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# Parametric Study on Dynamic Analysis of Composite Laminated Plate

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***Abstract*****—**A laminated plate composite of graphite/epoxy has been analyzed dynamically in the present work by using a quadratic element (8-node diso-parametric), and by depending on 1st order shear deformation theory, every node in this element has 6-degrees of freedom (displacement in x, y, and z axis and twist about x, y, and z axis). The dynamic analysis in the present work covered parametric studies on a composite laminated plate (square plate) to determine its effect on the natural frequency of the plate. The parametric study is represented by set of changes (plate thickness, number of layers, support conditions, layer orientation), and the plates have been simulated by using ANSYS package 12. The boundary conditions considered in this study, at all four edges of the plate, are simply supported and fixed boundary condition. The results obtained from ANSYS program show that the natural frequency for both fixed and simply supported increases with increasing the number of layers, but this increase in the natural frequency for the first five modes will be neglected after 10 layers. And it is observed that the natural frequency of a composite laminated plate will change with the change of ply orientation, the natural frequency increases and it will be at maximum with angle 45 of ply for simply supported laminated plate, and maximum natural frequency will be with cross-ply (0/90) for fixed laminated composite plate. It is also observed that the natural frequency increase is approximately doubled when the thickness is doubled.

***Keywords*****—**Laminated plate, orthotropic plate, square plate, natural frequency, graphite/epoxy.

## I. Introduction

TECHNOLOGICAL progress results in the continuous expansion of structural material types and in an improvement of their properties. The most frequently used structural materials can be categorized in four primary groups: metals, polymers, composites, and ceramics. The needs of the aerospace industry and high raise building drive to the use and improvement of composite materials [1]. The progress in the industrialization process and technology of composites materials has improved the use of the composites from secondary structural components to such a primary component. Through the last decade, plates made by composite materials are being progressively used in many engineering purposes. The high stiffness/weight ratio coupled with the flexibility of the chosen of the lamination sketch, which can be sewed to match the design demand, makes the laminated plate an interesting structural component for many manufactures. The frequent use of laminated plates in different areas has encouraged the use of plates as a constructional element for the present research. The results of the analysis of natural frequency for the laminated composite plates in the structural designing are so remarkable to avoid the resonant action of the laminated structures; whether the use of composite materials is in civil, aerospace, and marine, they are applied to dynamic loads.

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## II. Review of Literature

The natural frequencies and mode shapes of a number of graphite/epoxy and graphite/epoxy-aluminum plates and shells were experimentally determined by Crawley. Natural frequency and mode shape results were compared with finite element method [2].

Kim and Gupta [3] investigated the effects of lamination and extension–bending coupling, shear and twist-curvature couplings on the lowest frequencies and corresponding mode shapes for free vibration of laminated anisotropic composite plates using a finite element method with quadratic interpolation functions and five degrees of freedom (DOF).

Qatu and Leissa [4] analyzed free vibrations of thin cantilevered laminated plates and shallow shells by Ritz method. Convergence studies are made for spherical circular cylindrical, hyperbolic, paraboloidal shallow shells and for plates. Results are compared with experimental value and FEM. The effect of various parameters (material number of layers, fiber orientation, curvature) upon the frequencies is studied.

Koo and Lee [5] used a finite element method based on the shear deformable plate theory to investigate the effects of transverse shear deformation on the modal loss factors as well as the natural frequencies of composite laminated plates. The complex modules of an orthotropic lamina were employed to model damping effect.

Rikards [6] presented a sandwich composite beam and plate finite super elements with viscoelastic layers for vibration and damping analysis of laminated composite beams or plates. Each layer was considered as simply Timoshenko's beam or Mindlin-Reissner plate finite element.

Soares et al. [7] described an indirect identification technique to predict the mechanical properties of composites, which makes use of eigenfrequencies, experimental analysis of a composite plate specimen, corresponding numerical eigenvalue analysis, and optimization techniques.

## III. Material & Methodology Composite Materials

A composite material can be defined as a combination of a matrix and reinforcement, which yields properties superior to the properties of the individual components [8]. In the case of a composite, the reinforcement is the fibers and it is used to fortify the matrix in terms of strength and stiffness [9].

A reinforcement material is called fiber, and a base material is called matrix to achieve better engineering properties than the conventional material. Composite materials are ideal for structures that require high strength to weight and stiffness to weight ratios. The positive characteristic of using composite material is the control capability of fiber alignment by changing the fiber orientation and layers according to the required properties (strength and stiffness) [10].

For the present study, a square orthotropic plate consists of graphite/epoxy material, and accumulate series considered for the laminates is [ϴ/ϴ], with change of ϴ. The symmetry angle-ply composite plate is simulated for various stacking series for two boundary conditions, simply supported and fixed. Detail of the plate: Length = 1 m, width = 1 m, thickness = 0.01 m, stacking sequence angle (0,15,30,45,60, 75,90), Layer number = (2,4,6,8,10,12,14,16) ply, material properties are: E1 = 175x$10^9$ N/$m^2$, E2 = 7x$10^9$ N/$m^2$, E3 = 7 x$10^9$ N/$m^2$, V12 = V13 = 0.25, V23 = 0.01, G12 = G13 = 3.5x$10^9$ N/$m^2$, G23 = 1.4x$10^9$ N/$m^2$, Unit weight= 1550 kg/$m^3$.

### *A. Free Vibration*

Free vibration means the motion of a structure without any external forces (dynamic force) or support motion. The linear SDF systems motion without damping can be specialized to

$$m\frac{d^2u}{dt^2} + ku = 0 \tag{1}$$

Free vibration is initiated by disturbing the system from its static equilibrium position by imparting the mass some displacement u (0) and velocity ů (0) at time zero, and the motion is initiated

u = u(0), ů = ů(0)

So, solution to the equation is obtained by standard methods:

$$u(t) = u(0)cos\omega_n t + \frac{\dot{u}(0)}{\omega_n}\sin\omega_n t \tag{2}$$

where natural circular frequency of vibration in unit radians per second is

$$\omega_n = \sqrt{\frac{k}{m}} \tag{3}$$

The time required for the undamped system to complete one cycle of free vibration is the natural period of vibration of the system. Natural cyclic frequency of

$$T_n = \frac{2\pi}{\omega_n} \tag{4}$$

vibration is denoted by Fn= 1/ Tn, unit in Hz (cycles per second).

### *B. Mode Shape*

We introduce the eigenvalue problem whose solution gives the natural frequencies and modes of a system. The free vibration undamped system in one of its natural vibration modes can be described by

$$u(t) = q_n(t)\emptyset_n \tag{5}$$

where $\emptyset_n$ does not vary with time.

The time variation of the displacements is described by the simply harmonic function

$$q_n(t) = A_n\cos\omega_n t + B_n\sin\omega_n t \tag{6}$$

An, Bn are constants of integration. Combining the two equations, there will be

$$u(t) = \emptyset_n(A_n\cos\omega_n t + B_n\sin\omega_n t) \tag{7}$$

Putting in equation of undamped free vibration, we have

$$[-\omega_n{}^2 m\emptyset_n + k\emptyset_n]q_n(t) = 0 \tag{8}$$

Either, $q_n(t) = 0$, →u(t) = o, trivial solution. Or, $k\emptyset_n = \omega_n{}^2\, m\emptyset_n$. This is called matrix eigenvalue problem. This equation can be written as:

$$\lfloor k - \omega_n{}^2\, m\,\rfloor\emptyset_n = 0 \tag{9}$$

A set of “n” homogeneous algebraic equations is obtained. This set has always the trivial solution $\emptyset_n$=0, which implies no motion. The nontrivial solution is:

det$\lfloor k - \omega_n{}^2\, m\rfloor = 0$, this is called frequency equation.

### *C. Ansys Modeling*

By using ANSYS 12.0 package, the dynamic analysis has been done. For layered applications of a structural shell model, the element SHELL99 can be used. So, the element SHELL99 linear layer is used to model the laminated composite plates. SHELL99 goes ahead up to 250 layers. This element has 6-DOF in every node: displacement in the x, y, and z directions and twist about the nodal x, y, and z axes.

## IV. Results and Discussion

The natural frequency of laminated composite plates is investigated for various numbers of layers and stacking sequences (with the same thickness) in the laminated composite plates. Couples of cases for boundary conditions were analyzed (simply supported and fixed). The cases investigated during the present study are defined below.

Case 1: A composite laminated plate consisting of graphite/epoxy material for two boundary conditions (simply supported and fixed) has been investigated for the current study. In this problem, the natural frequency has been studied for various numbers of layers for the composite plate for the same thickness.

TABLE I
NATURAL FREQUENCY OF A FIXED SYMMETRY ANGLE-PLY [45 / 45-]S SQUARE COMPOSITE LAMINATED PLATE

| | Natural Frequency | | | | |
|---|---|---|---|---|---|
| No. of layer | mode 1 | mode 2 | mode 3 | mode 4 | mode5 |
| 2 | 67.05 | 134.58 | 134.58 | 211.51 | 232.25 |
| 4 | 99.65 | 176.11 | 219.21 | 268.91 | 334.75 |
| 6 | 104.95 | 195.73 | 222.04 | 308.11 | 355.21 |
| 8 | 106.11 | 201.25 | 221.3 | 319.33 | 359.48 |
| 10 | 106.81 | 204.85 | 220.69 | 326.39 | 362.22 |
| 12 | 107.1 | 206.75 | 219.98 | 330.06 | 363.27 |
| 14 | 107.32 | 208.18 | 219.51 | 332.65 | 364.15 |
| 16 | 107.44 | 209.13 | 219.01 | 334.32 | 364.55 |

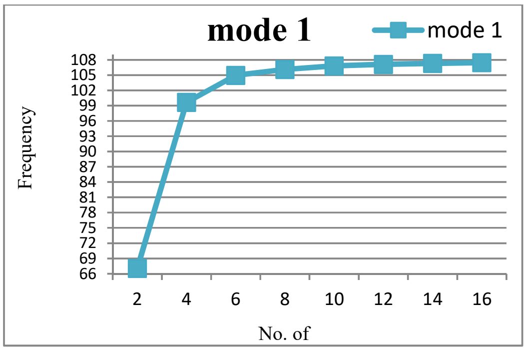


Fig. 1 Natural frequency (first mode shape) of fixed composite laminated plate with various numbers of layers

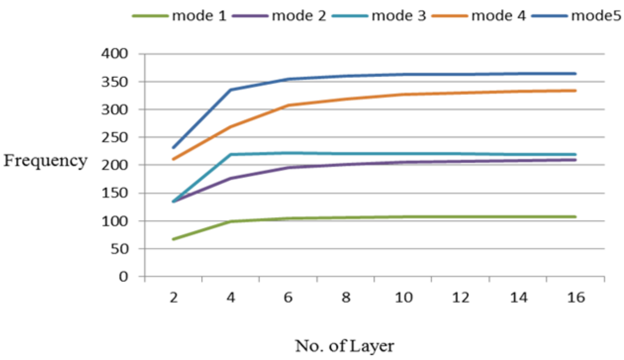


Fig. 2 Natural frequency (first five mode shape) of fixed composite laminated plate with various numbers of layers

TABLE II
NATURAL FREQUENCY OF A SYMMETRY ANGLE-PLY [-45 / 45] S SIMPLY SUPPORTED SQUARE LAMINATED COMPOSITE PLATE

| | Natural Frequency | | | | |
|---|---|---|---|---|---|
| No. of layer | mode 1 | mode 2 | mode 3 | mode 4 | mode5 |
| 2 | 51.25 | 105.79 | 105.79 | 179.26 | 181.48 |
| 4 | 59.83 | 129.88 | 157.82 | 199.19 | 256.5 |
| 6 | 64.73 | 137.35 | 160.64 | 234.99 | 275.01 |
| 8 | 65.87 | 142.41 | 160.25 | 245.6 | 279.2 |
| 10 | 66.5 | 145.61 | 159.68 | 252.33 | 281.61 |
| 12 | 66.79 | 147.35 | 159.13 | 255.96 | 282.67 |
| 14 | 66.99 | 148.62 | 158.68 | 258.5 | 283.43 |
| 16 | 67.1 | 149.48 | 158.29 | 260.23 | 283.84 |

From Table I, it is observed that the natural frequency increased clearly when the number of layers increased for the same thickness of the plate with the same case of boundary condition, which was fixed, but the amount of increase in natural frequency will be unremarkable for all mode shapes after augmenting layer numbers above 10 layers for the same thickness. So, it can be deduced from that the maximum natural frequency for this plate by using 10 layers for symmetric clamped composite laminated plate. The effects of number of layer in the laminate are clearly shown Figs. 1 and 2.

From Table II, it is observed that the natural frequency increased clearly when the number of layers increased for the same thickness of the plate with the same case of boundary condition, which was simply supported, but the amount of increase in natural frequency will be unremarkable for all mode shapes after increasing layer numbers above 10 layers for the same thickness. So, it can be deduced from the maximum natural frequency for this plate by using 10 layer for symmetric clamped composite laminated plate. The effects of number of layer in the laminate are clearly shown Figs. 3 and 4.

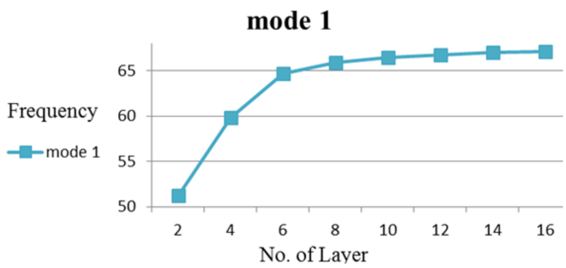


Fig. 3 Natural frequency (first five mode shape) of simply supported composite laminated plate with various numbers of layers

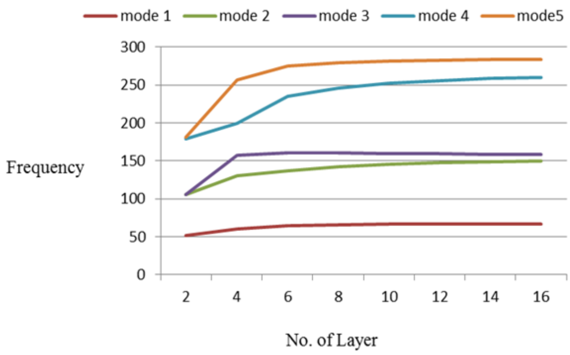


Fig. 4 Natural frequency (first five mode shape) of simply supported composite laminated plate with various numbers of layers

Case 2:Natural frequency of a fixed symmetry cross-ply and angle-ply square laminated composite plate for 10 layers was studied. The composite laminated plate consisting of graphite/epoxy material for two boundary conditions (simply supported and fixed) has been considered for the current case. In this case, the natural frequency has been studied for various layers orientation for the composite laminated plate.

TABLE III
NATURAL FREQUENCY OF A 10-LAYER FIXED SYMMETRY ANGLE-PLY SQUARE LAMINATED COMPOSITE PLATE

| Angle of layers | mode 1 | mode 2 | mode 3 | mode 4 | mode5 |
|---|---|---|---|---|---|
| [-15/15/-15/15/-15]s | 110.53 | 140.55 | 193.63 | 269.21 | 285.42 |
| [-30/30/-30/30/-30]s | 107.97 | 170.99 | 254.72 | 262.69 | 329.57 |
| [-45/45/-45/45/-45]s | 106.81 | 204.85 | 260 | 326.39 | 362.22 |
| [-60/60/-60/60/-60]s | 107.97 | 170.99 | 254.72 | 262.69 | 329.57 |
| -[75/75/-75/75/-75]s | 110.53 | 140.55 | 193.63 | 269.21 | 285.42 |
| -[90/90/-90/90/-90]s | 112.02 | 205.64 | 251.29 | 308.78 | 368.13 |

From the tables, it is observed that the natural frequency for 10-layer cross-ply and angle-ply composite laminated plate with clamped boundary condition in symmetric arrangement of layers decreases when the angle of ply changes from 0° to 45°, then it increases up to 90°. So, the maximum value of natural frequency for all mode shapes will be at angles 0/90 of ply orientation. And it is also observed that the natural frequency in case of angles 15° and 75° is the same, and for 30 and 60, it is the same. The effects of number of layer in the laminate are clearly shown Figs. 5 and 6.

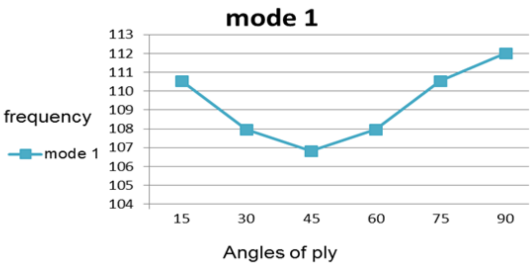


Fig. 5 Natural frequency (first mode shape) of fixed composite laminated plate with various layer orientations

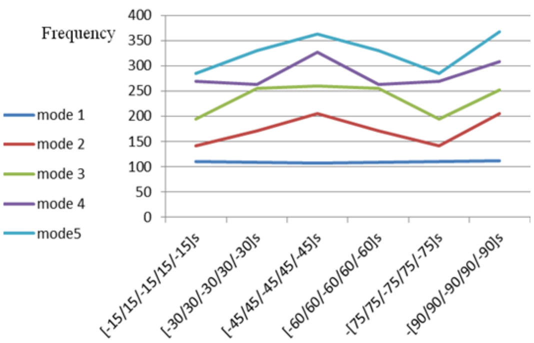


Fig. 6 Natural frequency (first five mode shape) of fixed composite laminated plate with various layer orientations

TABLE IV
NATURAL FREQUENCY OF A 10-LAYER SIMPLY SUPPORTED SYMMETRY ANGLE-PLY SQUARE LAMINATED COMPOSITE PLATE

| Angle of layers | mode 1 | mode 2 | mode 3 | mode 4 | mode5 |
|---|---|---|---|---|---|
| [-15/15/-15/15/-15]s | 55.62 | 86.79 | 138.1 | 189.46 | 209.45 |
| [-30/30/-30/30/-30]s | 63.134 | 120.48 | 176.53 | 202.08 | 250.52 |
| [-45/45/-45/45/-45]s | 66.5 | 145.61 | 169.68 | 252.33 | 281.61 |
| [-60/60/-60/60/-60]s | 63.134 | 120.48 | 176.53 | 202.08 | 250.52 |
| -[75/75/-75/75/-75]s | 55.62 | 86.79 | 138.1 | 189.46 | 209.45 |
| -[90/90/-90/90/-90]s | 51.344 | 127.82 | 161.25 | 204.13 | 270.09 |

From the tables, it is observed that the natural frequency for 10-layer cross-ply and angle-ply composite laminated plate with simply supported boundary condition in symmetric arrangement of layers increases when the angle of ply changes from 0° to 45°, then it decreases up to 90°. So, the maximum value of natural frequency for all mode shapes will be at angle 45° of ply orientation. And it is also observed that the natural frequency in case of angles 15 and 75 is the same, and for 30 and 60 also, it is the same. The effects of number of layer in the laminate are clearly shown Figs. 7 and 8.

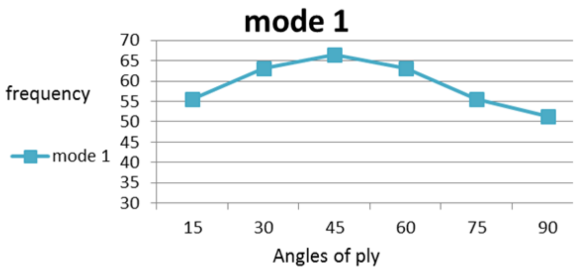


Fig. 7 Natural frequency (first mode shape) of simply supported composite laminated plate with various layer orientations

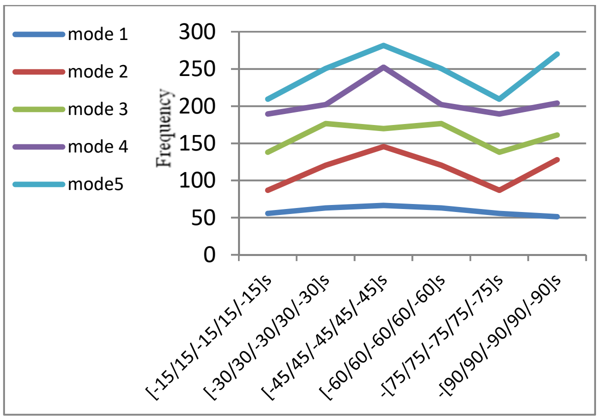


Fig. 8 Natural frequency (first five mode shape) of simply supported composite laminated plate with various layer orientations

Natural frequency of a symmetry cross-ply and angle-ply square laminated composite plate for 10 layers was studied. The composite laminated plate consisting of graphite/epoxy material for two boundary conditions, simply supported with angle of plies [-45/45] and fixed with angle of [0/90], has been considered for the current case. In this case, the natural frequency has been studied for various thicknesses of the composite laminated plate.

TABLE V
NATURAL FREQUENCY OF A 10 LAYER SIMPLY SUPPORTED [-45/45]s ANGLE-PLY SQUARE LAMINATED COMPOSITE PLATE

| Thickness (m) | Mode 1 | Mode 2 | Mode 3 | Mode 4 | Mode 5 |
|---|---|---|---|---|---|
| a/200 = 0.005 | 33.659 | 73.659 | 80.892 | 128.20 | 142.70 |
| a/150 = 0.0066 | 44.712 | 97.878 | 107.44 | 170.15 | 189.55 |
| a/100 = 0.01 | 66.499 | 145.61 | 159.68 | 252.33 | 281.61 |
| a/75 = 0.01333 | 87.660 | 191.88 | 210.15 | 330.90 | 370.08 |
| a/50 = 0.02 | 128.43 | 280.34 | 306.29 | 477.69 | 536.24 |
| a/25 = 0.04 | 237.46 | 508.24 | 550.88 | 827.18 | 935.49 |

The table shows that, by increasing the thickness of the plate the natural frequency will increase, but the amount of increase in the natural frequency is approximately 100% when the thickness is doubled. The increase in natural frequency is clearly shown in Fig. 9.

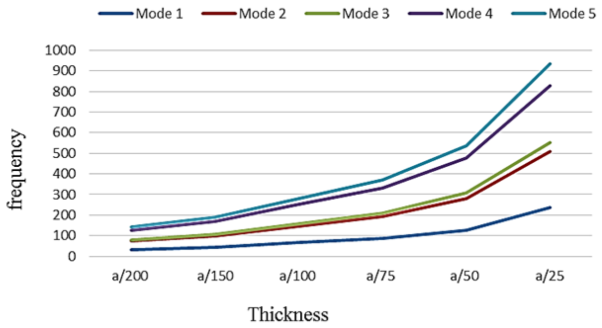


Fig. 9 Natural frequency (first five mode shape) of simply supported composite laminated plate with various thickness

TABLE VI
NATURAL FREQUENCY OF A 10 LAYER FIXED [0/90]S ANGLE-PLY SQUARE LAMINATED COMPOSITE PLATE

| Thickness(m) | Mode 1 | Mode 2 | Mode 3 | Mode 4 | Mode 5 |
|---|---|---|---|---|---|
| a/200 = 0.005 | 56.355 | 103.81 | 127.40 | 156.47 | 187.09 |
| a/150 = 0.0066 | 74.275 | 136.70 | 167.58 | 205.85 | 245.95 |
| a/100 = 0.01 | 112.02 | 205.64 | 251.29 | 308.78 | 368.13 |
| a/75 = 0.01333 | 148.21 | 270.96 | 329.42 | 405.00 | 481.23 |
| a/50 = 0.02 | 217.80 | 393.79 | 472.59 | 581.82 | 685.51 |
| a/25 = 0.04 | 394.53 | 682.33 | 783.04 | 968.06 | 1110.6 |

The same thing can be obtained from the table that the natural frequency will increase with the increase of plate thickness, and also the amount of increase in the natural frequency is approximately 100% when the thickness is doubled. The increase in natural frequency is clearly shown in Fig. 10.

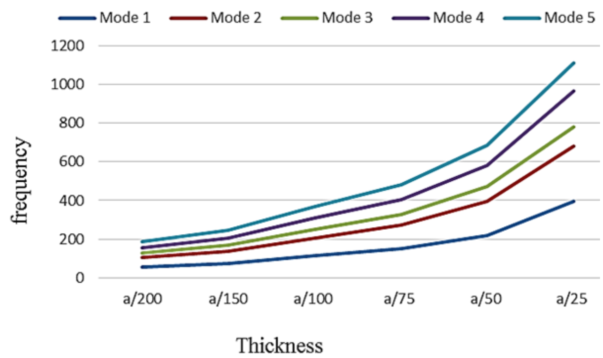


Fig. 10 Natural frequency (first five mode shape) of fixed composite laminated plate with various thickness

## V. CONCLUSION

The free vibration of a composite laminated plate has been stated. From different boundary condition (simply supported and fixed), it is found that the natural frequency for composite laminated plate is higher in fixed boundary condition than in simply supported boundary condition. By increasing the number of layers of the laminated plate for the same thickness, it is observed that the maximum natural frequency can be obtained using 10 layers laminated plate for both simply supported and fixed boundary conditions. And it is observed that the maximum natural frequency for angle-ply simply supported composite plate can be found using angle [-45 / 45] for fiber orientation, and using cross-ply [0/90] fixed composite plate. And the natural frequency for both simply supported and fixed plate increases with the increment of thickness, approximately 100% when the thickness is doubled.

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