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Supplier Selection by Bi-Objectives Mixed Integer Program Approach

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***Abstract*—**In the past, there was a lot of excellent research studies conducted on topics related to supplier selection. Because the considered factors of supplier selection are complicated and difficult to be quantified, most researchers deal supplier selection issues by qualitative approaches. Compared to qualitative approaches, quantitative approaches are less applicable in the real world. This study tried to apply the quantitative approach to study a supplier selection problem with considering operation cost and delivery reliability. By those factors, this study applies Normalized Normal Constraint Method to solve the dual objectives mixed integer program of the supplier selection problem.

***Keywords*—**Bi-objectives MIP, normalized normal constraint method, supplier selection, quantitative approach.

I. Introduction

SUPPLIER selection is one of essential issues in a logistics chain. The results of supplier selection may have the significant influence on a company's competence, which draws attentions of researchers to study the problem. Reference [1] conducted literature reviews and showed solution approaches of 123 papers, which included 26 solution approaches; of those, there are six popular approaches. These are AHP (Analytic Hierarchy process), ANP (Analytic network process), TOPSIS (Technique for order performance by similarity to ideal solution), DEA (Data Envelopment Analysis), LP (Linear Programming), MOP (Multiobjective programming). Reference [2] analyzed 221 papers from 1990 to 2015 to determine the evolutions of supplier selection studies. The main research areas of supplier selection include supplier selection approaches, selection criteria, green/sustainable, strategy oriented, R&D oriented, and operations oriented. Reference [3] summarized seven criteria and 14 essential attributes of choosing suppliers for a company, in which cost and risk are two distinct criteria. This study elaborates to establish a bi-objectives mixed integer program with considering minimizing cost and risk for solving supplier selection problem. Normalized Normal Constraint Method is the main tool to solve the NP-hard model. This study uses the exact approach, i.e. Normalized Normal Constraint Method, which belong to the MOP approach. However, in order to avoid solving the NP-hard problem, most studies adopted combined multiple objectives into one single objective, which results in optimal or near-optimal solution cannot be guaranteed [4]. This approach is called the weighted-sum or scalarization method. Reference [5] proposed ε-constraints method to solve multi-objectives problems. Later on, [6] proposed two modifications to improve ε-constraints method. Reference [7] developed Normalized Normal Constraint Method to solve the multiobjective optimization problem. Although this approach has the drawback of generating non-Pareto solutions under certain circumstances, those solutions can be filtered by the Pareto filter proposed by [7]. The following shows the seven steps of Normalized Normal Constraint Method for a bi-objectives program proposed by [7]. This study revises the original texts of [7] to make the meanings of all the steps easily understood for potential programmers for the approach of Normalized Normal Constraint Method.

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A. Symbol Definition

- P: original problem with objectives $\mu_1(x)$ and $\mu_2(x)$.
- PU1: sub-problem of P with objective $\mu_1(x)$.
- PU2: sub-problem of P with objective $\mu_2(x)$.
- P2: Extension problem derived by PU2
- x : variable vector, i.e. $x = (x^1, x^2)$.
- $\mu_1(x)$: objective function of PU1.
- $\mu_2(x)$: objective function of PU2.
- μ : objective function vector, i.e. $\mu(x) = (\mu_1(x), \mu_2(x))$ or $\mu = (\mu_1, \mu_2)$.
- $\bar{\mu}$: normalized form of μ.
- μ^u : utopia point.
- μ^{1*} : optimal objective value of PU1 with optimal solutions from PU2, $\mu^{1*} = \mu_1(x^{2*})$.
- μ^{2*} : optimal objective value of PU2 with optimal solutions from PU1, $\mu^{2*} = \mu_1(x^{1*})$.
- x^{1*} : optimal solution of PU1.
- x^{2*} : optimal solution of PU2.
- l_1 : distance between μ^{1*} and μ^u.
- l_2 : distance between μ^{2*} and μ^u.
- $\bar{N}_1$: the direction from $\bar{\mu}^{2*}$ to $\bar{\mu}^{1*}$

B. Mathematical Model

Problem P

$$\min_x \mu(x) \tag{1}$$

subject to

$$g(x) \le 0 \quad (2)$$

$$h(x) = 0 \quad (3)$$

Problem PU1

$$\min_x \mu_1(x) \quad (4)$$

subject to (2), (3).

Problem PU2

$$\min_x \mu_2(x) \quad (5)$$

subject to (2), (3).

Step 1. Identify Anchor Points

Estimate the two anchor points, μ^{1*} and μ^{2*}, by solving Problem PU1 and PU2, respectively. The line joining these two points is the Utopia line, which is shown in Fig. 1. In Fig. 1, the definitions of μ^{1*} in this study is μ^{2*} in [7], and μ^{2*} in this study is μ^{1*} in [7]. The reason for revision is [7] has a symbol definition conflict between figures and algorithms.

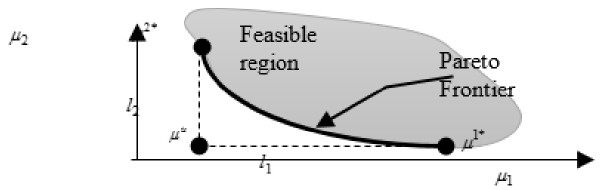

Fig. 1 The flowchart of the solution approach

Step 2. Objectives Mapping/Normalization

To avoid the scale issue of different problems, normalization has to be performed in advance. $\overline{\mu}$ is estimated from μ^u, μ^{1*}, μ^{2*}, x^{1*}, x^{2*}, l_1, and l_2.

$$\mu^u = [\mu_1(x^{1*}), \mu_2(x^{2*})]^T \quad (6)$$

$$l_1 = \mu_1(x^{2*}) - \mu_1(x^{1*}) \quad (7)$$

$$l_2 = \mu_2(x^{1*}) - \mu_2(x^{2*}) \quad (8)$$

$$\overline{\mu} = [\frac{\mu_1(x) - \mu_1(x^{1*})}{l_1}, \frac{\mu_2(x) - \mu_2(x^{2*})}{l_2}]^T \quad (9)$$

Step 3. Utopia Line Vector

$$\overline{N}_1 = \overline{\mu}^{1*} - \overline{\mu}^{2*} \quad (10)$$

Step 4. Normalized Increments

Compute a normalized increment, δ_1, along the direction Utopia line vector, $\overline{N}_1$, m_1 of (11) is number of section between two point, $\overline{\mu}^{1*}$ and, $\overline{\mu}^{2*}$ needs to be pre-determined.

$$\delta_1 = \frac{1}{m_1 - 1} \quad (11)$$

Step 5. Generate Utopia LINE Points

Estimate a set of evenly distributed points on the Utopia line, (12)–(14) show the formula of Step 5, and Fig. 2 shows the visualizations of Step 3 to Step 5.

$$\overline{X}_{pj} = \alpha_1 \overline{\mu}^{1*} + \alpha_2 \overline{\mu}^{2*} \quad (12)$$

$$\alpha_{1j} + \alpha_{2j} = 1 \quad (13)$$

$$j \in \{1, 2, \cdots, m_1\}$$
$$0 \le \alpha_{1j}, \alpha_{2j} \le 1 \quad (14)$$

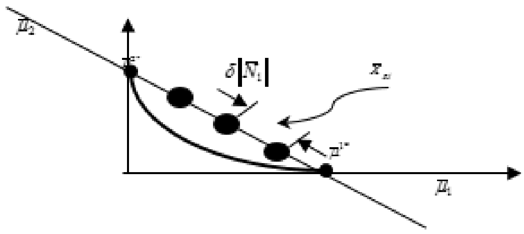

Fig. 2 Demonstrations of Step 3 to Step 5 for showing a set of evenly spaced points on the Utopia line for a bi-objective problem

Step 6. Pareto Points Generation

Equation (12) is used to determine the set of evenly distributed points on the Utopia line. For each point, Problem P2 is solved accordingly to generate a corresponding set of Pareto points.

Problem P2 (For the j^{th} point)

$$\min_x \overline{\mu}_2(x) \quad (15)$$

subject to

$$g(x) \le 0 \quad (16)$$

$$h(x) = 0 \quad (17)$$

$$\overline{N}_1(\overline{\mu} - \overline{X}_{pj}) \le 0 \quad (18)$$

Step 7. Pareto Design Metrics Values

Once P2 is solved, $\overline{\mu}_1(x^*)$ and $\overline{\mu}_2(x^*)$ can be determined. (9) can be applied to estimate $\mu_1(x^*)$ and $\mu_2(x^*)$, which is shown in (18) and (19):

$$\mu_1(x^*) = l_1 \overline{\mu}_1(x^*) + \mu_1(x^{1*}) \quad (19)$$

$$\mu_2(x^*) = l_2 \overline{\mu}_2(x^*) + \mu_2(x^{2*}) \quad (20)$$

This study adopts an exact approach, bi-objectives MIP model, to solve the supplier selection problem. Two essential factors, cost and risk, are chosen as objectives (or suppliers'

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performance indicators). To the best of the author's knowledge, this study is the first trial of bi-objectives MIP approach to solve the supplier selection problem by Normalized Normal Constraint Method. Consequently, this study mainly establishes solution approach to examine the effectiveness of the approach by case: 10 suppliers and each suppler sell the same 10 products. MIP model is implemented and solved with GAMS with Cplex solver. The organizations of this study are as follows: Section I mentions the research motivation, and explains the importance of the supplier selection problem. Compared to the popular solution approaches, Normalized Normal Constraint Method for a bi-objectives program is widely used might be because of the algorithm is not friendly. Section I also re-writes the algorithm proposed by [7] and makes the algorithm clear. Section II introduces the solution framework, including the bi-MIP model and the steps to solve the problem. Section III demonstrates the results of a test case and Section IV concludes this study.

II. Solution Framework

A. Assumptions

The model parameters are static not the same as the real world case; the parameters might be dynamic. The factors of influencing the decisions of the supplier selections are diverse. This study only considers two essential factors, cost and risk, which might cause minor derivations of the results. However, the main purpose of this study is to verify the solution approach, and whether this approach is practical or not needs more research evidence in the future.

B. Mathematical Model

The details of the mathematical model are as follows, including indices, parameters, decision variables, and equations.

1. Index

- i : supplier index
- j : supplier's product index

2. Parameter

- N : number of suppliers
- K : number of products
- A_{ij} : setup cost of product j of supplier i
- B_{ij} : shortage cost of product j of supplier i
- C_{ij} : unit cost of product j of supplier i
- U_{ij} ; upper limit quantities of product j that supplier i can sell.
- R_{ij} : risk of product j from supplier i
- D_j : Demand of product j

3. Decision Variables

- z_1 : total risk
- z_2 : total cost
- x_{ij} : quantities of product j from supplier i .
- y_{ij} : a binary variable. $y_{ij}=1$, when product j from supplier i , $y_{ij}=0$, otherwise
- s_{ij} : shortage quantities of product j from supplier i .

4. Equation

$$z_1 = \min \sum_{i=1}^{N}\sum_{j=1}^{K} R_{ij} y_{ij} \tag{21}$$

$$z_2 = \min \sum_{i=1}^{N}\sum_{j=1}^{K} (C_{ij} x_{ij} + A_{ij} y_{ij} + B_{ij} s_{ij}) \tag{22}$$

$$x_{ij} \le U_{ij} y_{ij} \quad \forall i,j \tag{23}$$

$$\sum_{i=1}^{N} (x_{ij} + s_{ij}) \ge D_j \quad \forall j \tag{24}$$

Equations (21)-(24) represent objective functions, total risk and total cost, respectively. Risk is estimated from the on-time delivery history of suppliers. Total cost includes three items. The first item is product cost, the second item is purchasing setup cost, and the third is the shortage cost due to the over-demand. (23) indicates the relationship between the product demand quantity and decisions on supplier selection. (24) represents the customer's demand has to be fulfilled. If over-demand happens, a shortage is allowed.

C. GAMS Code

This section is for those who are familiar GAMS language to know how to implement GAMS code. For confidential reason, the follows show parts of the complete GAMS codes.

Step 1.

```
obj1..z1=e=sum((i,j),R(i,j)*y1(i,j))

supply1(i,j).. x1(i,j) =l= U(i,j) * y1(i,j);

demand1(j).. sum(i,x1(i,j)+s1(i,j)) =g= D(j);

obj2..z2=e=sum((i,j),A(i,j)*y2(i,j)+C(i,j)*x2(i,j))+10e7*sum((
i,j),s2(i,j));

supply2(i,j).. x2(i,j) =l= U(i,j) * y2(i,j);

demand2(j).. sum(i,x2(i,j))+sum(i,s2(i,j)) =g= D(j);
```

In the obj2 equation, the purpose that shortage term times a big number is to let the solver not choose shortage variable as possible, which makes a little difference from (20). Once running the program, the optimal solutions of PU1 and PU2 can be acquired.

Step 2.

```
L1 = sum((i,j),R(i,j)*y2.l(i,j))-sum((i,j),R(i,j)*y1.l(i,j))

L2=sum((i,j),A(i,j)*y1.l(i,j)+C(i,j)*x1.l(i,j))+10e7*sum((i,j),s
1.l(i,j))-sum((i,j),A(i,j)*y2.l(i,j)+C(i,j)*x2.l(i,j))+100*sum((i,j)
,s2.l(i,j));
```

Once L1 and L2 are estimated, the Normalized term can be acquired for later steps.

Step 3.

finalObj..finalZ=e=(sum((i,j),A(i,j)*y(i,j)+C(i,j)*x(i,j))+10e7* sum((i,j),s(i,j))-zzz2)/L2;

supply(i,j).. x(i,j) =l= U(i,j) * y(i,j);

demand(j).. sum(i,x(i,j))+sum(i,s(i,j)) =g= D(j);

ubar1..(sum((i,j),R(i,j)*y(i,j))-zzz1)/L1-(sum((i,j),A(i,j)*y(i,j) +C(i,j)*x(i,j))+10e7*sum((i,j),s(i,j))-zzz2)/L2 + 1 - 2 * jj/mmmm =l= 0;

D. Test Case

The test case is not a real case, but is for the purpose of demonstrating the effectiveness of the solution approach. The case descriptions are as follows: A company produces some kinds of products, which have 10 common components. At a specific time, components 1 to 5 need 50 units each, and components 6 to 10 need 100 units each. Risks are generated by the uniform distribution of the values between 0 and 1. Costs are generated by the uniform distribution of the values between 0 and 100. The setup cost is 1000 times of the unit cost of each product. Shortage cost is 100 times of risk value. The upper limit quantities of product are generated by the uniform distribution of the values between 0 and 100.

III. Computational Results

According to the test case setting, Normalized Normal Constraint Method is applied to solve the bi-objectives MIP model of the supplier selection problem.

Because the bi-objective MIP model belongs to NP-hard problems, it is hard to estimate the real computational time complexity when solving the model. However, Normalized Normal Constraint Method makes bi-objective MIP into several single objective MIP, which decreases computational time dramatically. In the testing case, m_1 is set to 20, which indicates the 20 model runs needed to be performed. For this easy single objective MIP model, it takes around 0.15 seconds on average for one run by an INTEL i7 computer. That is, for the test case, it takes approximately 3 seconds to acquire solutions that can be provided as a supplier selection decision reference.

Figs. 3 and 4 show the computational results of the test case. Fig. 3 shows the non-dimension results of the test case. In some situations, it might be that the Normalized Normal Constraint Method generate non- Pareto solutions. If this happens, [7] suggests that the computational results need corrections with Pareto filter. Reference [7] provided the Pareto filter algorithm. The issue might come from the discontinuity of the discrete variables of a specific problem. However, in the test case, none of the deficiencies happens. Fig. 3 shows the dotted figure is discrete convex; therefore, no Pareto filter is needed in the test case. Fig. 4 is a transformation of Fig. 3 and shows the real values calculated from the bi-objective MIP model. The result shows that low cost makes high risk, and high cost makes for a low risk. That makes reasonable sense because if a company willing to pay more money for their suppliers, suppliers can offer benefits, which encourage suppliers to be willing to achieve an on-time delivery goal.

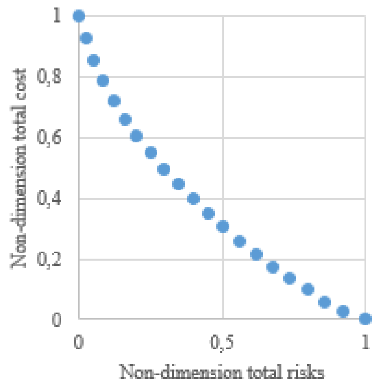

Fig. 3 Non-dimension Pareto Frontier of the test case

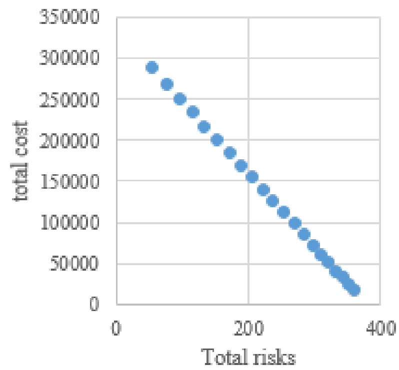

Fig. 4 Pareto Frontier of the test case

The points on the Pareto front have the equivalent effects for a company, whether a company wants to pay attention to costs or risks still depends on the company decision-maker. Although the final decisions might be difficult to make, the approach this study proposes is still worthwhile to the more accurate quantitative information from the bi-objectives MIP model than those from the single-objective MIP model.

IV. Conclusions

In this study, a bi-objective MIP model of supplier selection is proposed and solved by Normalized Normal Constraint Method. The Normalized Normal Constraint Method is used in solving other problems, but was not widely used for the supplier selection problem before. Consequently, this study applies the Normalized Normal Constraint Method on the supplier selection by considering cost and risk factors. The solution approach is trying to solve the problem quantitatively rather than qualitatively. The test case computational results indicate that the Normalized Normal Constraint Method can be applied to the supplier selection problem potentially. However, more numerical experiments need to be performed to reach solid conclusions. The other possible future study might include uncertain parameter settings to make this solution approach practical to real world applications.

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