

# The Light-Effect in Cylindrical Quantum Wire with an Infinite Potential for the Case of Electrons: Optical Phonon Scattering

Hoang Van Ngoc, Nguyen Vu Nhan, Nguyen Quang Bau

**Abstract**—The light-effect in cylindrical quantum wire with an infinite potential for the case of electrons, optical phonon scattering, is studied based on the quantum kinetic equation. The density of the direct current in a cylindrical quantum wire by a linearly polarized electromagnetic wave, a DC electric field, and an intense laser field is calculated. Analytic expressions for the density of the direct current are studied as a function of the frequency of the laser radiation field, the frequency of the linearly polarized electromagnetic wave, the temperature of system, and the size of quantum wire. The density of the direct current in cylindrical quantum wire with an infinite potential for the case of electrons – optical phonon scattering is nonlinearly dependent on the frequency of the linearly polarized electromagnetic wave. The analytic expressions are numerically evaluated and plotted for a specific quantum wire, GaAs/GaAsAl.

**Keywords**—The light-effect, cylindrical quantum wire with an infinite potential, the density of the direct current, electrons - optical phonon scattering.

## I. INTRODUCTION

THE light-effect of charge carriers by electromagnetic waves is explained by the possibility of using this phenomenon for detecting intense electromagnetic radiation [1], [2], as well as for characterizing kinetic properties of semiconductors [3], [4]. In semiconductor systems, the presence of intense laser radiation can influence the electrical conductivity and kinetic effects in material [5]-[8]. The photon drag effect has been researched in semiconductors [9], in superlattice [10], in quantum wire with a parabolic potential [11], but the photon drag effect in quantum wire with an infinite potential is still open to study. In this paper, we use the quantum kinetic equation to study the drag of charge carriers in cylindrical quantum wire with an infinite potential for case of electrons – optical phonon. Electrons system is placed in a direct electric field, an electromagnetic wave, and the presence of an intense laser field. The constant current density of the light-effect with an infinite potential for the case of electrons – optical phonon is calculated. The difference between the light-effect with parabolic potential for the case of electrons – acoustic phonon scattering and the light-effect with infinite potential for the case of electrons – optical phonon scattering

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is in potential barrier, form factor, wave function, energy and electrons - phonon scattering. These will be indicated in this paper.

## II. CALCULATING THE CONSTANT CURRENT DESTINY OF THE LIGHT-EFFECT IN CYLINDRICAL QUANTUM WIRE WITH INFINITE POTENTIAL FOR THE CASE OF ELECTRONS – OPTICAL PHONON SCATTERING

The Hamiltonian of the electron - phonon system with the presence of the laser radiation field in the quantum wire can be written as [5]-[12] (using with  $\hbar=1$  unit and we suppose the axis 0z along the length of the wire):

$$H = H_0 + U = \sum_{n,l,\vec{p}_z} \epsilon_{n,l,\vec{p}_z} (\vec{p}_z - \frac{e}{\hbar c} \vec{A}(t)) a_{n,l,\vec{p}_z}^+ a_{n,l,\vec{p}_z} + \sum_{\vec{q}} \omega_{\vec{q}} b_{\vec{q}}^+ b_{\vec{q}} + \sum_{n,l,n',l',\vec{p}_z,\vec{q}} C_q \cdot I_{n,l,n',l'}(\vec{q}) a_{n',l',\vec{p}_z+\vec{q}}^+ a_{n,l,\vec{p}_z} (b_{\vec{q}} + b_{-\vec{q}}^+) \quad (1)$$

where  $\vec{A}(t)$  is the vector potential of laser field (only the laser field affects the probability of scattering):  $-\frac{1}{c} \vec{A}(t) = \vec{F}_0 \sin \Omega t$ ;

$a_{n,l,\vec{p}_z}^+$  and ( $b_{\vec{q}}^+$  and  $b_{\vec{q}}^-$ ) are the creation and annihilation operators of electron (phonon);  $\vec{p}_z$  is the electron wave momentum along axis 0z;  $\vec{q}$  is the phonon wave vector;  $\omega_{\vec{q}}$  is the frequency of optical phonon;  $C_q$  is the electrons – optical

phonon interaction constant:  $|C_q|^2 = \frac{2\pi e^2 \hbar \omega_q}{\epsilon q_z^2} \left( \frac{1}{\epsilon_\infty} - \frac{1}{\epsilon_0} \right)$  (this

is a difference with electrons – acoustic scattering);  $\epsilon$ ,  $\epsilon_\infty$ , and  $\epsilon_0$  are dielectric constant, high – frequency dielectric constant and low – frequency dielectric constant; (n, l) and (n', l') are the quantum numbers of electron;  $I_{n,l,n',l'}(q_z)$  is the form factor for electrons in quantum wire with infinite potential and it is different from the form factor for electrons in quantum wire with parabolic potential.

The electron energy takes the simple form:

$$\epsilon_{n,l,\vec{p}_z} = \frac{p_z^2}{2m} + \frac{B_{n,l}^2}{2mR^2} \quad (\text{this is a difference with parabolic potential})$$

( $n = 0, \pm 1, \pm 2, \dots$ ,  $l = 1, 2, 3, \dots$ ) where R is the radius of wire, and  $B_{n,l}$  is the solution of the Bessel function of real argument  $J_n(B_{n,l}) = 0$ .

We use general quantum equations for electron distribution

function:

$$i\hbar \frac{\partial f_{n,l,p_z}(t)}{\partial t} = \langle [a_{n,l,p_z}^+ a_{n,l,p_z}, H] \rangle_t \quad (2)$$

where  $f_{n,l,p_z}(t) = \langle a_{n,l,p_z}^+ a_{n,l,p_z} \rangle_t$  is the distribution function.

From (1) and (2), we obtain the quantum kinetic equation for electrons in quantum wire with the case of electrons – optical phonon scattering (after supplement: a linearly polarized electromagnetic wave field and a direct electric field  $\vec{E}_0$ ):

$$\begin{aligned} \frac{\partial f_{n,l,p_z}(t)}{\partial t} + \left( e\vec{E}(t) + e\vec{E}_0 + \omega_c [\vec{p}_z, \vec{h}(t)] \right) \frac{\partial f_{n,l,p_z}(t)}{\partial \vec{p}_z} = \\ = 2\pi \sum_{n',l',\vec{q}} |D_{n,l,n',l'}(\vec{q})|^2 \cdot \sum_{L=-\infty}^{\infty} J_L^2\left(\frac{e\vec{F}_0 \vec{q}}{m\Omega^2}\right) N_q \times \\ \times \left\{ \left[ f_{n',l',\vec{p}_z+\vec{q}}(t) - f_{n,l,\vec{p}_z}(t) \right] \delta(\varepsilon_{n',l',\vec{p}_z+\vec{q}_z} - \varepsilon_{n,l,\vec{p}_z} - \omega_q - L\Omega) + \right. \\ \left. + \left[ f_{n',l',\vec{p}_z-\vec{q}_z}(t) - f_{n,l,\vec{p}_z}(t) \right] \delta(\varepsilon_{n',l',\vec{p}_z-\vec{q}_z} - \varepsilon_{n,l,\vec{p}_z} + \omega_q - L\Omega) \right\} \end{aligned} \quad (3)$$

where  $\vec{h} = \frac{\vec{H}}{H}$  is the unit vector of the magnetic field direction,

$J_L\left(\frac{e\vec{F}_0 \vec{q}}{m\Omega^2}\right)$  is the Bessel function of real argument;  $N_q$  is the time-independent component of distribution function of optical phonon:  $N_q = \frac{k_B T}{\omega_q}$  (this is a difference with electrons – acoustic scattering).

The constant current density is in the form [9] of

$$\vec{j}_0 = \vec{R}_0(\varepsilon) d\varepsilon \quad (4)$$

with  $\vec{R}_0(\varepsilon) = -\frac{e}{m} \sum_{n,l,\vec{p}_z} \vec{p}_z f_{l0}(\vec{p}_z) \delta(\varepsilon - \varepsilon_{n,l,p_z})$  which is the partial current density.

For simplicity, we limit the problem to the case of  $L = 0, \pm 1$  and we multiply both sides of (3) with  $(-e/m) \vec{p}_z \delta(\varepsilon - \varepsilon_{n,l,p_z})$ . After a few mathematical transformations, we obtained:

$$(-i\omega + \frac{1}{\tau(\varepsilon)}) \vec{R}(\varepsilon) = \vec{Q}(\varepsilon) + \vec{S}(\varepsilon) + \omega_c [\vec{R}_0(\varepsilon), \vec{h}] \quad (5)$$

$$(-i\omega + \frac{1}{\tau(\varepsilon)}) \vec{R}^*(\varepsilon) = \vec{Q}(\varepsilon) + \vec{S}^*(\varepsilon) + \omega_c [\vec{R}_0(\varepsilon), \vec{h}] \quad (6)$$

$$\frac{\vec{R}_0(\varepsilon)}{\tau(\varepsilon)} = \vec{Q}_0(\varepsilon) + \vec{S}_0(\varepsilon) + \omega_c [\vec{R}(\varepsilon) + \vec{R}^*(\varepsilon), \vec{h}] \quad (7)$$

where  $\omega_c$  is the cyclotron frequency,  $\tau(\varepsilon)$  is the momentum relaxation time in absence of laser radiation [9], and

$$\vec{R}(\varepsilon) = -\frac{e}{m} \sum_{n,l,\vec{p}_z} \vec{p}_z f_l(\vec{p}_z) \delta(\varepsilon - \varepsilon_{n,l,p_z}) \quad (8)$$

$$\vec{Q}(\varepsilon) = -\frac{e^2 \vec{E}}{m^2 k_B T} \sum_{n,l,p_z} p_z^2 f_0(\varepsilon_{n,l,p_z}) \delta(\varepsilon - \varepsilon_{n,l,p_z}) \quad (9)$$

$$\vec{Q}_0(\varepsilon) = -\frac{e^2 \vec{E}_0}{m^2 k_B T} \sum_{n,l,p_z} p_z^2 f_0(\varepsilon_{n,l,p_z}) \delta(\varepsilon - \varepsilon_{n,l,p_z}) \quad (10)$$

$$\begin{aligned} \vec{S}_0(\varepsilon) = \frac{2\pi e}{m} \sum_{n,l,n',l',p_z,q_z} C_q^2 |I_{n,l,n',l'}(q)|^2 N_q \vec{q}_z f_{l0}(\vec{p}_z) \frac{e^2 F^2 q_z^2}{4m^2 \Omega^4} \times \\ \times \left\{ \left[ \delta(\varepsilon_{n',l',\vec{p}_z+\vec{q}_z} - \varepsilon_{n,l,\vec{p}_z} - \omega_q + \Omega) + \delta(\varepsilon_{n',l',\vec{p}_z+\vec{q}_z} - \varepsilon_{n,l,\vec{p}_z} - \omega_q - \Omega) \right] - \right. \\ \left. - \left[ \delta(\varepsilon_{n',l',\vec{p}_z-\vec{q}_z} - \varepsilon_{n,l,\vec{p}_z} + \omega_q + \Omega) + \delta(\varepsilon_{n',l',\vec{p}_z-\vec{q}_z} - \varepsilon_{n,l,\vec{p}_z} + \omega_q - \Omega) \right] \right\} \times \\ \times \delta(\varepsilon - \varepsilon_{n,l,\vec{p}_z}) \end{aligned} \quad (11)$$

where  $f_l(\vec{p}_z) = -\vec{p}_z \vec{\chi}_0 \frac{\partial f_0}{\partial \varepsilon_{n,l,p_z}}$ ;  $f_0(\varepsilon_{n,l,p_z}) = n_0 \exp(-\frac{\varepsilon_{n,l,p_z}}{k_B T})$ ;  $n_0$  is the particle density;  $k_B$  is the Boltzmann constant;  $T$  is the temperature of system;  $\vec{\chi}_0 = \frac{e}{m} \vec{E}_0 \tau(\varepsilon_{n,l,p_z})$ .

$$\begin{aligned} \vec{S}(\varepsilon) = \frac{2\pi e}{m} \sum_{n,l,n',l',p_z,q_z} C_q^2 |I_{n,l,n',l'}(q)|^2 N_q \vec{q}_z f_l(\vec{p}_z) \frac{e^2 F^2 q_z^2}{4m^2 \Omega^4} \times \\ \times \left\{ \left[ \delta(\varepsilon_{n',l',\vec{p}_z+\vec{q}_z} - \varepsilon_{n,l,\vec{p}_z} - \omega_q + \Omega) + \delta(\varepsilon_{n',l',\vec{p}_z+\vec{q}_z} - \varepsilon_{n,l,\vec{p}_z} - \omega_q - \Omega) \right] - \right. \\ \left. - \left[ \delta(\varepsilon_{n',l',\vec{p}_z-\vec{q}_z} - \varepsilon_{n,l,\vec{p}_z} + \omega_q + \Omega) + \delta(\varepsilon_{n',l',\vec{p}_z-\vec{q}_z} - \varepsilon_{n,l,\vec{p}_z} + \omega_q - \Omega) \right] \right\} \times \\ \times \delta(\varepsilon - \varepsilon_{n,l,\vec{p}_z}) \end{aligned} \quad (12)$$

with

$$f_{l0}(\vec{p}_z) = -\vec{p}_z \vec{\chi} \frac{\partial f_0}{\partial \varepsilon_{n,l,p_z}}; \quad \vec{\chi} = \frac{e}{m} \vec{E} \frac{\tau(\varepsilon_{n,l,p_z})}{1 - i\omega\tau(\varepsilon_{n,l,p_z})}$$

Solving (4)-(6), we obtain:

$$\vec{R}_0(\varepsilon) = \tau(\varepsilon) (\vec{Q}_0 + \vec{S}_0) + \frac{2\omega_c \tau^2(\varepsilon)}{1 + \omega^2 \tau^2(\varepsilon)} [\vec{Q}, \vec{h}] + 2\omega_c \tau^2(\varepsilon) \text{Re} \left\{ \frac{[\vec{S}, \vec{h}]}{1 - i\omega\tau(\varepsilon)} \right\} \quad (13)$$

The density of direct current:

$$\vec{j}_0 = \int_0^\infty \vec{R}_0(\varepsilon) d\varepsilon = [AC + D] \vec{E}_0 + \frac{2\omega_c \tau(\varepsilon_F)}{1 + \omega^2 \tau^2(\varepsilon_F)} \left[ \frac{1 - \omega^2 \tau^2(\varepsilon_F)}{1 + \omega^2 \tau^2(\varepsilon_F)} AC + D \right] [\vec{E}, \vec{h}] \quad (14)$$

with

$$A = \frac{n_0 e^6 \hbar F^2 \tau(\epsilon_F)}{8m^4 \epsilon \Omega^4} \left( \frac{1}{\epsilon_\infty} - \frac{1}{\epsilon_0} \right) \sum_{n,l,n',l'} I^2 \exp \left\{ -\beta \frac{B_{n,l}^2}{2mR^2} \right\} \quad (15)$$

$$I = \int_{-\infty}^{+\infty} |I_{n,l,n',l'}(q_z)|^2 dq_z \quad (16)$$

$$D = \frac{n_0^2 e^2 \tau^2(\epsilon_F)}{4\pi m^2 k_B T} \left( \frac{\beta}{2m} \right)^{-2} \sum_{n,l} \left\{ -\beta \frac{B_{n,l}^2}{2mR^2} \right\} \quad (17)$$

$$C = \frac{1}{2} (N_1 + N_2 - N_3 - N_4) \left( \frac{\beta}{2m} \right)^{-2} - N_1^{5/2} \left[ \psi_{(2,5/2;\beta \frac{\hbar^2 N_1}{2m})} + 2\psi_{(3,7/2;\beta \frac{\hbar^2 N_1}{2m})} \right] -$$

$$- N_2^{5/2} \left[ \psi_{(2,5/2;\beta \frac{\hbar^2 N_2}{2m})} + 2\psi_{(3,7/2;\beta \frac{\hbar^2 N_2}{2m})} \right] - N_3^{5/2} \left[ \psi_{(2,5/2;\beta \frac{\hbar^2 N_3}{2m})} + 2\psi_{(3,7/2;\beta \frac{\hbar^2 N_3}{2m})} \right] -$$

$$- N_4^{5/2} \left[ \psi_{(2,5/2;\beta \frac{\hbar^2 N_4}{2m})} + 2\psi_{(3,7/2;\beta \frac{\hbar^2 N_4}{2m})} \right] \quad (18)$$

$$N_1 = -\frac{1}{\hbar^2} (B_{n,l'}^2 - B_{n,i}^2) + 2m(\omega_q - \Omega) \quad (19)$$

$$N_2 = -\frac{1}{\hbar^2} (B_{n,l'}^2 - B_{n,i}^2) + 2m(\omega_q + \Omega) \quad (20)$$

$$N_3 = -\frac{1}{\hbar^2} (B_{n,l'}^2 - B_{n,i}^2) - 2m(\omega_q + \Omega) \quad (21)$$

$$N_4 = -\frac{1}{\hbar^2} (B_{n,l'}^2 - B_{n,i}^2) - 2m(\omega_q - \Omega) \quad (22)$$

$$\Psi_{(a,b,z)} = \frac{1}{\Gamma(a)} \int_0^\infty e^{-zx} x^{a-1} (1+ax)^{b-a-1} dx \text{ is the hypergeometric}$$

function.

Equation (14) shows the dependence of the direct current density on the parameters of the system such as: the frequency, wave function, energy spectrum, form factor  $I_{n,l,n',l'}$  and potential barrier (that is the difference between cylindrical quantum wire with an infinite potential), cylindrical quantum wire with a parabolic potential, superlattices and bulk semiconductor.

### III. NUMERICAL RESULTS AND DISCUSSION

In this section, we will survey, plot and discuss the expressions for  $j_{0z}$  for the case of a specific GaAs/GaAsAl quantum wire (we select:  $\vec{E} \uparrow \uparrow 0x$ ;  $\vec{h} \uparrow \uparrow 0y$ ):

$$j_{0z} = [AC + D] E_0 + \frac{2\omega_c \tau(\epsilon_F)}{1 + \omega^2 \tau^2(\epsilon_F)} \left[ \frac{1 - \omega^2 \tau^2(\epsilon_F)}{1 + \omega^2 \tau^2(\epsilon_F)} AC + D \right] E$$

The parameters used in the calculations are as follows [12]:

$m = 0.0665m_0$  ( $m_0$  is the mass of free electron);  $\epsilon_F = 50$  meV; and  $\tau(\epsilon_F) \sim 10^{-11} \text{ s}^{-1}$ ;  $n_0 = 10^{23} \text{ m}^{-3}$ ;  $\rho = 5.3 \times 10^3 \text{ kg/m}^3$ ;  $\omega_0 = 5 \times 10^{13} \text{ s}^{-1}$ ;  $E_{0z} = 0.5 \times 10^6 \text{ (V/m)}$ ;  $F = 1.2 \times 10^6 \text{ (N)}$ ,  $\omega_q = 2 \times 10^6 \text{ s}^{-1}$ ;  $q = 2 \times 10^6 \text{ (kg.m/s)}$ .

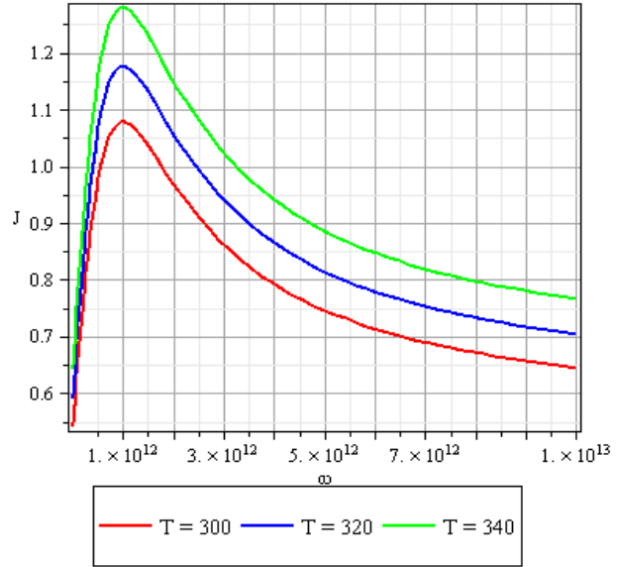


Fig. 1 The dependence of  $j_{0z}$  on the frequency of electromagnetic field with different values of  $T$

Fig. 1 shows the dependence of the constant current density on the frequency of electromagnetic wave. From this figure, we can see that the direct current density increases strongly with increasing the frequency of electromagnetic wave for the area of values  $10^{11} < \omega < 10^{12} \text{ (s}^{-1}\text{)}$ . The direct current density decreases when  $\omega > 10^{12}$  and reaches saturation as the frequency  $\omega$  continues to increase. That is the difference between bulk semiconductor [9], superlattices [10] and quantum wire with a parabolic potential [11].

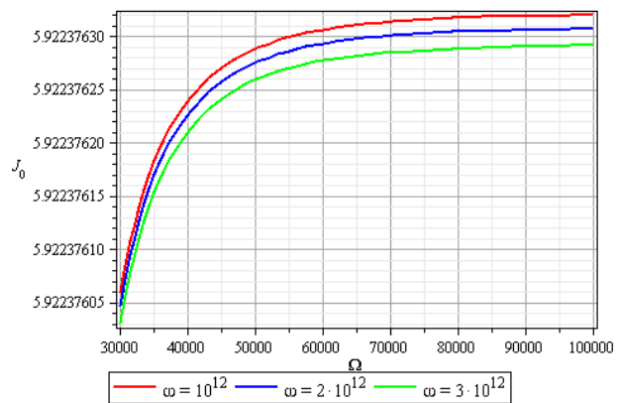


Fig. 2 The dependence of  $j_{0z}$  on the frequency of radiation laser field with different values of  $\omega$

Fig. 2 shows the dependence of the constant current density

on the frequency of laser radiation field. We can see that the value of the direct current density increases strongly with increasing the frequency of radiation laser field for the area of values  $3 \cdot 10^4 < \Omega < 5 \cdot 10^4$  and reaches saturation as the frequency  $\omega$  continues to increase. That is the difference between bulk semiconductor [9], superlattices [10] and quantum wire with a parabolic potential [11].

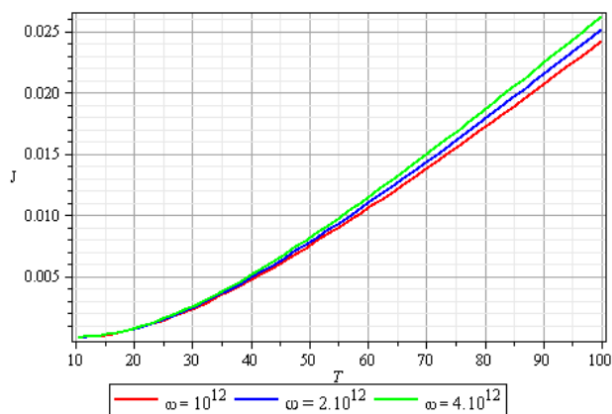


Fig. 3 The dependence of  $j_{0z}$  on the temperature with different values of  $\omega$

Fig. 3 shows that when temperature  $T$  of the system rises up, the constant current density along the  $Oz$  axis goes up too. When  $T < 40$  (K), then the constant current density depends nonlinearly on temperature, when  $T > 40$  (K) then the constant current density has almost linear dependence on temperature. That is the difference between bulk semiconductor [9], superlattices [10] and quantum wire with a parabolic potential [11].

#### IV. CONCLUSIONS

In this paper, we have studied the light-effect in cylindrical quantum wire with an infinite potential for the case of electrons – optical phonon. We obtain the expressions of  $\vec{j}_0$ ,  $j_{0z}$  and show the dependence of  $\vec{j}_0$  on the parameters of the system and on the basic elements of quantum wire with an infinite potential. The analytical results are numerically evaluated and plotted for a specific quantum wire GaAs/GaAsAl.

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