

Influence of Pile Radius on Inertial Response of Pile Group in Fundamental Frequency of Homogeneous Soil Medium

Faghihnia Torshizi Mostafa, Saitoh Masato

Abstract—An efficient method is developed for the response of a group of vertical, cylindrical fixed-head, finite length piles embedded in a homogeneous elastic stratum, subjected to harmonic force atop the pile group cap. Pile to pile interaction is represented through simplified beam-on-dynamic-Winkler-foundation (BDWF) with realistic frequency-dependent springs and dashpots. Pile group effect is considered through interaction factors. New closed-form expressions for interaction factors and curvature ratios atop the pile are extended by considering different boundary conditions at the tip of the piles (fixed, hinged). In order to investigate the fundamental characteristics of inertial bending strains in pile groups, inertial bending strains at the head of each pile are expressed in terms of slenderness ratio. The results of parametric study give valuable insight in understanding the behavior of fixed head pile groups in fundamental natural frequency of soil stratum.

Keywords—Winkler-foundation, fundamental frequency of soil stratum, normalized inertial bending strain, harmonic excitation.

I. INTRODUCTION

THE deformations of a structure during earthquake generate inertial forces atop the pile foundation systems. Investigation on lateral response of single piles and pile groups due to induced inertial forces has attracted a vast amount of researches. Various types of techniques have been proposed to investigate the behavior of pile-soil-structure under dynamic loads in recent years, such as continuum approach [1], [2], boundary element method [3], [4] finite element solutions [5], [6]. A simplified approach was also presented by [7] for calculating the dynamic response and internal forces caused by harmonic loading atop the pile cap. That method is based on generalized Winkler model in conjunction with a three-step wave interference solution for pile to pile effect. Although those studies had led to sufficient understanding in the behavior of inertial response of pile-soil-structure systems, the predictions of inertial bending remain questionable. For design purposes, it is necessary to determine pile radius because the size of the radius directly affects the bending stiffness of the pile EI . When inertial loading is significant, increasing the pile radius is a proper technique to decrease bending strains. Saitoh [8] proposed a closed form

formula in order to obtain optimal radius of vertical, cylindrical fixed-head single pile embedded in a homogeneous elastic soil layer and supported by rotationally compliant bedrock. Particularly the frequency of horizontal excitation was assumed to be equal to the natural frequency of the soil medium. The variations in normalized inertial bending strains as a function of the slenderness ratio r/H were investigated. Despite this effort, research on the influence of the pile radius on bending strains in soil-pile group systems, where inertial interaction is predominant, has not reported yet, therefore to establish criteria for optimal pile radius in pile group, variations of inertial bending strains with respect to pile radius should be quantified in a systematic way. This paper attempts to offer comprehensive relations between radius and the inertial bending strains at the head of vertical, cylindrical pile group embedded in a homogeneous soil layer, pile group is assumed to be under harmonic loading at the head, and different constraint conditions at the pile group tip (hinged and fixed) is considered. Analytical results will be assessed through BDWF model. Mylonakis and Nikolaou [9] implied that, in dominance of inertial responses in fundamental frequency of soil-pile system, the inertial bending would be significant, particularly at upper part of the piles. Therefore, to get insight into the physics of the problem and basic characteristics, it would be beneficial to investigate inertial bending strains in the fundamental frequency of soil layer.

A. Analytical Solution of Kinematic Bending of Pile Group

The soil-pile-structure system is shown in Fig. 1: a group of vertical cylindrical piles of length L , diameter d , pile cross-sectional moment of inertia I_p , mass density ρ_p , mass per unit length of the piles m_p and Young's modulus E_p is embedded in a homogeneous soil layer of thickness $H(=L)$ resting on a rigid base. Soil is modelled as a linear elastic material of Poisson's ratio ν , mass density ρ_s , frequency-independent material damping β_s , expressed through a complex-valued shear modulus $G_s^* = G_s(1 + 2i\beta_s)$ and as a Winkler foundation resisting the lateral pile motion by continuously-distributed frequency-dependent linear springs k_x and dashpots c_x along the pile length. Based on the latter model, the energy losses due to radiation of waves and due to hysteretic dissipation. The pile group is excited by harmonic horizontal load at the head. The frequency of horizontal excitation is assumed to be equal to the natural frequency of the soil medium.

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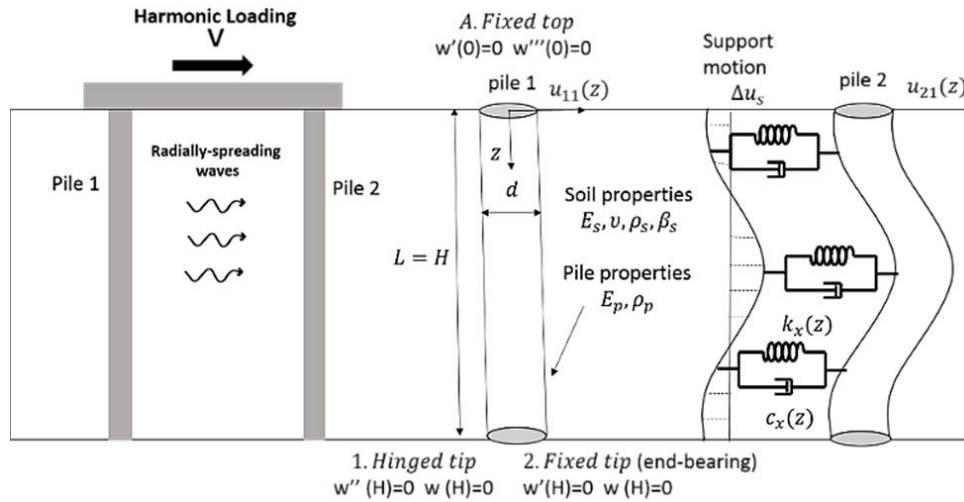


Fig. 1 Problem considered and different boundary conditions at pile head and tip

B. The Dynamic Winkler Modulus in Fundamental Frequency of Soil Layer

In this study, Winkler parameters of Mylonakis [10] which are efficient to capture the fundamental frequency effects in soil layer are utilized as:

$$k_x^*(z, \omega) = \pi G_s^* s^2 \frac{4K_1(q)K_1(s) + sK_1(q)K_0(s) + qK_0(q)K_1(s)}{qK_0(q)K_1(s) + sK_1(q)K_0(s) + qK_0(q)K_0(s)} \quad (1)$$

$$s = \frac{1}{2} \sqrt{a_c^2 - \frac{a_0^2}{1+2i\beta_s}} \quad (2a)$$

$$q = \frac{s}{\eta_u} \quad (2b)$$

$$\eta_u = \sqrt{\frac{2-\nu_s}{1-\nu_s}} \quad (3)$$

$$a_c = b_u d \quad (4a)$$

$$b_u = \frac{\int_0^H \left(\frac{d\chi(z)}{dz}\right)^2 dz}{\int_0^H (\chi(z))^2 dz} \quad (4b)$$

In (2a), $a_0 = \omega d / V_s$ denotes the dimensionless frequency factor. The a_c stands for a dimensionless fundamental frequency (termed “cutoff frequency”) below which no waves can be emanate from the pile-soil interface to propagate horizontally in soil medium. $k_x^*(z, \omega)$ in (1) constitutes the lateral local dynamic impedance of the Winkler foundation. $\chi(z)$ is the shape function to describe the lateral vibrations along the pile length. For simplicity, a sinusoidal shape function is selected, in which the cutoff frequency coincides with the fundamental natural frequency of homogenous soil layer in shearing vibrations.

$$\chi(z) = \cos\left(\frac{\pi z}{2H}\right) \quad (5a)$$

$$b_u = \frac{\pi^2}{4H^2} \quad (5b)$$

$$a_c = \frac{\pi r}{H} \quad (5c)$$

1. Deflection of Active Pile (Source Pile)

Let $u_{11}(z, t) = u_{11}(z)e^{i\omega_g t}$ denote the harmonic pile deflection. With reference to Fig. 1, dynamic equilibrium under harmonic steady-state conditions yields:

$$\frac{d^4 u_{11}(z)}{dz^4} + 4\lambda^4 u_{11}(z) = 0 \quad (6)$$

$$\lambda = \left(\frac{k_x + i c_x \omega_g - m_p \omega_g^2}{4E_p I_p}\right)^{1/4} \quad (7)$$

where λ is the characteristics wave number governing the attenuation functions of pile displacement with depth. The solution will yield harmonic horizontal deflection of the active pile $u_{11}^I(z, t) = u_{11}^I(z)e^{i\omega_g t}$ in terms of inertial integration constants $A_{11}^I, B_{11}^I, C_{11}^I, D_{11}^I$ which are dependent on the boundary conditions.

$$u_{11}^I(z) = e^{\lambda z} (A_{11}^I \cos(\lambda z) + B_{11}^I \sin(\lambda z)) + e^{-\lambda z} (C_{11}^I \cos(\lambda z) + D_{11}^I \sin(\lambda z)) \quad (8)$$

2. Attenuation of Soil Displacement Away from Active Pile (Source Pile)

This step starts by calculating the difference between single pile deflections and free-field soil displacements, Δu_{11} . In inertial loading, this difference is equal to the deflection of the active single pile: $\Delta u_{11} = u_{11}$, new cylindrical waves emanate from the periphery of the vibrating active pile while spreading outward in all directions. In this study, attenuation functions of Mylonakis [10] are used.

$$\psi_{21}(s, 0) = \left(\frac{2s}{d}\right)^{-1/2} \exp\left[-\left(\frac{s}{d} - \frac{1}{2}\right) \sqrt{a_c^2 - \frac{a_{g0}^2}{1+2i\beta_s}}\right] \quad (9)$$

$$\psi_{21}\left(s, \frac{\pi}{2}\right) = \left(\frac{2s}{d}\right)^{-1/2} \exp\left[-\left(\frac{s}{d} - \frac{1}{2}\right) \eta_u^{-1} \sqrt{a_c^2 - \frac{a_{g0}^2}{1+2i\beta_s}}\right] \quad (10)$$

$$\psi_{21}(s, \theta) = \psi_{21}(s, 0) \cos^2(\theta) + \psi_{21}\left(s, \frac{\pi}{2}\right) \sin^2(\theta) \quad (11)$$

According to previous model, at a distance s from the vibrating pile and angle θ from the direction of loading in Fig. 2, the displacement field can be expressed as:

$$u_s(s, z, \theta) = \psi_{21}(s, \theta) \Delta u_{11} = \psi_{21}(s, \theta) u'_{11}(z) \quad (12)$$

where $u_s(s, z, \theta)$ = horizontal soil displacement generated by active pile (source pile); $\psi_{21}(s, 0)$ and $\psi_{21}\left(s, \frac{\pi}{2}\right)$ = attenuation functions corresponding to wave travelling along and perpendicular to the direction of loading; θ = angle between the direction of loading and the line connecting the pile centers; $a_{g0} = \pi r/H$ is dimensionless frequency associated with the fundamental frequency of the soil layer.

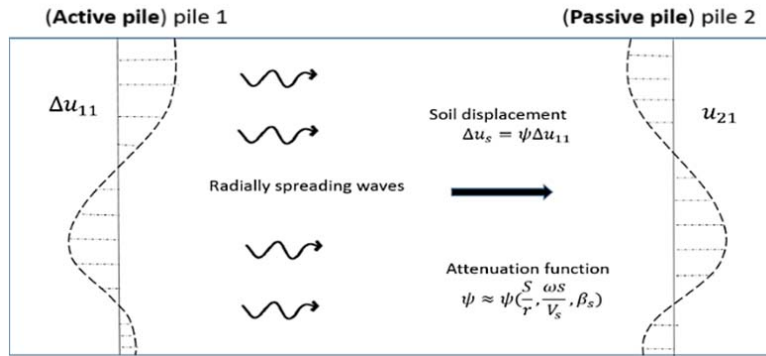


Fig. 2 Schematic illustration for computing influence of active pile on adjacent passive pile

3. Interaction of the Passive Pile (Receiver) with Arriving Waves

The diffracted wave field generated by the active pile in (8) propagates to strike the passive pile at a distance s from the active pile. The passive pile does not exactly follow the diffracted wave field, and its inertial and flexural rigidity tends to resist this induced displacement. Therefore, the result of this strike will be a modified motion at soil-passive pile interface. In order to determine the additional displacement that passive pile experiences, we consider the dynamic equilibrium of an infinitesimal pile segment which yields the following equation governing the deflection $u_{21}(z)$ of the passive pile.

$$\frac{d^4 u_{21}(z)}{dz^4} + 4\lambda^4 u_{21}(z) = \frac{k_x + i\omega_g c_x}{E_p I_p} \psi_{21}(s, \theta) u'_{11}(z) \quad (13)$$

when the active pile is excited by the lateral harmonic loading $u_0(z, t) = u_0 e^{i\omega_g t}$ at the head, the solution of (13) will give us the additional inertial displacement of the passive pile. This displacement consists of two parts; $(u'_{21}(z))_1$ as homogeneous solution and $(u'_{21}(z))_2$ as particular solution.

$$u'_{21}(z) = (u'_{21}(z))_1 + (u'_{21}(z))_2 \quad (14)$$

$$(u'_{21}(z))_1 = e^{\lambda z} (A_{21}^I \cos(\lambda z) + B_{21}^I \sin(\lambda z)) + e^{-\lambda z} (C_{21}^I \cos(\lambda z) + D_{21}^I \sin(\lambda z)) \quad (15)$$

$$(u'_{21}(z))_2 = \frac{k_x + i c_x \omega_g}{16 E_p I_p \lambda^3} \psi_{21}(s, \theta) [z e^{\lambda z} (A' \cos(\lambda z) + B' \sin(\lambda z)) + z e^{-\lambda z} (C' \cos(\lambda z) + D' \sin(\lambda z))] \quad (16)$$

$$A' = -(A_{11}^I + B_{11}^I) \quad (17a)$$

$$B' = (A_{11}^I - B_{11}^I) \quad (17b)$$

$$C' = (C_{11}^I - D_{11}^I) \quad (17c)$$

$$D' = (C_{11}^I + D_{11}^I) \quad (17d)$$

In particular solution, (16), A', B', C' and D' are integration constants in which $A_{11}^I, B_{11}^I, C_{11}^I$ and D_{11}^I are known inertial integration constants (i.e. they have already been determined from the boundary conditions of the active pile). In homogeneous solution $A_{21}^I, B_{21}^I, C_{21}^I$, and D_{21}^I are inertial integration constants that should be determined from the boundary conditions of the passive pile.

4. Inertial Interaction Factor

The inertial interaction factor $\alpha^I_{21}(s, \theta)$ between the active pile (pile 1) and the passive pile (pile 2) is defined as the response of the degree of freedom (DOF) i at pile 2 due to displacement (or rotation) of the j th DOF of pile 1 caused by its own load. In this study, pile cap group is assumed to be rotationally fixed, therefore inertial interaction factor between active pile (pile 1) and passive pile (pile 2) can be simplified as:

$$\alpha^I_{21}(s, \theta) = \frac{u'_{21}(0)}{u'_{11}(0)} = \frac{A'_{21} + B'_{21}}{A'_{11} + B'_{11}} \quad (18)$$

C. Inertial Response of Pile-Soil Systems

A pile group with identical N piles was considered to be connected by a rigid cap restricted against rotation and subjected to lateral vibration $u(t) = U^{(G)} e^{i\omega_g t}$ at the head of pile group. The total horizontal response of N pile at the head may be calculated as the sum of the following components: (1) The horizontal displacement at the head of single (solitary) pile due to its own head loading with the amplitudes P_1, \dots, P_N ; (2) The additional horizontal displacement at the head of the pile is transmitted from the other $N-1$ piles due to

their head-loading with the amplitudes $P_1 \dots, P_N$. When the horizontal head displacement of the foundation is expressed by $U^{(G)}$, the compatibility condition can be described by

$$U^{(G)} = \sum_{j=1}^N \alpha_{ij}^I \frac{P_j}{k_x^{(1)}} \quad (19)$$

$$\sum_{j=1}^N P_j = V \quad (20)$$

This system of equations can be set into a matrix form as

$$\begin{bmatrix} 1 & -\alpha_{11}^I & -\alpha_{12}^I & \dots & -\alpha_{1N}^I \\ 1 & -\alpha_{21}^I & -\alpha_{22}^I & \dots & -\alpha_{2N}^I \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & -\alpha_{N1}^I & -\alpha_{N2}^I & \dots & -\alpha_{NN}^I \\ 0 & 1 & 1 & \dots & 1 \end{bmatrix} \begin{Bmatrix} \frac{P_1}{k_x^{(1)}} \\ \frac{P_2}{k_x^{(1)}} \\ \vdots \\ \frac{P_N}{k_x^{(1)}} \\ \frac{V}{k_x^{(1)}} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \frac{V}{k_x^{(1)}} \end{Bmatrix} \quad (21)$$

where α_{ij}^I are the interaction factors for inertial loading in the case where $i = j$ $\alpha_{ij}^I = 1$ and $k_x^{(1)}$ is the dynamic stiffness at the head of single pile.

1. Inertial Curvature Ratios

(1) Inertial curvature ratio of the active pile is defined as the ratio of the active pile head curvature as a single solitary pile due to its own inertial head loading to the active pile-top displacement [11]; (2) Inertial curvature ratio of the passive pile is defined as the ratio of the passive pile head curvature due to the additional inertial head displacement of the passive pile to the active pile-top displacement due to the inertial head loading [11];

$$\beta_{11}^I = \frac{u_{11}''(0)}{u_{11}^I(0)} = \frac{2\lambda^2(B_{11}^I - D_{11}^I)}{A_{11}^I + C_{11}^I} \quad (22)$$

$$\beta_{21}^I = \frac{u_{21}''(0)}{u_{21}^I(0)} = \frac{\frac{2\lambda^2}{3}(3(B_{21}^I - D_{21}^I) - 2\alpha_{21}(B_{11}^I - D_{11}^I))}{A_{11}^I + C_{11}^I} \quad (23)$$

Finally by using superposition method, the total curvature can be expressed as

$$U_i''(0) = \sum_{j=1}^N \beta_{ij}^I \frac{P_j}{k_x^{(1)}} \quad (24)$$

$$\begin{Bmatrix} U_1''(0) \\ U_2''(0) \\ \vdots \\ U_N''(0) \end{Bmatrix} = \begin{bmatrix} \beta_{11}^I & \beta_{12}^I & \dots & \beta_{1N}^I \\ \beta_{21}^I & \beta_{22}^I & \dots & \beta_{2N}^I \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{N1}^I & \beta_{N2}^I & \dots & \beta_{NN}^I \end{bmatrix} \begin{Bmatrix} \frac{P_1}{k_x^{(1)}} \\ \frac{P_2}{k_x^{(1)}} \\ \vdots \\ \frac{P_N}{k_x^{(1)}} \end{Bmatrix} \quad (25)$$

Based on (21), vector of forces $\left\{ \frac{P_i}{k_x^{(1)}} \right\}$ can be obtained as

$$\left\{ \frac{P_i}{k_x^{(1)}} \right\} = \{\gamma_j\} \frac{V}{k_x^{(1)}} \quad (26)$$

Vector $\{\gamma_j\}$ is displacement group factor which can be obtained after solving (21), $k_x^{(1)}$ is the horizontal stiffness at

the head of hinged and end-bearing pile respectively and details on that are given in [12]. By replacing vector forces in (25), bending strains can be calculated therefore bending moments at the head of each pile in the group in vector form can be calculated as following expression

$$\{\varepsilon_{pi}^I(0)\} = \left\{ -\frac{d}{2} \frac{d^2 U_i(z=0)}{dz^2} \right\} \quad (27)$$

Next, the inertial bending strain ε_{pi}^I at the head of the pile is normalized with respect to a mean shear strain of the soil medium γ_s as the same treatment in Saitoh [8]. Therefore, the closed form formula of the normalized bending strains can be written as follows

$$\left\{ \frac{\varepsilon_{pi}^I(0)}{\gamma_s} \right\} = [\beta_{ij}^I] \{\gamma_j\} \frac{fr}{\frac{\pi}{8} \gamma_s^3 (\lambda H)^3 F(\lambda H)} \quad (28)$$

$$F(\lambda H) = \begin{cases} 4 \frac{\cos(2\lambda H) + \cosh(2\lambda H)}{-\sin(2\lambda H) + \sinh(2\lambda H)} & \text{hinged} \\ 4 \frac{\sin(2\lambda H) + \sinh(2\lambda H)}{\cos(2\lambda H) + \cosh(2\lambda H) - 2} & \text{end - bearing} \end{cases} \quad (29)$$

Factor fr is a dimensionless factor which is related to the effect of the lateral load relative to the deformation of the soil layer. This factor is a complex value since there is a phase lag between lateral load V , and the mean shear strain of the soil medium γ_s , therefore this factor can be rewritten again by the following formula:

$$fr = Fr e^{i\varphi_r} \quad (30)$$

$$Fr = \left| \frac{V}{E_p \gamma_s H^2} \right| \quad (31)$$

The factor Fr can be calculated by estimating the maximum values of the lateral load V and the mean shear strain γ_s . The determination of phase lag φ_r is difficult because there have been few investigations into the phase lag between the lateral load V and mean shear strain γ_s . In this study, it is assumed that the phase lag is equal to zero.

II. NUMERICAL RESULTS

In order to examine the applicability of the preceding expressions for the normalized inertial bending strains, two cases of fixed head piles (2x2, 3x3) will be studied. Here, special attention is paid to the effects of slenderness ratio r/H , spacing and the number of piles in the group, different boundary conditions at the tip (fixed or hinged), and the results are compared with the single pile's result. The normalized inertial bending strains in single are also divided by N the number of piles in each case of pile groups to be comparable with pile groups' results. Results of parametric studies indicate that, in both cases of end-bearing and hinged pile groups, normalized inertial bending strains show almost the same values as single pile's results.

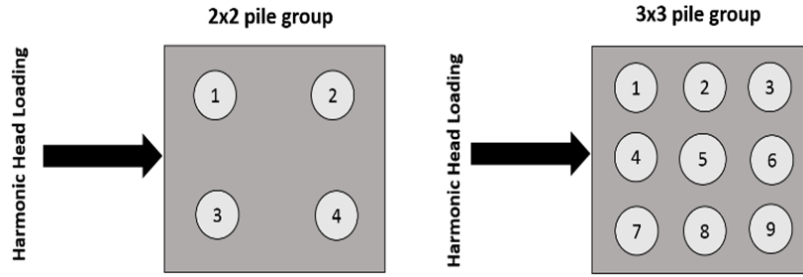


Fig. 3 (2x2) and (3x3) pile groups under study

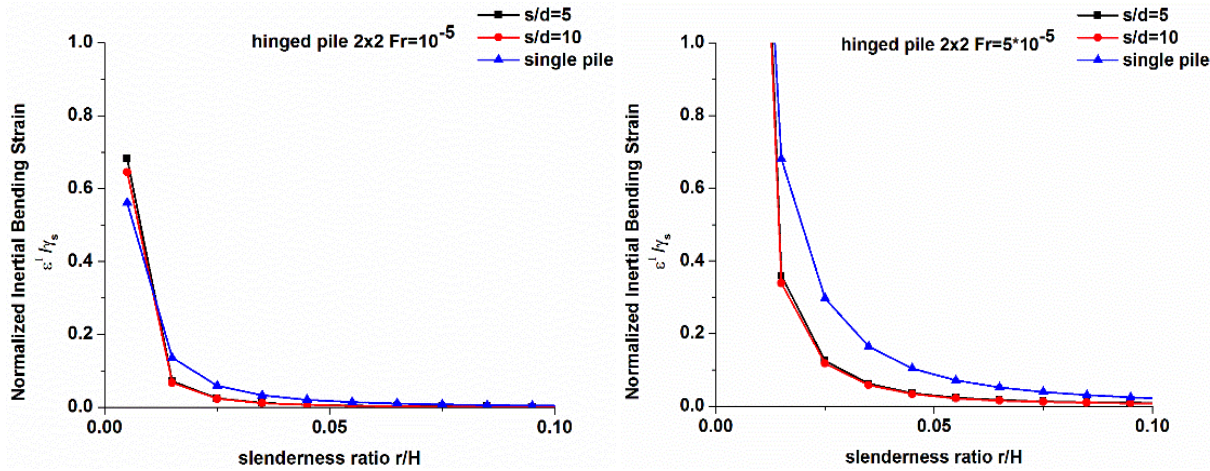


Fig. 4 Normalized inertial bending strain of (2x2) hinged pile group with respect to single pile ($\frac{\rho_p}{\rho_s} = 1.43, \nu = 0.4, \beta = 0.05, \frac{E_p}{E_s} = 1000$)

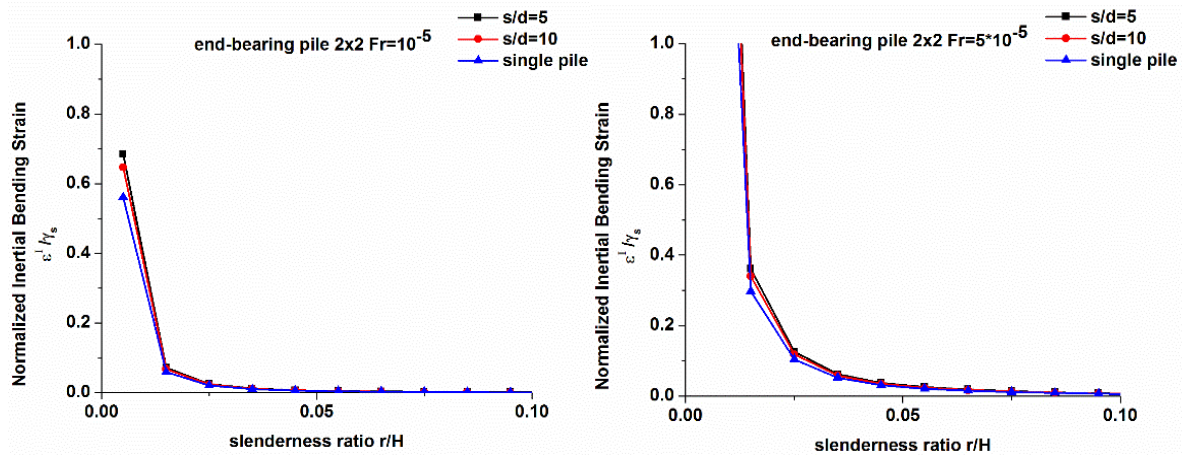


Fig. 5 Normalized inertial bending strain of (2x2) end-bearing pile group with respect to single pile ($\frac{\rho_p}{\rho_s} = 1.43, \nu = 0.4, \beta = 0.05, \frac{E_p}{E_s} = 1000$)

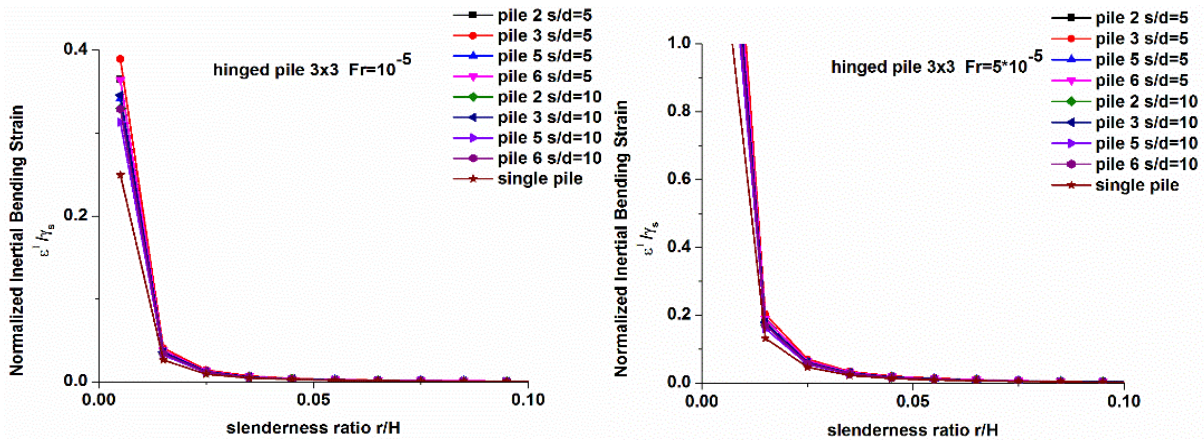


Fig. 6 Normalized inertial bending strain of (3x3) hinged and end-bearing pile group with respect to single pile ($\frac{\rho_p}{\rho_s} = 1.43$, $\nu = 0.4$, $\beta = 0.05$, $\frac{E_p}{E_s} = 1000$)

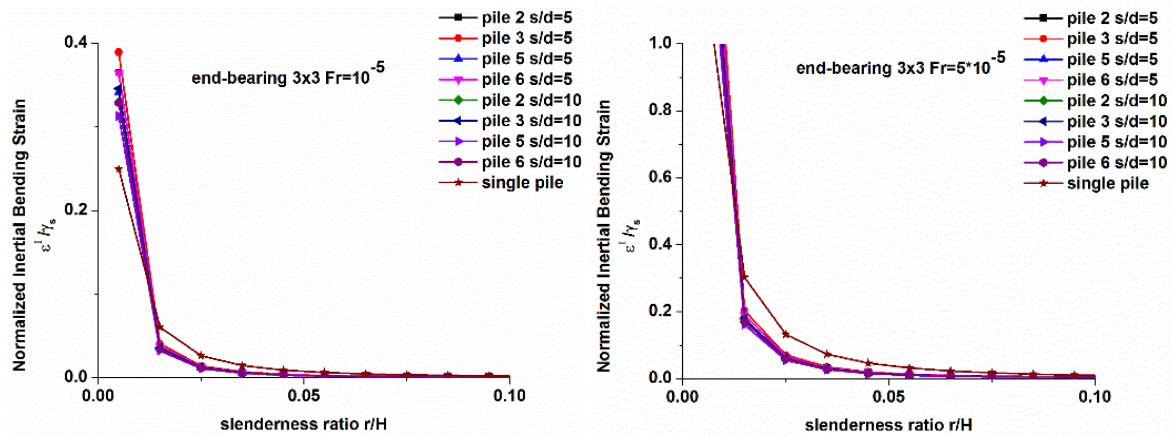


Fig. 7 Normalized inertial bending strain of (3x3) hinged and end-bearing pile group with respect to single pile ($\frac{\rho_p}{\rho_s} = 1.43$, $\nu = 0.4$, $\beta = 0.05$, $\frac{E_p}{E_s} = 1000$)

III. CONCLUSION

An efficient method has been extended to compute the bending strains of fixed-head pile groups of finite length embedded in a homogeneous soil layer, where inertial interaction dominates. This method allows the inertial bending strains to be obtained in closed form formula while using dynamic Winkler model in conjunction with an extension to three dimensional of Novak's plain-strain model. This model is free of the drawbacks of the two dimensional plain-strain model reproducing cutoff frequency of the soil-pile system. Pile group effect is considered through interaction factors, and the inertial bending strains are normalized with respect to a mean shear strain of a soil stratum γ_s , then the variations of normalized inertial bending strains against slenderness ratio r/H are investigated, which gives valuable insight into the characteristics of the inertial bending strains in pile groups. Homogeneous solutions are considered in active and passive piles deflections for appropriately considering various boundary conditions when estimating bending strains.

Solutions for pile group response are performed in fundamental frequency of soil strata.

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