state uncertainty, further reduction of the state uncertainty is possible. Let $\chi' \subset X$ be the previous state uncertainty, that is, Σ has experienced a stable transition from a state with the state uncertainty χ' to the current state. What we deduce from χ' is that the current state is the next stable state of a state in χ' with the external input v. $E_3(\chi', v, z)$ represents those states as follows.

$$E_3(\chi', v, z) := \{(x, y) \in E_2(v, z) | \exists (\hat{x}, \hat{y}) \in \chi' \text{ s.t. } s(\hat{x}, \hat{y}, v) = (x, y)\}. \tag{16}$$

Access to the output burst b further reduces the state uncertainty in association with χ'. To this end, define another mapping $E_4 : P(X) \times A_n \times Z \times Z^+ \to P(X)$ as follows.

$$E_4(\chi', v, z, b) := \{(x, y) \in E_3(\chi', v, z) | \exists (\hat{x}, \hat{y}) \in \chi' \text{ s.t. } B(\hat{x}, \hat{y}, v) = b\}. \tag{17}$$

Summing up the above analysis, we address the formulation of χ, uncertainty about the current state of Σ, with respect to the previous uncertainty χ', the external input v, the output z, and the output burst b.

$$\chi = E_4(\chi', v, z, b). \tag{18}$$

Assume now that in the control configuration of Fig. 1 with output feedback, Σ has been staying at a stable combination with the state uncertainty χ'. Assume further that the output is observed to change to z with the output burst b, while the external input v remains unchanged. Then one of adversarial inputs w_1 and w_2 must have occurred, causing an unauthorized state transition. The controller C can perceive the fault occurrence by observing a change of the output. Further, C can estimate that the current state of Σ is one of $\chi = E_4(\chi', v, z, b)$ as presented above.

To preserve fundamental mode operations, the input must not change while Σ undergoes any transitions. Thus C must determine not only state uncertainty updated after the end of an unauthorized transition, but also whether Σ has reached a next stable state by the adversarial input. Assume again that Σ experiences an unauthorized state transition in which the state uncertainty changes from χ' to χ with $v \in A_n$, $z \in Z$, and $b \in Z^+$. This unauthorized transition is said to be *fault detectable* if it can be determined from inputs and outputs of Σ whether the (unknown) next stable state in χ has been reached. Once χ' and χ are identified, we can induce all the

possible output bursts that may occur in this unauthorized state transition. Denote by

$$B(\chi',\chi) \subset Z^+ \tag{19}$$

such a set of bursts. It is known that to determine the termination of a transition only with output burst, the last character of the burst must differ from the one that is generated right before the last one [8]. In formal terms, we must ensure $b^{-1} \neq b$ to discern the end of a transition having the output burst b. Since this condition must be valid for any possible outcome associated with χ' and χ, we induce the following condition for fault detectability of the unauthorized transition from χ' to χ.

$$\forall b \in B(\chi',\chi),\ b^{-1} \neq b. \tag{20}$$

IV. Example

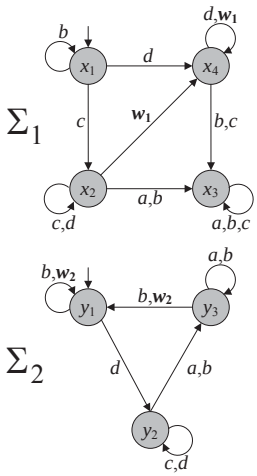

Fig. 2 $\Sigma = \Sigma_1||\Sigma_2$

Consider an instance of the composite asynchronous machine $\Sigma = \Sigma_1||\Sigma_2$ shown in Fig. 2 where $X = \{x_1, x_2, x_3, x_4\}$ with $x_0 = x_1$, $Y = \{y_1, y_2, y_3\}$ with $y_0 = y_1$, $A_n = \{a, b, c, d\}$, $Z = \{1, 2, \ldots, 5\}$, and $A_d = \{w_1, w_2\}$. For simplicity, we set $f_i = s_i$, $\forall i = 1, 2$. The output function h is defined in Table I.

Since fault detectability is self-evident in the case of state feedback, let us investigate fault detectability of the closed-loop system with output feedback. As an example instance, assume that Σ has been staying at a stable combination with $\chi' = \{(x_1, y_3), (x_3, y_3)\}$ and $v = b$ when the output feedback z changes from 3 to 2 with output burst 32. Using the derived formula of state uncertainty, we easily derive that

$$\chi = E_4(\chi', b, 2, 32) = \{(x_1, y_1), (x_3, y_1)\}. \tag{21}$$

Further, we have

$$B(\chi',\chi) = \{b'\} = \{32\}. \tag{22}$$

Since $b'^{-1} \neq b'$, this unauthorized state transition is fault detectable. Referring to Fig. 2, we know that w_2 is the adversarial input that causes this transition. The other unauthorized transition caused by w_1 is also fault detectable, derivation of which is omitted.

TABLE I
Output Function h

(x, y)	(x_1, y_1)	(x_2, y_1)	(x_3, y_1)	(x_4, y_1)
$h(x, y)$	2	1	2	3
(x, y)	(x_1, y_2)	(x_2, y_2)	(x_3, y_2)	(x_4, y_2)
$h(x, y)$	5	1	5	4
(x, y)	(x_1, y_3)	(x_2, y_3)	(x_3, y_3)	(x_4, y_3)
$h(x, y)$	3	2	3	1

V. Summary

We have investigated fault diagnosis of a class of composite asynchronous sequential machines with parallel composition. We have examined whether an unauthorized state transition can be identified in the closed-loop system of composite asynchronous sequential machines endowed with state or output feedback. Specifically, in the case of output feedback with output burst, uncertainty about the state of the machine is updated according to the available information of the machine. The condition for fault detectability is also derived in the framework of corrective control. The proposed scheme has been validated using a simple example instance.

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