

# Analysis of Simple Mechanisms to Continuously Vary Mach Number in a Supersonic Wind Tunnel Facility

Prateek Kishore, T. M. Muruganandam

**Abstract**— Supersonic wind tunnel nozzles are generally capable of producing a constant Mach number flow in the test section of the wind tunnel. As a result, most of the supersonic vehicles are widely designed using steady state flow characteristics which may have errors while facing unsteady situations. This study aims to explore the possibility of varying the Mach number of the flow during wind tunnel operation. The nozzle walls are restricted to be inflexible for cooling near the throat due to high stagnation temperature requirement of the flow to simulate the conditions as experienced by the vehicle. Two simple independent mechanisms, rotation and translation of nozzle walls have been analyzed and the nozzle ranges have been optimized to vary the Mach number from Mach 2 to Mach 5 using minimum number of nozzles in the wind tunnel.

**Keywords**—Method of characteristics, Nozzle, supersonic wind tunnel, variable Mach number.

## NOMENCLATURE

$\Omega$	Rotation of nozzle wall, positive if area ratio increases
$\Delta y$	Translation of nozzle wall, positive if area increases
$M_D$	Design Mach number
$M$	Mach number
$y_{\max}$	Minimum height of nozzle wall for any orientation
$y_r$	y-coordinate relative to $y_{\max}$
$\Delta M$	Maximum variation in Mach number from $y_r = 0$ to 0.4
$\theta$	Flow angle
$P_o$	Stagnation pressure
$T_o$	Stagnation temperature
$P_a$	Ambient pressure

## I. INTRODUCTION

WIND tunnel nozzle designs are mostly carried out for a specific Mach number and there exist established methods to design a nozzle for any given Mach number using method of characteristics (MOC) [3]. However, very few attempts have been made in the field of variable Mach number nozzle designs. Among the various techniques used to produce variable Mach number, flexible nozzles and asymmetric nozzle shapes are most widely used [1]-[3]. Flexible walls present disadvantages such as inability to attach a cooling system on nozzle walls for high stagnation temperature flows and high complexity of mechanical systems to operate a small nozzle. This design aims to simulate unsteady supersonic flow at similar operating conditions as experienced by a high-speed vehicle. A high-speed vehicle experiences high stagnation pressure and temperature during operation and thus the stagnation temperature of the wind tunnel needs to be

maintained high. The wall of the nozzle needs to be constructed of rigid material for a cooling system to be attached on it. The nozzle designs are aimed to produce varying Mach number ranging from Mach 2 to Mach 5, to study acceleration of vehicles such as scramjet which accelerates up to Mach 5.

This paper shows the analysis of two separate techniques used to produce continuously varying Mach number in the nozzle. The methods employ a simple change in orientations of the nozzle wall to change the overall nozzle area ratio. The first method is to rotate the nozzle wall downstream of the throat keeping the throat area to be the same thus increasing/decreasing the area ratio corresponding to the direction of rotation. The second method is to translate the nozzle walls perpendicular to the flow. The methods do not need the nozzle walls to be flexible and therefore can be cooled if operated at high temperatures.

## II. METHOD OF CHARACTERISTICS

The nozzle walls are designed using MOC for two-dimensional rotational isentropic flow [4]. The nozzle throat height is set to be 2 cm for all designs. The ambient pressure at nozzle exit is set to be 1 atm and the stagnation pressure is set higher than the third critical pressure ratio for all the cases to achieve supersonic flow at nozzle exit. The nozzle wall near the throat is a circular contour of radius 5 cm. The initial value line for MOC is obtained using Sauer's method [4]. The flow properties at points downstream of the initial value line are solved using characteristic and compatibility equations of left running and right running characteristics and the streamline characteristic passing through the point.

The variation in Mach numbers in the designed nozzle is produced by using two simple techniques, the rotation of nozzle wall and the translation of nozzle wall. The first method requires a hinge above the nozzle throat such that the rotation of nozzle wall does not vary the throat height. This can be achieved if the hinge is placed at the center of the circular contour which forms the nozzle throat. The rotation is considered positive if the exit area increases and negative when the exit area decreases. The schematic for nozzle rotation is displayed in Fig. 1 (a).

The second method employed is to translate the nozzle wall perpendicular to the flow direction either towards each other or away from each other. This creates a change in throat area as well as the exit area of the nozzle. If the walls move away from each other, it is considered in this paper to be positive translation and the exit area increases causing a decrease in the area ratio of the nozzle. Similarly, during a negative

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translation, the nozzle walls move towards each other causing a decrease in the nozzle exit area and an increase the area ratio of the nozzle. The schematic of the nozzle translation is shown in Fig. 1 (b). The vertical line near the symmetry line of the nozzles at nozzle exit in Figs. 1-3 represents the 40% of the nozzle exit height during minimum rotation angle and maximum negative translation. The model that is to be tested using the nozzle must have a size less than the 40% of the nozzle exit. This 40% of the nozzle exit will be henceforth referred as the region of interest in the paper.

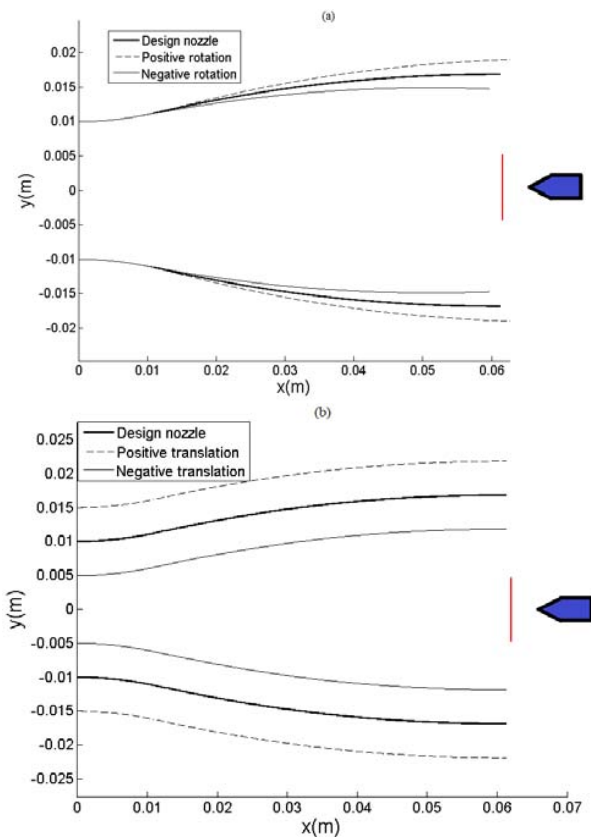


Fig. 1 Schematic of the techniques used to change the orientation of nozzle wall (a) Rotation of nozzle wall (b) Translation of nozzle wall

During any orientation change of the nozzle walls, there is a change in the nozzle exit area. For this reason, the nozzle is designed to be operated in an enclosed free jet or a free jet facility. The analysis of a nozzle designed to produce Mach 2 is shown in Section III along with the effects of rotation and translation of nozzle wall on the flow at nozzle exit. The model that is to be tested must be smaller than the region of interest. The region of interest is taken to be 40% of the nozzle exit height when the nozzle has the minimum exit height among all possible orientations. This feature is illustrated in Sections IV and V.

### III. MOC IMPLEMENTATION

MOC is implemented to design a nozzle to produce Mach 2 in its unperturbed state. The characteristic lines for the nozzle

wall rotated by  $\Omega = 3.5^\circ$  is shown in Fig. 2 (a) and  $\Omega = -2.5^\circ$  in Fig. 2 (b). The dark thick lines in the nozzle show expansion characteristics when the characteristic lines move away from each other. The light thin lines in the nozzle show compression characteristics when the lines move towards each other. The vertical dark thick line at the nozzle exit near the symmetry line shows the region of interest and the model placed behind the region of interest.

In the design condition, the waves from the expansion of flow downstream of the throat get cancelled, but when the walls are positively rotated, there is more expansion downstream of the throat than in the design condition. This causes an increase in Mach number of the flow near the symmetry line. As can be seen in Fig. 2 (a), the expansion waves from downstream of the throat accelerate the flow. The region of interest still lies in the region of expansion and is not affected by compression waves. When the nozzle is rotated inwards (negative rotation), as seen in Fig. 2 (b), there are lesser expansion waves reflected than in the case of design condition and stronger compression waves are reflected from the wall which cause a decrease in overall Mach number of the flow.

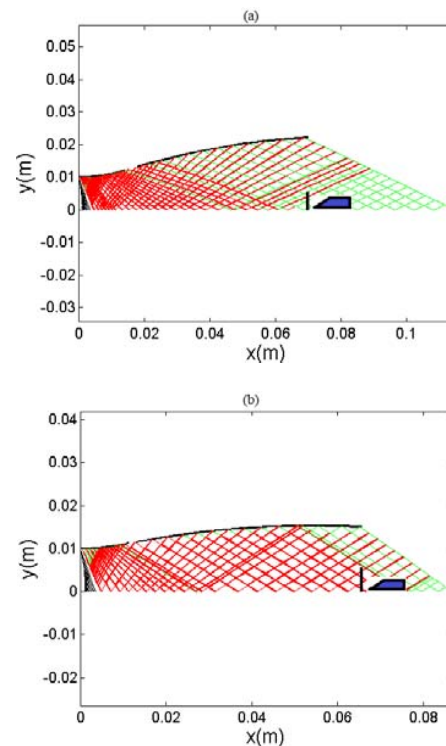


Fig. 2 Rotation of Mach 2 nozzle wall keeping the throat constant, and the wall is rotated by an angle (a)  $\Omega = 3.5^\circ$  and (b)  $\Omega = -2.5^\circ$ .

Similarly, Fig. 3 (a) shows the characteristic lines for a negatively translated nozzle wall,  $\Delta y = -0.25\text{cm}$ . Negative translation is moving the walls towards each other, thus decreasing the exit area and increasing the nozzle area ratio. As the walls move closer to each other, the expansion characteristics reflect multiple times inside the nozzle which

expands the flow multiple times and hence causes an increase in the exit Mach number of the nozzle. The vertical dark line near symmetry shows the region of interest.

When the nozzle walls are moved away from each other, i.e. a positive translation, as can be seen in Fig. 3 (b) for  $\Delta y = 0.25\text{cm}$ , the region of interest experiences high compression from the reflected compression waves from the wall causing a decrease in exit Mach number of the nozzle.

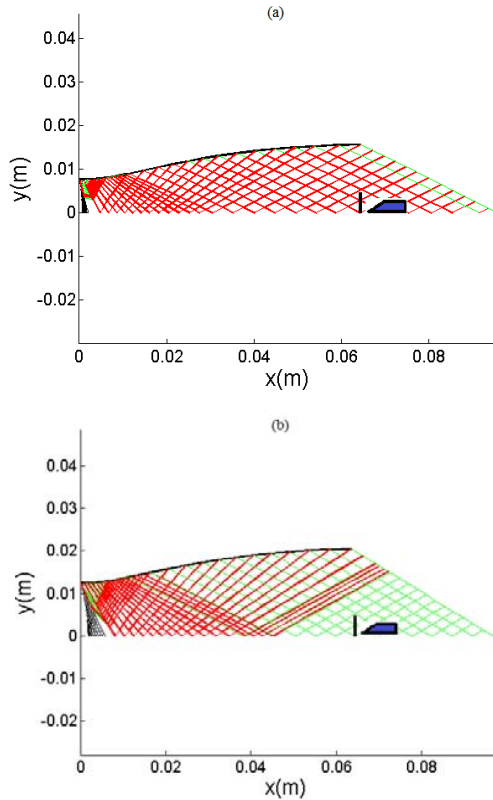


Fig. 3 Translation of Mach 2 nozzle wall for (a)  $\Delta y = -0.25\text{cm}$  and (b)  $0.25\text{ cm}$

The flow in Mach 2 nozzle is computed from MOC and the Mach number profile at the nozzle exit is displayed in Fig. 4. The nozzle wall is rotated from a minimum rotation angle of  $-2.5$  degrees to a positive rotation angle of  $3.5$  degrees, For the minimum rotation angle, the nozzle exit height is taken to be  $y_{\max}$ . The exit height of the nozzle is thus non-dimensionalized with respect to  $y_{\max}$  and a variable  $y_r$  is introduced such that that value of  $y_r$  is 0 at the nozzle symmetry line and  $y_r = 1$  at  $y = y_{\max}$ . Fig. 4 shows the Mach number at nozzle exit in the range  $y_r = 0$  to  $0.4$ .

As seen in Fig. 4, the region of interest has an almost uniform flow for the all values of rotation and translation. But, due to the change in nozzle orientation the flow is not uniform throughout the nozzle exit plane and for high rotation angles or translation distances.

Due to rotation of nozzle walls, the waves do not get cancelled as the inflection point does not remain same and hence compression or expansion waves reach the nozzle exit

to distort the flow. For this reason, the region of 40% of the exit height is selected and any orientation the nozzle is considered acceptable if the flow is uniform in the 40% region. The limit for uniformity of the flow is set such that the maximum flow variation  $\Delta M$  in the region of interest must be less than  $0.02$  for any orientation of the nozzle. Maximum flow variation is defined to be the difference between the maximum and the minimum Mach number in the region of interest.

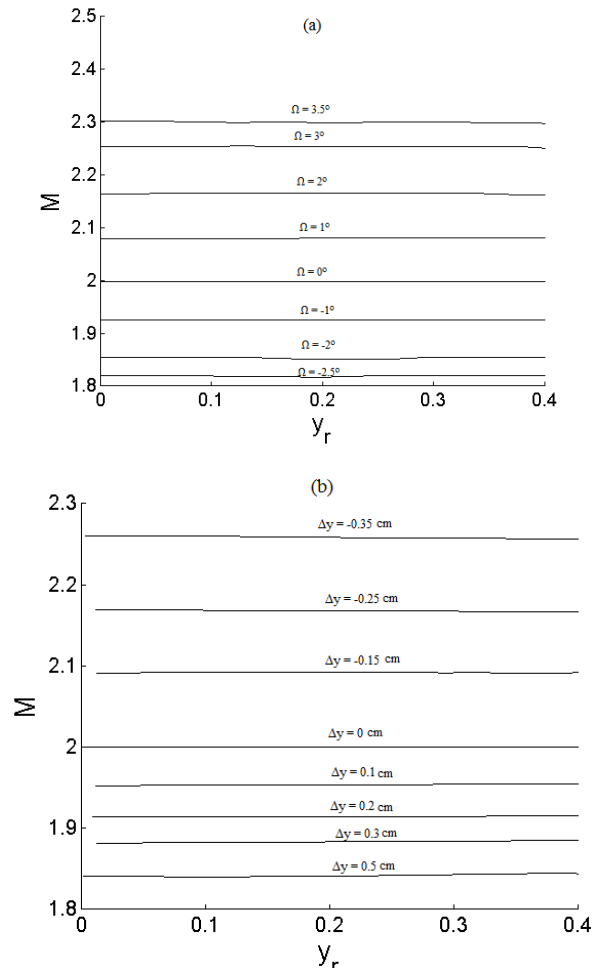


Fig. 4 Mach number at nozzle exit for nozzle design of  $M_D = 2$  for (a) Rotation and (b) Translation, for  $0 \leq y_r \leq 0.4$

The model that is to be tested must be of a size smaller than  $y_r = 0.4$  or smaller than  $0.4 * y_{\max}$ . The acceptable ranges of both wall rotation and wall translation are taken such that maximum variation in Mach number does not exceed  $0.02$  and no oblique shocks observed inside the nozzle. Therefore,  $y_{\max}$  is chosen when the nozzle has maximum negative rotation or maximum negative translation and  $y_r$  is non-dimensionalized using  $y_{\max}$ . The value of  $y_{\max}$  changes for different nozzles with different design Mach numbers as each of them have a different minimum rotation angle and translation distance. However, the minimum  $y_{\max}$  of the smallest nozzle is chosen

to determine range of the nozzle. The results in Section IV display the range of variation in Mach number that can be achieved using the nozzles by the method of rotation and translation.

#### IV. NOZZLE RANGE

To illustrate the effect of using different nozzles to test the same model, a schematic is shown in Fig. 5. Fig. 5 (a) shows a Mach 2 nozzle testing a model smaller than the size of its region of interest. If the same model is now to be tested in a Mach 4 nozzle, due to relatively larger size of the Mach 4 nozzle, the region of interest is quite small as seen in Fig. 5 (b). This requires the flow to be uniform in a very small region and therefore increases the range of high Mach number nozzles by a large amount.

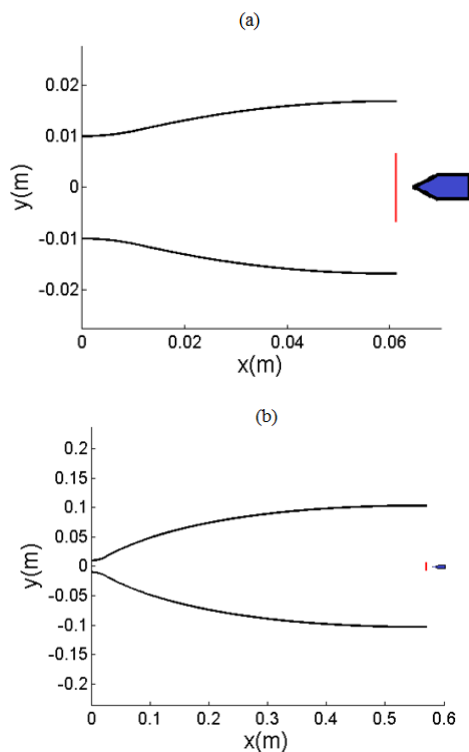


Fig. 5 Schematic of nozzles testing the same size model (a)  $M_D = 2$  and (b)  $M_D = 4$

Several different nozzle designs are tested and the nozzle designs are selected to vary the Mach number from Mach 2 to Mach 5. The nozzle ranges are illustrated in Fig. 6. Fig. 6 (a) shows the Mach number plot against the nozzle rotation angle plotted for design Mach numbers of 2.25, 2.5, 3, 3.5 and 4. The overall range of all nozzles put together is from Mach 2 to Mach 5.

As can be seen from Fig. 6 (a), the nozzle ranges increase if the model size is kept same in all the nozzles. Similarly, Fig. 6 (b) shows the plot of three different nozzles of Mach numbers 2, 3 and 3.8 obtained from method of translation and the Mach number ranges are much higher than in the case of rotation.

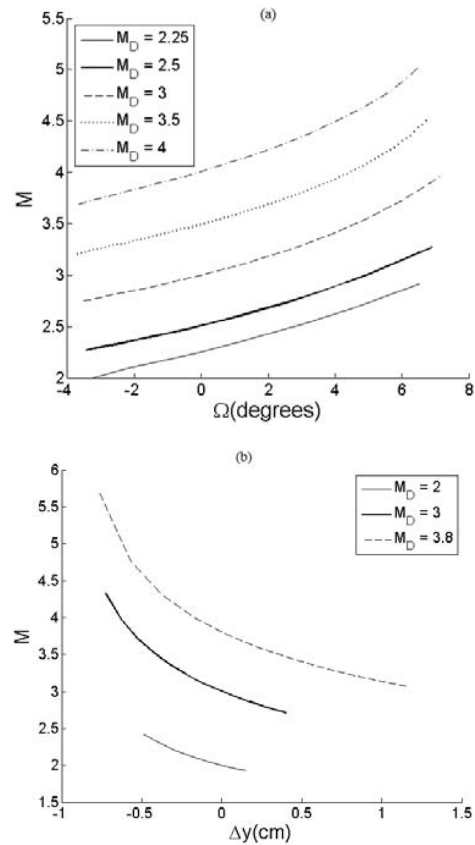


Fig. 6 Mach number range at nozzle exit for various design Mach numbers  $M_D$ , as a function of (a) rotation angle, and (b) translation distance, for the same model size used in all nozzles

The selection of nozzles can be optimized such that minimum number of nozzles are used to achieve the desired range from Mach 2 to Mach 5. As can be seen from the Fig. 7 (a), 3 different nozzle designs with design Mach numbers of 2.25, 3.1 and 4 can be used to vary the Mach number in the wind tunnel from Mach 2 to Mach 5. The horizontal dotted lines in Fig. 6 shows the cut-off Mach number for a particular nozzle. The nozzle range limits are set such that, at any orientation of the nozzle within the limits, the maximum flow variation  $\Delta M$  does not exceed 0.02 in the region of interest and no oblique shocks are observed in the nozzle. The same criteria have been used for the case of translation as well.

As can be seen in Fig. 7 (b), for the case of translation, just 2 nozzle designs are required to vary the Mach number from Mach 2 to Mach 5.7. This clearly indicates that a nozzle can produce higher Mach number range in the case of nozzle wall translation than in the case of rotation. This indicates that translation might be a better method to vary the Mach number of the flow than the method of rotation of the nozzle walls.

The flow angle,  $\theta$ , is defined as the angle that the flow velocity vector makes with the horizontal at a particular point in the flow. The flow angle is another important parameter for this analysis as the compression and expansion waves do not cancel and hence turn the flow at the nozzle exit. Fig. 8 displays the maximum flow angle in the region of interest for

a particular orientation of the nozzle wall, for various design Mach numbers that are shown in Fig. 6. Fig. 8 (a) shows the maximum flow angles for nozzles with design Mach numbers of 2.25, 3.1 and 4. It can be observed that the maximum flow angle decreases with increase in design Mach number. This is due to the smaller region of interest which is considered for larger nozzles to test the same model as shown previously in this section. Also, high Mach number nozzles need smaller rotation to produce large variation in Mach numbers due to their longer lengths. In the case of translation, in Fig. 8 (b), it can be seen that for design Mach number of 3.8, the flow angles are lower throughout. This can be due to the longer length of the nozzle which enable the waves to reflect multiple times inside the nozzle and then cancel out. Overall, the maximum flow angle observed in the figures in 0.6 degrees which is well within the acceptable limit, so the methods of translation and rotation are considered acceptable to vary the Mach number of the flow.

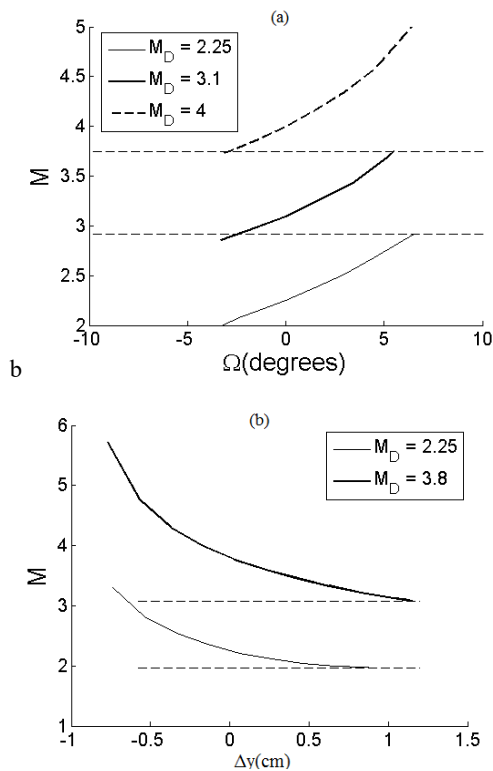


Fig. 7 Mach number range at nozzle exit for various design Mach numbers  $M_D$ , as a function of (a) rotation angle, and (b) translation distance to achieve optimized Mach number range from Mach 2 to Mach 5

Since, MOC is an inviscid technique, the nozzle walls are further corrected for boundary layer using approximate turbulent boundary layer correction technique for supersonic wind tunnels [5], to account for viscosity if the nozzles are to be used in wind tunnels.

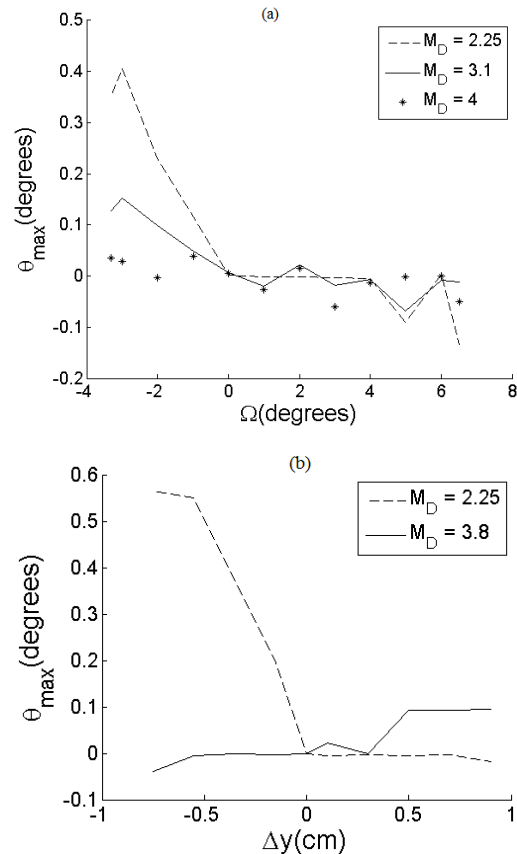


Fig. 8 Maximum flow angle in the region  $0 \leq y_r \leq 0.4$ , for various design Mach numbers as a function of (a) rotation angle, and (b) translation distance

## V. CONCLUSION

The experimental simulation of unsteady flows at high speeds requires a wind tunnel facility which can produce supersonic variable Mach number at the nozzle exit. The nozzle needs to operate at high stagnation pressure and high stagnation temperature to simulate a similar condition as experienced by any high-speed vehicle which needs the nozzle walls to be rigid for the cooling system to be attached on the nozzle walls. Two separate techniques are proposed in this paper which explores the feasibility of variable Mach number operation in an enclosed free jet or a free jet facility.

The methods of rotation and translations have been successfully shown to produce variable Mach numbers with the maximum variation of 0.02 in Mach number of the flow. It has been shown that with the help of 3 nozzles of design Mach numbers 2.25, 3.1 and 4 in the case of rotation and 2 nozzles of design Mach numbers of 2.25 and 3.8 in the case of translation, the complete range of flows from Mach 2 to Mach 5 can be studied. The method of translation can thus be selected to be the better of the two methods in terms of range of Mach numbers that can be achieved from a single nozzle design. The method of translation is also a better method in terms of low flow angles. This study attempts to explore the feasibility of variable Mach number nozzles and generate a

working wind tunnel facility for testing of high speed accelerating bodies. Further studies are underway to arrive at better methods to produce variable Mach number nozzles.

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