Tsunami Inundation Modeling in a Boundary Fitted Curvilinear Grid Model Using the Method of Lines Technique

M. Ashaque Meah, M. Shah Noor, M Asif Arefin, Md. Fazlul Karim

Abstract**—**A numerical technique in a boundary-fitted curvilinear grid model is developed to simulate the extent of inland inundation along the coastal belts of Peninsular Malaysia and Southern Thailand due to 2004 Indian ocean tsunami. Tsunami propagation and run-up are also studied in this paper. The vertically integrated shallow water equations are solved by using the method of lines (MOL). For this purpose the boundary-fitted grids are generated along the coastal and island boundaries and the other open boundaries of the model domain. A transformation is used to the governing equations so that the transformed physical domain is converted into a rectangular one. The MOL technique is applied to the transformed shallow water equations and the boundary conditions so that the equations are converted into ordinary differential equations initial value problem. Finally the 4th order Runge-Kutta method is used to solve these ordinary differential equations. The moving boundary technique is applied instead of fixed sea side wall or fixed coastal boundary to ensure the movement of the coastal boundary. The extent of intrusion of water and associated tsunami propagation are simulated for the 2004 Indian Ocean tsunami along the west coast of Peninsular Malaysia and southern Thailand. The simulated results are compared with the results obtained from a finite difference model and the data available in the USGS website. All simulations show better approximation than earlier research and also show excellent agreement with the observed data.

Keywords**—**Open boundary condition, moving boundary condition, boundary-fitted curvilinear grids, far field tsunami, Shallow Water Equations, tsunami source, Indonesian tsunami of 2004.

I. Introduction

THE 2004 Indian Ocean tsunami hit the coastal belts of Phuket Island in southern Thailand and caused extreme inundation. As the mega thrust tsunami hit around high tide, the damage was severe. The event also affected the Peninsular Malaysia and approximately 68 people were killed in Penang Island, Malaysia [7].

The maximum vertical extent of wave up rush on a beach above is the tsunami run-up and the maximum horizontal extent is tsunami inundation. Modeling tsunami wave run-up and inundation for local or near field tsunami are equally important and challenging problem for the development of tsunami warning system. Numerical models are good enough to simulate tsunami. After knowing the tsunami source, the tsunami propagation and coastal behavior can be modeled by computer simulation [4], [9], [18]. Many researchers [1], [5], [7], [8], [10], [11], [20], [21] put their contributions on numerical simulation of tsunami modeling as well as the extent of inundation due to long wave.

The west coast of Peninsular Malaysia and Southern Thailand are curvilinear in nature. Boundary fitted curvilinear grid technique is used when the coast line is curvilinear in nature and the bending is high. It makes the equations and boundary conditions simple and better represents the complex geometry with a relatively less number of grid points [15]. The boundary fitted grid system improves the finite difference schemes.

During the simulation of tsunami inland inundation many researchers [1], [2], [5], [13], [16] used moving boundary instead of fixed vertical sea side wall to properly represent the dynamics of the intrusion process. These kinds of boundaries are considered as time dependent problems and their updated positions are determined by treating them as a function of space and time. Several types of moving boundary conditions have been proposed and used in the simulation of wave –front flooding on dry land [10]. All of them are based on the idea of so-called weir model in which a threshold depth is introduced to judge whether the water in the computational cell is moving or stopped. On the other hand, the MOL is a technique for solving partial differential equations (PDE) by discretizing in all as a system of ordinary differential equations (ODE). The main benefit of the method is that it allows the solution to take advantage of the sophisticated general-purpose methods that have been developed for numerically integrating ODEs. For the PDEs to which the MOL is applicable, the method is typically proved to be quite efficient [19].

Reference [6] used the MOL technique in a stair step model to simulate the tsunami propagation for 2004 Indian ocean tsunami along the coastal belt of Peninsular Malaysia and southern Thailand. Reference [14] used the same technique in a boundary fitted curvilinear grid model to simulate the propagation of 2004 tsunami. In these works the coastal belts were used as a fixed sea side vertical wall. Reference [10] developed a boundary fitted curvilinear model to simulate inland inundation due to 2004 tsunami in a finite difference framework.

In this paper, a numerical scheme based on MOL for solving the shallow water equations in a boundary fitted curvilinear grid system is developed to simulate the extent of inland inundation as well as the tsunami run up along the

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coastal belt of western coast of Peninsular Malaysia and southern Thailand. Through a literature review, it is found that not much work has been carried out to estimate the extent of inundation due to tsunami along the above coastal belts using the method of lines. During the simulation process, the moving boundary technique is implemented instead of vertical sea side wall. Propagation history of 2004 event is also carried out to show the effect of moving boundary condition in the simulation process. Extent of inundation and tsunami run up in various coastal locations and for different slope of the sea beach is also investigated.

II. Governing Equations and Boundary Conditions

The shallow water equations are commonly used for modeling the two phases of tsunami: tsunami propagation from deep sea to near shore and the inland inundation. The nonlinear shallow water equations, similar to [13], are

$$\frac{\partial(bL\zeta)}{\partial t}+\frac{\partial\widetilde{U}}{\partial\eta}+\frac{\partial\widetilde{V}}{\partial\lambda}=0 \tag{1}$$

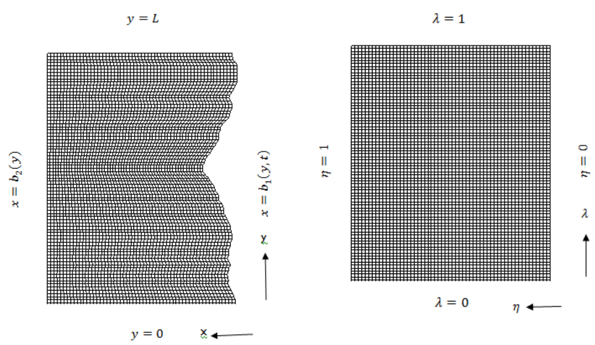
$$\frac{\partial\tilde{u}}{\partial t}+\frac{\partial(U\tilde{u})}{\partial\eta}+\frac{\partial(V\tilde{u})}{\partial\lambda}-f\,\tilde{v}=-gL(\zeta+h)\frac{\partial\zeta}{\partial\eta}-\frac{C_f\tilde{u}\left(u^2+v^2\right)^{1/2}}{\zeta+h} \tag{2}$$

$$\frac{\partial\widetilde{v}}{\partial t}+\frac{\partial(U\widetilde{v})}{\partial\eta}+\frac{\partial(V\widetilde{v})}{\partial\lambda}+f\,\widetilde{u}=-g(\zeta+h)\left[b\frac{\partial\zeta}{\partial\lambda}-L\left(\frac{db_1}{dy}+\eta\frac{db}{dy}\right)\frac{\partial\zeta}{\partial\eta}\right]-\frac{C_f\widetilde{v}\left(u^2+v^2\right)^{1/2}}{\zeta+h} \tag{3}$$

where

$$U=\frac{1}{b}\left[u-\left(\frac{db_1}{dy}+\eta\frac{db}{dy}\right)v\right],\quad V=\frac{v}{L},$$
$$\left(\widetilde{u},\widetilde{v},\widetilde{U},\widetilde{V}\right)=bL(\zeta+h)\left(u,v,U,V\right)$$

Similar model domain, boundary fitted grids and boundary conditions of [13] are also applied in this study. The shape of the physical and transformed computational model domains are shown in Figs. 1 (a), (b).



(a) Physical Domain (b) Computational Domain

Fig. 1 Boundaries and grid system; (a) curvilinear boundaries and the curvilinear grid system (Physical domain), (b) Transformed/ Computational domain

III. Method of Lines

A. Solution of the Governing Equations

In the MOL approach, a system of PDEs is transformed into a system of ODE with initial conditions by discretizing the spatial derivatives together with the boundary conditions. This system of ODEs, at every grid point and every instant time, is accompanied by a set of current values (initial values) of the dependent variables at every grid points and its surrounding points. Thus at every grid point and every instant time, we have an initial value problem involving a system of ODE's. The details are discussed in [6]. Now, each ODE of the system, with the initial conditions is solved using an ODE solver. The classical 4th order Runge-Kutta method is used to compute the updated values of the dependent variables.

B. Grid Generation

Here the physical domain or the analysis area is transformed into a computational domain: a rectangular coordinate system where the rectangular grid system is generated in the analysis area using a set of equidistant straight lines parallel to η-axis and a set of equidistant straight lines parallel to λ-axis. The distances between any two consecutive grid lines parallel to η-axis and λ-axis are assumed to be $\Delta\eta$ and $\Delta\lambda$ respectively. If there are *M* grid lines parallel to η-axis and *N* grid lines parallel to λ-axis, then there are *M* grid points in η -direction and *N* grid points in λ -direction. Therefore the total number of grid points is $M\times N$.

The grid points (η_i, λ_j) are defined by

$$\eta_i=(i-1)\Delta\eta,\qquad i=1,2,3,\cdots,M$$
$$\lambda_j=(j-1)\Delta\lambda,\qquad j=1,2,3,\cdots,N$$

We have used two types of independent variables, namely time and space, in our governing equations. Equations (13)-(15) together with the boundary conditions (9)-(12) are discretized by (central) finite difference scheme for space derivatives in the staggered grid system. Thus at the grid point (η_i,λ_j) we obtain only one first order ODE involving time derivative of one dependent variable together with values of all dependent variables at the present time. The 4th order Runge-Kutta method is used to solve the ODEs as in [14].

IV. Moving Boundary Technique

When there is an inland inundation the shore line moves with the same velocity with that of the approaching water. For inundation purpose moving boundary technique has been implemented during simulation process. Following [3], at $x=b_1(y,t)$ or $\eta=0$ the moving boundary condition is

$$u - \frac{\partial b_1}{\partial t} - v\frac{\partial b_1}{\partial y} = 0 \tag{4}$$

The kinematical boundary condition (4) ensures the continuously deforming position of the coastline. On the other hand, if the coastline consists of a vertical wall through which there is no flux of water, the boundary condition may be expressed as

$$u - v\frac{\partial b_1}{\partial y} = 0 \tag{5}$$

A further requirement for inland intrusion of water is that the total depth of water becomes zero at the coastline. Therefore

$$\zeta(\eta = 0, y, t) + h(x = b_1(y,t), y) = 0 \tag{6}$$

Differentiating (6) with respect to t to yields:

$$\frac{\partial \zeta(\eta = 0, y, t)}{\partial t} + S_j\frac{\partial b_1}{\partial t} = 0 \tag{7}$$

where $S_j = \left[\frac{\partial h}{\partial x}\right]_{x=b_1(y,t)}$ is the onshore slope of the coastal belt. Equation (7) gives the value of $b_1(y,t)$ at any time t. For a constant value of S_j, by integrating (7) we get,

$$b_1(y,t) = b_1(y,0) - \frac{1}{S_j}\left(\zeta(\eta = 0, y, t) - \zeta(\eta = 0, y, 0)\right) \tag{8}$$

The change in b_1 corresponding to the current advance of the solution through the time interval Δt is given by (8). From (8), it is evident that, if $\zeta(\eta = 0, y, t) < 0$, the water surface at the coastline is depressed and as a result the coastline will recede from its initial position. If $\zeta(\eta = 0, y, t) > 0$, water surface at the coastline rises above its equilibrium level and so there is a corresponding inland intrusion of water. The schematic of inland intrusion of water is presented in Fig. 2 and can be determined by:

$$\mu(y,t) = b_1(y,o) - b_1(y,t) \tag{9}$$

At the advanced time level the x-coordinates of the points in the physical domain can be calculated from

$$x = b_1(y,t) + \eta b(y,t) \tag{10}$$

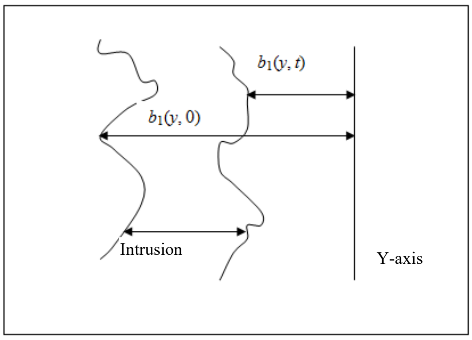


Fig. 2 Definition sketch of intrusion of water

Equation (10) gives the advanced values of h in the coastal boundary and which is given by

$$h(x = b_1(y,t), y) = -\zeta(\eta = 0, y, t) \tag{11}$$

Equation (11) shows that h is the negative of the updated elevation. Also depending on whether $b_1(y,t) > b_1(y,0)$ or $b_1(y,t) < b_1(y,0)$ interpolation or use new inland orographical data is used to fix the value of $h(x = b_1(y,t), y)$.

V. Model Data Set-up

The η- and the λ-axes are directed towards the west and north at 15° anticlockwise direction with the latitude and longitude line respectively. The model domain is extended from 2° N to 14 °N latitudes (incorporating the west coasts of Malaysia including Penang and Southern Thailand including Phuket) and 91° E to 100.5° E longitudes (Fig. 3) which includes the tsunami source region associated with 2004 Indonesian tsunami.

The origin of the Cartesian coordinate system is set to be O (3.125° N, 101.5° E). In this model the number of grids in north and west directions are respectively taken to be M = 230 and N = 319 so that there are 73370 grid points. The time step, in this computation, is set to be 10 s which ensures that there is no stability problem. The value of the friction coefficient C_f, following [12], is taken as 0.0033 throughout the model domain. Depth data are collected from Admiralty bathymetric charts for some representative grid point and the rest is calculated by interpolation process. The bathymetry of the modal domain is presented in Fig. 4.

VI. Tsunami Source Generation and Initial Conditions

Tsunami generation depends on the initial deformation of the undersea earthquake. The generation mechanism is modeled by the initialization of the water surface. The generation mechanism for the tsunami occurred on 26 December 2004 was mainly due to a static sea floor deformation caused by an abrupt slip at the India/Burma plate interface [12]. The estimated subduction zone with the amount of uplift and subsidence are presented in Table I.

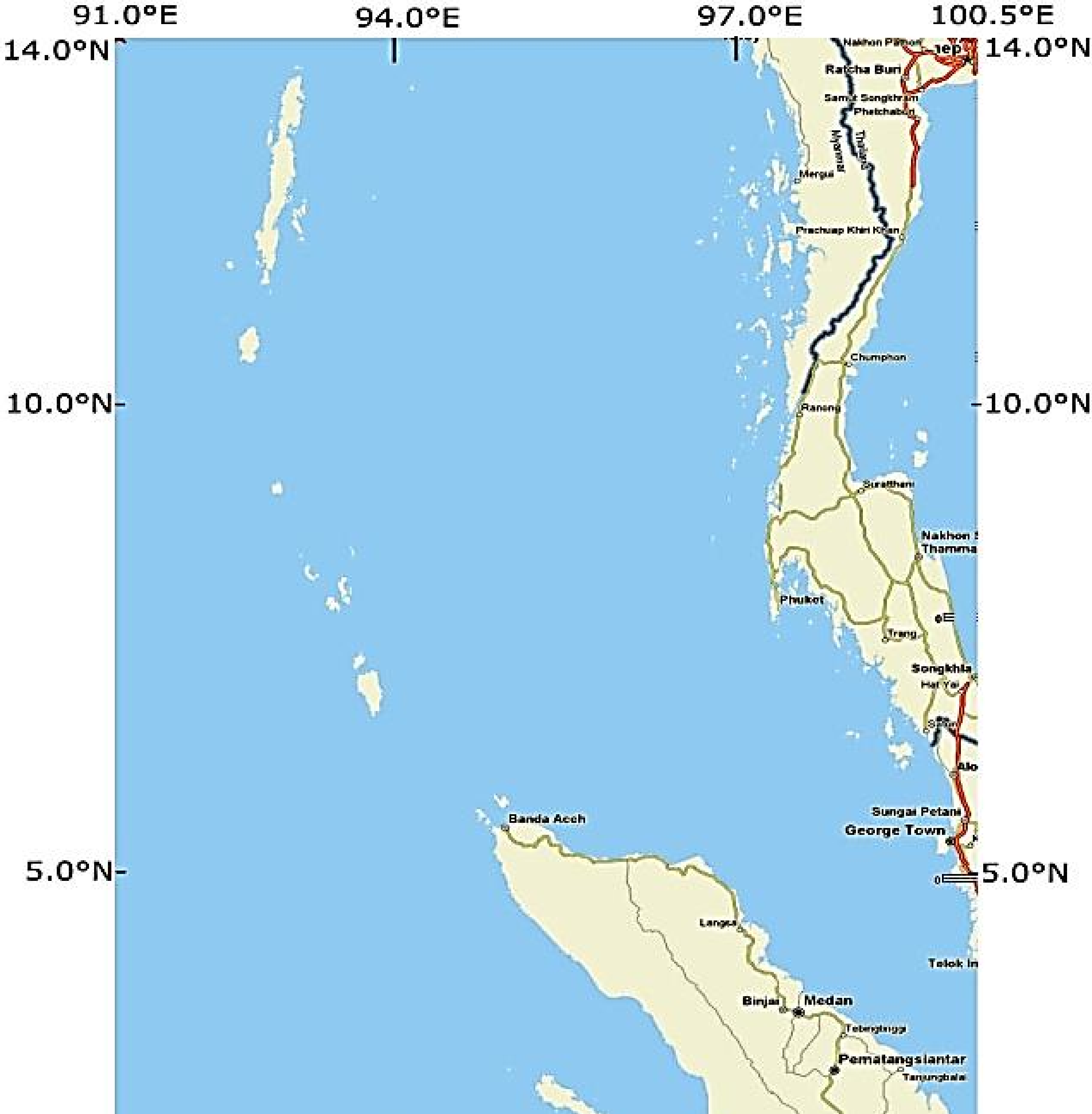


Fig. 3 Model Domain including the coastal geometry and the epicenter of the 2004 earthquake (as [7])

International Journal of Information, Control and Computer Sciences
ISSN: 2517-9942
Vol:11, No:1, 2017

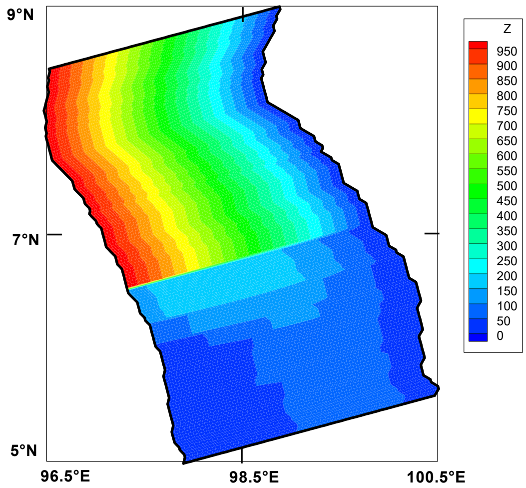


Fig. 4 Bathymetry used during the simulation process

TABLE I
TSUNAMI SOURCE PARAMETERS

Parameter	Value
Up Lift Zone Latitude	2^0N To 10^0N
Up Lift Zone Longitude	92^0E To 97^0E
Maximum Uplift	507 cm
Maximum Subsidence	474 cm
Direction of Maximum Uplift	West Side of The Source Zone
Direction of Maximum Subsidence	East Side of The Source Zone
Direction from Uplift to Subsidence	West to East of the Source Zone

Due to the incompressibility of the ocean water, we can assume that this sea surface displacement is same as the ocean bottom displacement. In this simulation, we consider the source zone extended along the fault line is same as [12] and the disturbance is in the form of rise and fall of sea surface. In the source zone the maximum rise and fall of the disturbance are considered as 500 cm and 475 cm respectively. Outside of the source zone, the initial disturbance of the water is set to be zero. In addition, the initial x and y components of the velocity are taken as zero throughout the model area.

VII. Simulation of Tsunami through the MOL

Tsunami traveling time from the source region to a particular location plays a vital role in the tsunami forecasting. The numerical computation of tsunami travelling time at every grid point in the model domain is presented in Fig. 5 that shows the contour plot of time, in minutes, for attaining +0.1 m sea level rise. If we consider 0.1 m sea level rise as the arrival of tsunami, it is observed that the generated tsunami propagates gradually towards Phuket and then after Penang.

The present numerical calculation shows that the tsunami travelling times to reach to the islands of Phuket and Penang are approximately 1 hr 50 min and 3 hr 50 min respectively. According to USGS report [7], we see that the tsunami waves to reach to Phuket needed less than two hours after the earthquake. The USGS website data also confirm the fact that the tsunami arrival time at Penang is between 3 hr 30 min and 4 hours. Therefore, the computed tsunami travelling times along the coastal belts of Peninsular Malaysia and southern Thailand agree well with USGS data.

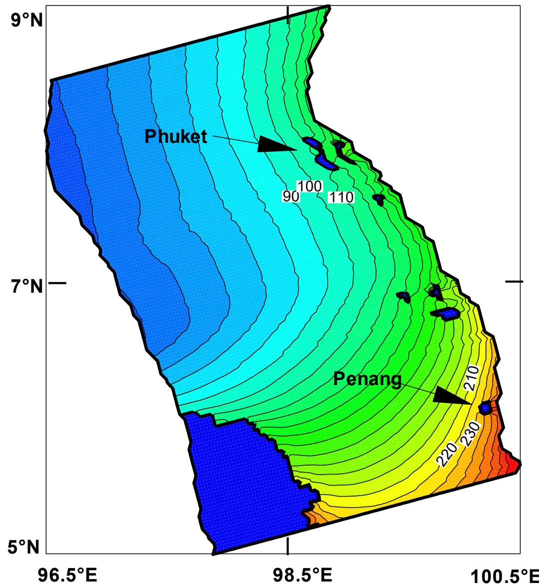


Fig. 5 Tsunami propagation time in minutes towards Phuket and Penang

The distribution of computed maximum water levels along the west coast of Peninsular Malaysia and Thailand is shown in Fig. 6. The maximum water level is from 6 m to 11.5 m approximately at the Phuket region (northern part) of the model area. The tsunami amplitudes are very large in the northwest direction toward Phuket. The maximum coastal surge varies from 18 to 20 m in some locations approximately 50 km north from the Phuket.

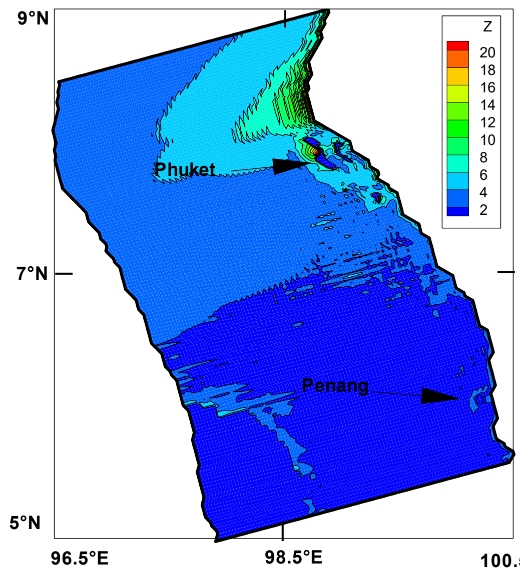


Fig. 6 Contour of maximum water elevation around the west coast of Thailand and Malaysia

The calculated tsunami amplitudes are relatively smaller along the Penang coast. The maximum amplitudes at Penang Island range from 2 m to 3.5 m with an increasing trend to the north. The north and the west coast are found to be vulnerable for stronger tsunami surge. On the east coast, the measured tsunami heights show a decreasing trend towards the north.

The maximum inland inundations across the west coast of Thailand and Peninsular Malaysia for the inclinations of 8 degree and 12 degree are presented in Figs. 7 and 8, respectively. The maximum inland disturbance across the west coast of Peninsular Malaysia is ~30 m while across the Thailand coast is ~170 m for 8 degree inclination. As the on-shore inclination increases from 8 degree to 12 degree the level of inundation reduces to ~20 m and ~115 m respectively for the Malaysian and Thailand coast (Fig. 8).

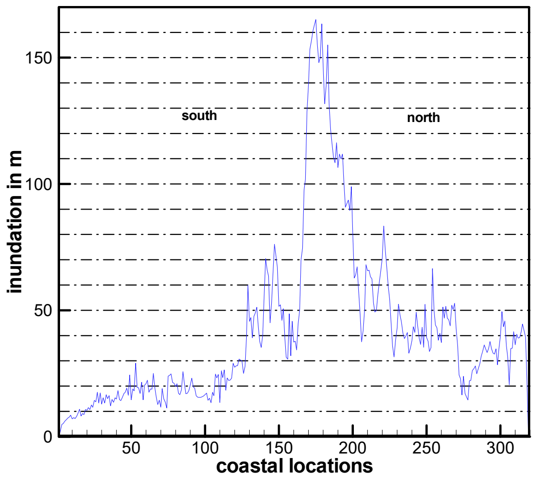


Fig. 7 Maximum Inland Inundation along the west coast of Peninsular Malaysia and Thailand for 8 degree inclination

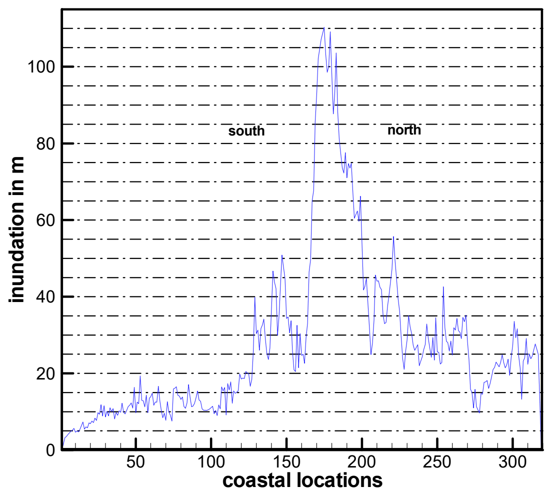


Fig. 8 Maximum Inland Inundation along the west coast of Peninsular Malaysia and Thailand for 12 degree inclination

If the slope of Malaysia and Thailand coast is not considered as constant and it varies from 8 degree to 12 degree then the inundation is ~40 m and ~200 m respectively, which are shown in Fig. 9. Therefore, we can conclude that the tsunami inundation is controlled by the configuration of the beach. After the 2004 tsunami, a post tsunami field survey has been carried out to measure its impact along the coastal belt of southern Thailand [17]. Eyewitnesses reported that the tsunami inundation in the Patong beach, Thailand, varied from 150 m to 750 m. Another report is that the inundation was of only 20 m. In Maya Bay the tsunami wave inundated for about 200 m to 300 m. However, the computed result shows good agreement with the field survey report of [17].

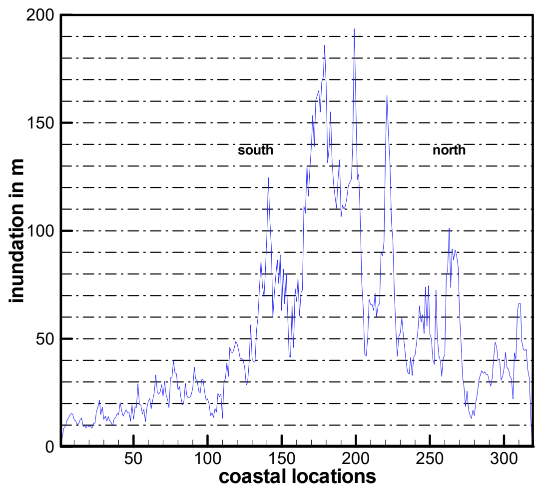


Fig. 9 Maximum Inland Inundation along the west coast of Peninsular Malaysia and Thailand for variable degree inclination

Fig. 10 depicts the computed time series of water level at a coastal location of Thailand about 140 km north to Phuket due to the initial tsunami generated at Sumatra on 26 December 2004. At the south coast of Phuket, the time series begins with recession that reaches a depression of –2.5 m before rising up; the water level then gradually increases to a maximum level of 11.6 m. Finally the water level oscillates continuously for several hours with low amplitude. Fig. 10 also shows the computed inundation and recession for the same coastal location due to 8 degree inclination associated with the tsunami wave. For 8 degree inclination the inland inundation/ recession curve is consistent with the water level fluctuation with a maximum inundation of 82 m and maximum recession of -35 m. Since the slope is considered as constant, the curve of inundation/recession should be consistent with that of sea surface oscillation. The computed inundation or recession increases due to the increase of the on-shore slope. On the other hand, both inundation and recession amplitudes decrease gradually if the on-shore slope decreases.

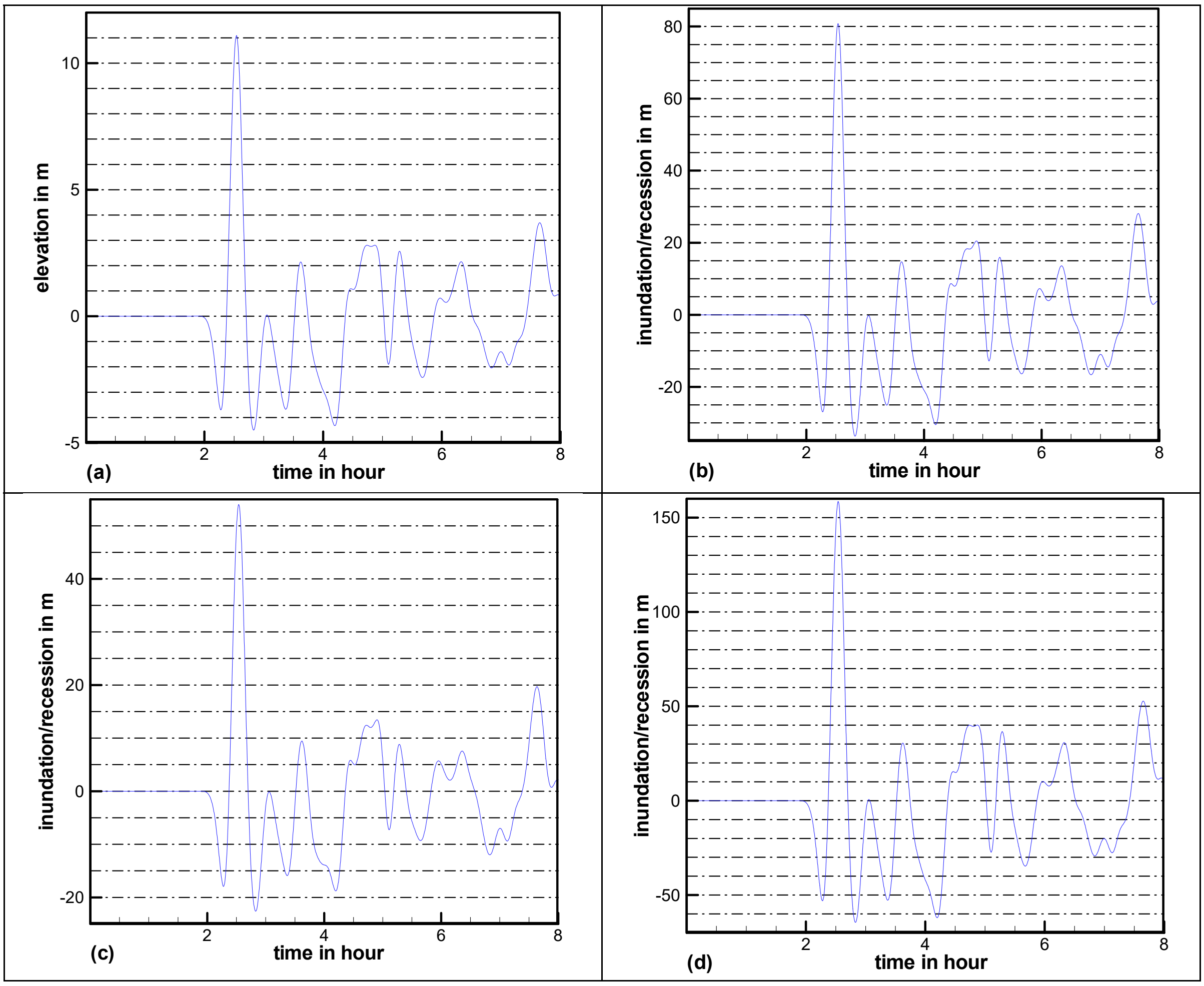


Fig. 10 Comparison of water elevation and inundation along a particular coastal location about 140 km north to Phuket

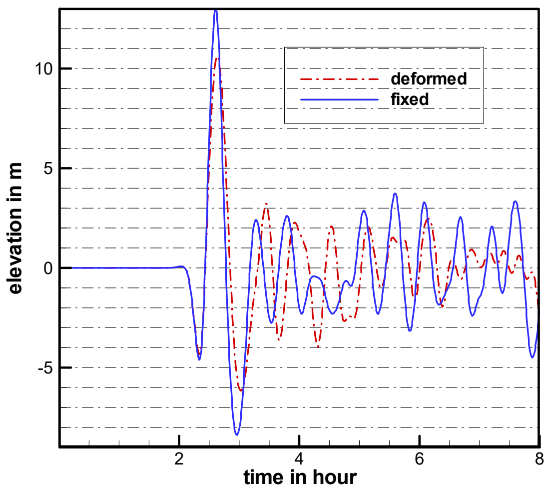


Fig. 11 Comparison of water elevation along a particular position of the coast due to fixed boundary and deformed/moving coastal boundary

Fig. 11 exhibits the comparison of water elevation at a particular coastal location closed to the south of Phuket due to moving and fixed coastal boundaries. It shows that the maximum water level is ~10.75 m along the coast due to the moving boundary and the peak surge is ~12 m due to the fixed coastal boundary. The surge height along the fixed coastal boundary is considerably higher than the surge height of the moving coastal boundary. This may occur due to the unrealistic piling up of water at the vertical sea side wall. If the moving boundary condition (considering the coastal boundary as moving) is applied, the water flows freely across the land so that it causes inland inundation and at the same time the sea surface elevation at the initial position of the coastal boundary (fixed) reduces. On the other sides of the coastal boundary, there is no significant difference in the maximum water elevations. On the storm surge modeling of [3], they observe a significant difference in maximum water elevation at a particular coastal location due to fixed and moving boundary condition. So our comparison is consistent with that of [3].

The present study is a modified version of the model of [10]. The simulated results are compared with the data of [10] and the data available in the USGS website. Here, we have solved the governing equations using the MOL technique in a boundary fitted curvilinear grid model whereas the governing equations, in the model of [10], are directly discretized by the finite difference approximations. In both the models the moving boundary condition is applied instead of fixed vertical wall to ensure the movement of the coastal boundary. The tsunami run-up along the coastal belt of Peninsular Malaysia, southern Thailand, and the extent of inland inundations are mutually compared and found to be similar in both the models. The simulated results are also compared with the data available in USGS website.

TABLE II
COMPUTED AND OBSERVED USGS TSUNAMI PROPAGATION TIME AND WATER LEVELS FOR PENANG AND PHUKET

		MOL	Finite Difference Model	USGS/ Observed
Run-Up (m)	Penang	2-3.5	-	2-3.5
	Phuket	6-11.5	-	7-11
Inundation (m)	Penang	40	40	-
	Phuket	200	200	200-300

From Table II and above discussion, it is clear that the tsunami run-up and extent of inundation along the island boundaries of Penang and Phuket computed by the models agree well with the observed data or the data available in the USGS website. As discussed earlier MOL has the advantages that it is more effective than regular finite difference method in terms of accuracy, computational time and numerical stability. So, hopefully, for the development of real time tsunami regional warning system, the present study will contribute a lot.

VIII. CONCLUSION

This study simulates the tsunami inundation for the event of 2004 Indian ocean tsunami along the coastal belt of Peninsular Malaysia and southern Thailand. The MOL technique has been applied to solve the vertically integrated shallow water equations in boundary-fitted curvilinear grids. The moving boundary technique is successfully implemented during the simulation process to ensure the movement of the shore lines. Tsunami propagation and run-up are also studied for the above mentioned coastal belts. Though the results are similar to a prior modeling study using finite difference scheme, the implementation of MOL technique makes the model more effective and realistic. The computed results of this study show that the MOL approach can be applied in tsunami regional mitigation efforts.

ACKNOWLEDGMENT

The research is partially funded by SUST research centre, Shahjalal University of Science and Technology, Sylhet, Bangladesh. The authors acknowledge the support.

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