

# Optimal Design of Composite Patch for a Cracked Pipe by Utilizing Genetic Algorithm and Finite Element Method

Mahdi Fakoor, Seyed Mohammad Navid Ghoreishi

**Abstract**—Composite patching is a common way for reinforcing the cracked pipes and cylinders. The effects of composite patch reinforcement on fracture parameters of a cracked pipe depend on a variety of parameters such as number of layers, angle, thickness, and material of each layer. Therefore, stacking sequence optimization of composite patch becomes crucial for the applications of cracked pipes. In this study, in order to obtain the optimal stacking sequence for a composite patch that has minimum weight and maximum resistance in propagation of cracks, a coupled Multi-Objective Genetic Algorithm (MOGA) and Finite Element Method (FEM) process is proposed. This optimization process has done for longitudinal and transverse semi-elliptical cracks and optimal stacking sequences and Pareto's front for each kind of cracks are presented. The proposed algorithm is validated against collected results from the existing literature.

**Keywords**—Multi objective optimization, Pareto front, composite patch, cracked pipe.

## I. INTRODUCTION

LONGITUDINALY seam welded pipes are frequently used in the oil and gas industries. One of the main concerns in designing and maintaining of these pipes is initiation and propagation of cracks which caused substantial decreases in strength. In the most common cases, crack initiation is from surface imperfections of welding line. These surface flaws are a fountain of infinitesimal semi-elliptical crack initiation that is partly growth and joined together to become larger semi-elliptical cracks. Composite patching is a common way for reinforcing these structures. Composite patching methods for repairing of structures initially were investigated in early 1970s by Baker and Jones [1]. They have been discussed many advantages of using this method for the repair of cracked and damaged metallic structures. Jones and Callinan [2] were the pioneers of using finite element method for modeling composite patch. Later on, extensive studies have been performed to develop this repairing method by various experimental and numerical approaches. In this regards, Bachir et al. [3] and Ayatollahi and Hashemi [4] have been studied the effect of composite patching on repairing of cracked structures by using Finite Element Method (FEM).

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Stacking sequence optimization of composite patch is an important issue in the applications of cracked pipes because the effects of composite patch reinforcement on stress intensity factor of a cracked pipe depends on a variety of parameters such as number of layers, angle, thickness and material of each layer. Over the past few years, stacking sequence optimization of composite material has been studied by many researchers. In this regards, Adali and Verijenko [5] have designed optimal stacking sequence of a symmetric hybrid laminates undergoing free vibrations. Chakraborty and Dutta [6] have worked on optimization of fiber-reinforced polymer (FRP) composites against impact induced failure using island model parallel genetic algorithm (GA). Todoroki and Ishikawa [7] studied design of experiments for stacking sequence optimizations with GA using response surface approximation.

Regarding the literature review, up to the present, a few papers have coupled Multi-Objective GA (MOGA) and FEM for stacking sequences optimization of composite patch in cracked pipes. In this paper, a coupled MOGA and FEM process is proposed to obtain the optimal stacking sequence for a composite patch, where its weight and stress intensity factor are minimized simultaneously. Unlike estimation methods like response surface, in the proposed optimization algorithm, objective functions for each individual are evaluated directly by FEM software which leads to precise results. Also, the results of the proposed algorithm are validated against collected results from the existing literature.

## II. FINITE ELEMENT MODELING OF CRACKED PIPE CONTAINING 3D SEMI-ELLIPTICAL CRACKS AND COMPOSITE PATCH

Generally, the surface cracks could be assumed as a semi elliptical crack. Because Lin and Smith [8] proved that any kind of cracks after a few propagations transforms to a semi-elliptical crack. Thus, in this section, several finite element models of cracked pipes containing 3D semi-elliptical cracks and composite patches were modeled in ABAQUS software. For modeling of the longitudinal and transverse semi-elliptical cracks in the pipe, we consider the length of crack along the major axis of ellipse and the depth of crack in accordance with ellipse minor axis. Modeled longitudinal and transverse semi-elliptical cracks have the length and depth of 10 and 4 mm, respectively. The created 3D models of cracked pipes were meshed using a total number of 85,496 solid C3D20 elements in the ABAQUS code. Also, composite patch was meshed using a total number of 2,264 shell S4R elements in the

ABAQUS code. Fig. 1 shows typical 3D mesh pattern generated for the cracked pipe and a zoomed view of the crack tip region. In order to model the singularity of stress and strain components of the crack face, a special element called a singular element was considered. This was performed by moving the first ring mid-side nodes around the crack front to the quarter distance near the crack front nodes.

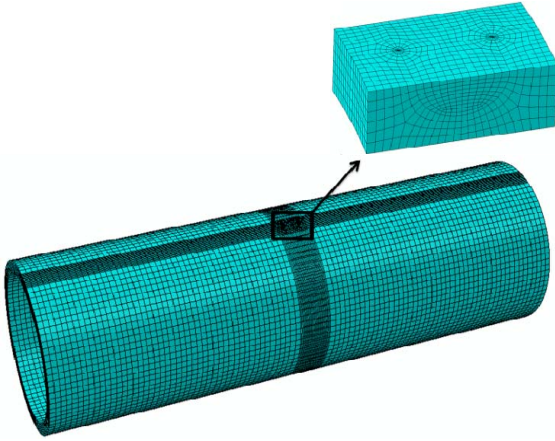


Fig. 1 3D mesh pattern generated for the cracked pipe and a zoomed view of the crack tip region

### III. CALCULATION OF STRESS INTENSITY FACTORS (SIFs)

As it is well known, stress intensity factors are the basic parameters for investigation of fracture behavior in cracked structures. These coefficients in cracked pipes (in various modes of loading) are function of load, crack length, crack depth, and geometry (radius and thickness of the pipes) and it could be expressed as (1) [9]:

$$\begin{aligned} K_I &= Y_I \left( \frac{a}{c}, \frac{a}{R}, t \right) \times \sigma \sqrt{\pi a} \\ K_{II} &= Y_{II} \left( \frac{a}{c}, \frac{a}{R}, t \right) \times \sigma \sqrt{\pi a} \\ K_{III} &= Y_{III} \left( \frac{a}{c}, \frac{a}{R}, t \right) \times \sigma \sqrt{\pi a} \end{aligned} \quad (1)$$

where  $Y_I$ ,  $Y_{II}$ , and  $Y_{III}$  are the geometrical coefficients in modes I, II and III, respectively. Cracked pipes are commonly encountered to the emergence of the only mode I stress intensity factor. In particular cases, whenever the geometry condition and loading is not elaborate, stress intensity factors could be evaluated by analytical methods, but in most cases, that problem is more complicated (such as three-dimensional cracks in pipes), and numerical methods (such as FEM method) are picked out. In the FEM method, some methods such as virtual crack growth [10] and  $J$  integral [11] are available for computing the stress intensity factors. The  $J$  integral on crack tip  $r$  contour is defined as (2) [12]:

$$J = \int_{\Gamma} \left( W dy - T_i \frac{\partial u_i}{\partial x} ds \right) \quad (2)$$

Due to the tiresome method of  $J$  integral evaluation, (2) converted to the surface integral as follows:

$$J = \int_A \left[ \left( \sigma_{ij} \frac{\partial u_i}{\partial x_1} - W \delta_{1j} \right) \frac{\partial q}{\partial x_j} \right] dA \quad (3)$$

where,  $A$  and  $q$  are surface between two contours (including crack) and weight function, respectively. Also, the weight function on inner contour and outer is equal to either 1 or 0. In addition,  $W$  in (3) represents the strain energy and is calculated as (4):

$$W = \int \sigma_{ij} d\epsilon_{ij} = \frac{1}{2} \sigma_{ij} \epsilon_{ij} \quad (4)$$

In this paper, a  $J$  integral based method built in ABAQUS was used for obtaining SIF directly from a static analysis of cracked pipe. Fig. 2 represents the crack face and the points on the crack face that the stress intensity factors are derived from. For simplification in determining of the points on crack face, at first,  $x$  coordinates are defined and then points on the crack face prescribed by a dimensionless distance of  $x/c$ .

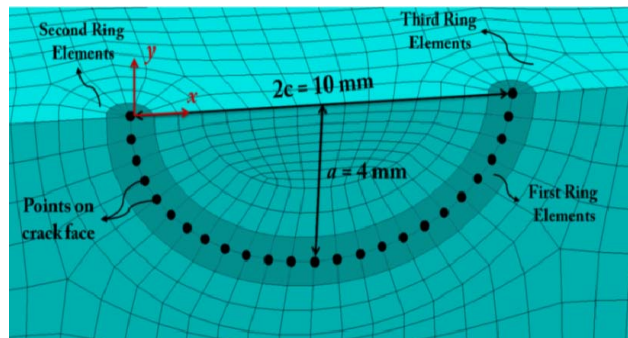


Fig. 2 A crack face and the points on the crack face for a semi-elliptical crack with aspect ratio 0.8

### IV. DESCRIPTION OF THE OPTIMIZATION ALGORITHM

GA is iterative search procedures for optimization of an objective function based on simulation of Darwin's theory of species evolution which describes the mechanics of natural genetics and natural selection [13]. GA is computationally simple, but powerful in their search for improvement and even does not need problem-specific properties, such as the derivatives, in order to complete a search. However, GA does not present an absolute result but provides a set of answers that the designer could select one or some of them according to the problem's criteria.

A GA uses four elements [14]: cross-over, mutation, a stopping criterion, and selection. Crossover is a reproduction method that mixes genes of two chromosomes to produce new chromosomes. Mutation modified slightly some genes of

selected chromosomes to cover diversity among the population, for finding global optimum of the function [15]. The application of GA also needs the generation of a population of candidate solutions as a starting point which evolves iteratively new, gradually better populations from the previous ones until the stopping criterion is satisfied. To do this, each chromosome of a population should be evaluated and compared with the others to achieve a better value of the

objective function. New population is developed from previous one by using cross-over and mutation operators. The individuals who lead to a better objective function have a greater chance of being selected to pass their genes to next population. There are several stopping criteria that have been checked at this step, and the optimization process will converge if one of them is satisfied otherwise the algorithm will start from evaluation new genes.

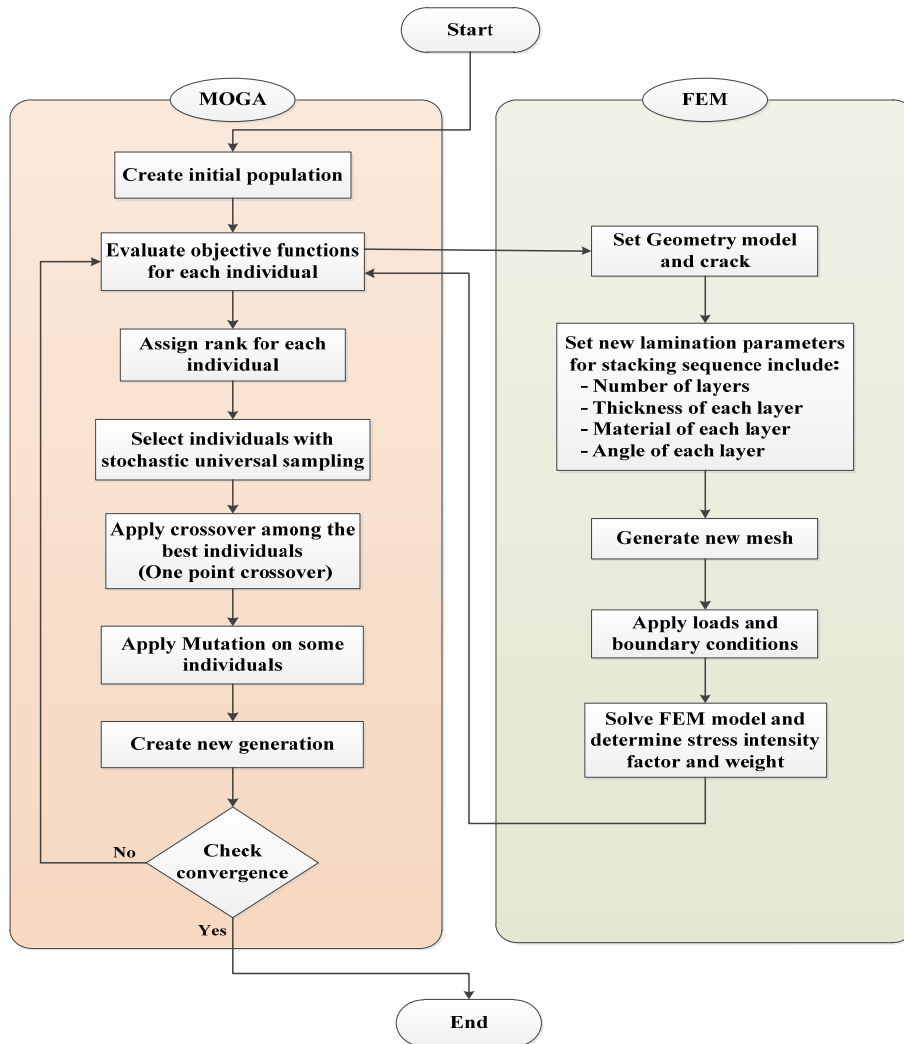


Fig. 3 Flowchart of the coupled MOGA and FEM

In this paper, MOGA and FEM are coupled to obtain the optimum solutions of defined objective functions. It means that, in MOGA process, objective functions are calculated directly by FEM in each evaluation. Completing the process, MOGA creates new individuals which are new lamination parameters for stacking sequence including number of layers, thickness, material and angle of each layer and it transfers them to the FEM software. In the FEM software, geometric modeling, meshing and applying loads and boundary conditions are generated automatically, and the results of the FEM software include maximum stress intensity factor and

weight which are retransferred to the MOGA process. This process continues till convergence criteria is satisfied. Fig. 3 illustrates flowchart of the coupled MOGA and FEM. Unlike estimation methods like response surface and simple analytic methods, in the proposed optimization algorithm, objective functions for each individual are evaluated directly by FEM software which leads to precise results.

The goals of the optimization are minimizing the stress intensity factor and weight simultaneously. The design independent variables for optimization are number of layers, thickness, material and angle of each layer. Allowable bounds

of each parameter are presented in Table I. The mechanical properties of used materials are inserted in Table II. The MOGA parameters for optimization are initial populations of 100, elitist choice of 3%, Mutation probability of 2%, crossover probability of 50%, and convergence Pareto percentage of 99%.

TABLE I  
ALLOWABLE BOUNDS OF PARAMETERS IN OPTIMIZATION

Parameter	Lower bound	Upper bound	Step	Number of possible variables
Number of layers	1	6	1	6
Thickness of layers	0.1 mm	0.25 mm	0.05 mm	4
Material	-	-	-	3
Angle of layers	-90 deg.	+90 deg.	15 deg.	13

TABLE II  
MECHANICAL PROPERTIES OF USED MATERIALS

Material	$E_1$ (GPa)	$E_2$ (GPa)	$G_{12}$ (GPa)	$\nu_{12}$	$\rho$ (kg/m <sup>3</sup> )
Carbon/Epoxy	121	8.6	4.7	0.27	1490
Kevlar/Epoxy	80	5.5	2.2	0.34	1400
Glass/Epoxy	45	10	5	0.3	2000

## V. RESULTS AND DISCUSSION

The coupled MOGA and FEM that are described in the previous sections were applied to perform the optimization of stacking sequence for a composite patch in the cracked pipe which is shown in Fig. 1. The optimization has done for longitudinal and transverse semi-elliptical cracks. Maximum stress intensity factors in the crack front versus the number of evaluations for longitudinal and transverse semi-elliptical cracks have been shown in Figs. 4 and 5, respectively.

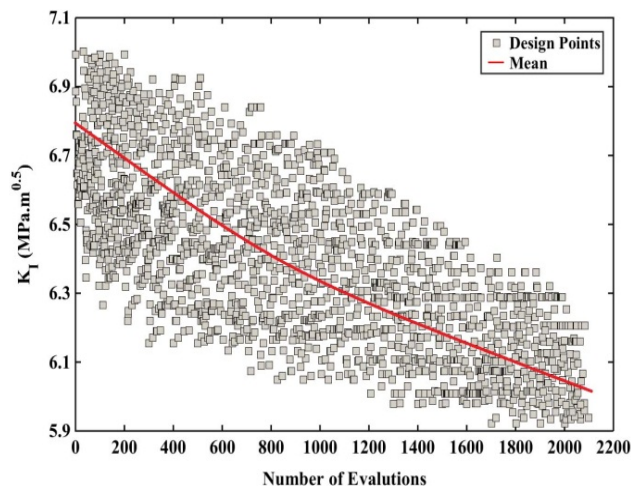


Fig. 4 Maximum stress intensity factor in the crack front versus number of evaluations for longitudinal semi-elliptical cracks

The maximum stress intensity factors in the crack front vs. weight of composite patch (Pareto's front curves) that are calculated in optimization process for longitudinal and transverse semi-elliptical cracks have been shown in Figs. 8 and 9, respectively. The figures show entire of design space in

which optimization could search. In the figures, points divide the space into two feasible and infeasible parts. It means that, in a certain weight, there is no possibility to obtain a lower stress intensity factor than what it is found in the optimal points.

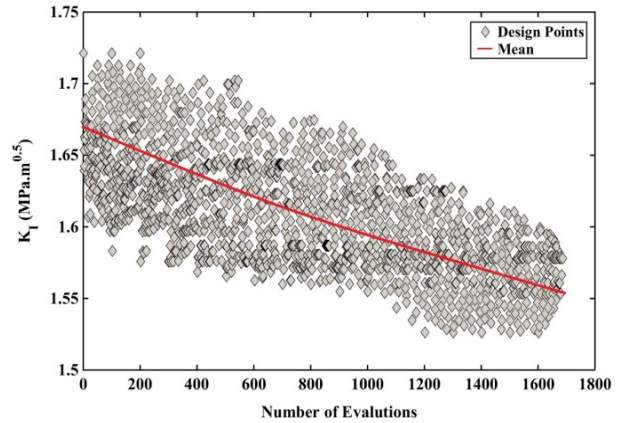


Fig. 5 Maximum stress intensity factor in the crack front versus number of evaluations for transverse semi-elliptical cracks

Weights of designed composite patch versus the number of evaluations for longitudinal and transverse semi-elliptical cracks have been shown in Figs. 6 and 7, respectively.

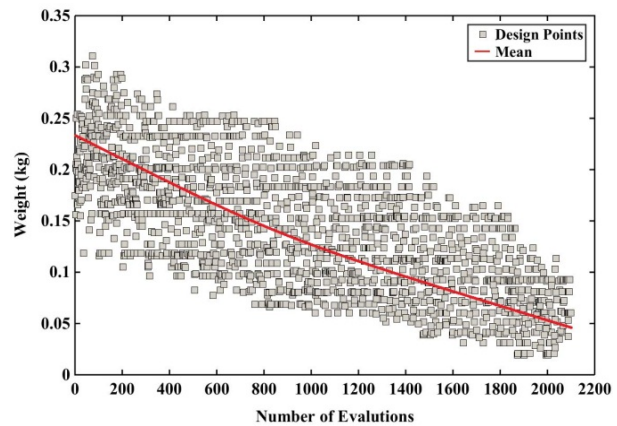


Fig. 6 Weight of designed composite patch versus number of evaluations for longitudinal semi-elliptical cracks

By analyzing Pareto's front, some decisions could be taken. The first decision is obtained by considering the highest point in Pareto's front. In this point, the highest value of stress intensity factor and lowest value of weight are obtained. The lamination parameters for stacking sequence that correspond to this point are appropriate in some applications where weight is the most important goal. In these applications, resistance to propagation of cracks is not an important target. On the lowest point of the Pareto's front, the lowest value of stress intensity factor is reached. Just as it was expected, the highest value in weight is achieved in this point too. The lamination parameters for stacking sequence that correspond to this point, are appropriate in some applications where resistance to



propagation of cracks is the most important aim. In these applications, weight is not an important target. All the other points in Pareto's front are intermediate cases. They must be used when a certain stress intensity factor and weight are needed.

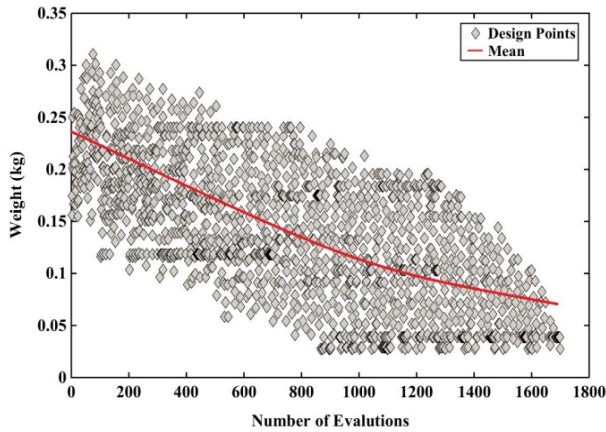


Fig. 7 Weight of designed composite patch versus number of evaluations for transverse semi-elliptical cracks

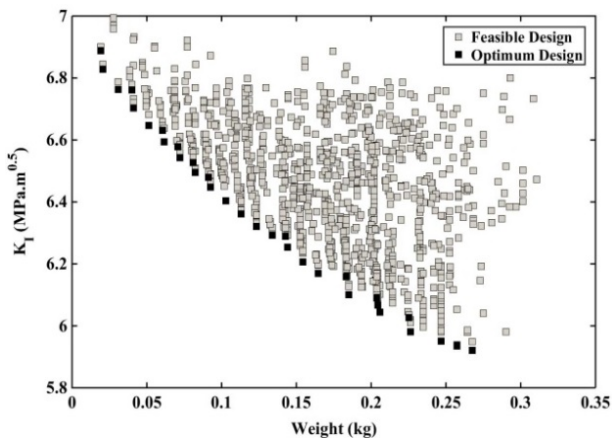


Fig. 8 Pareto's front for longitudinal semi-elliptical cracks

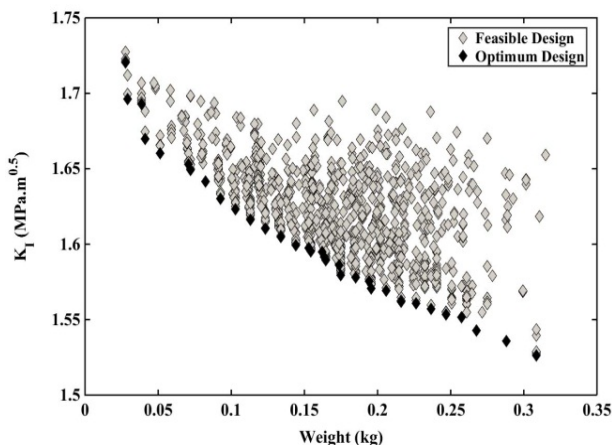


Fig. 9 Pareto's front for transverse semi-elliptical cracks

## VI. VERIFICATION OF THE PROPOSED ALGORITHM

To verify the proposed algorithm which is described in the previous section, the obtained Pareto's front by the proposed algorithm is compared with those presented by Shi et al. [16]. They employed a hybrid GA for optimal design of the advanced grid-stiffened (AGS) carbon-fiber triangular grid conical shells under external pressure. Comparison of the results between the proposed algorithm and [16] is illustrated in Fig. 10. The obtained Pareto's front with those presented by Shi et al. [16] showed to be in a good agreement with each other and have negligible differences in most cases, which can conclude the accuracy of the proposed algorithm in predicting the optimal solutions.

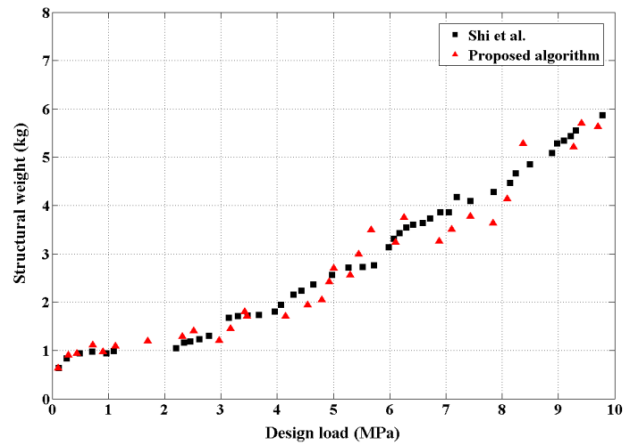


Fig. 10 Comparison between obtained Pareto's front by proposed algorithm and Shi et al. [16]

## VII. CONCLUSION

The present study proposed a coupled MOGA and FEM process for stacking sequence optimization of composite patches to minimizing weight and maximizing the resistance in propagation of cracks. The proposed algorithm was capable of obtaining a set of solutions, which are uniformly distributed, in order to arrange the Pareto's front at a low computational cost. Pareto's front graph helps designer to make good decision regarding all design's criteria. Unlike the estimation methods like response surface and simple analytic methods, in this algorithm, objective functions for each individual are evaluated directly by FEM software which leads to precise results. Therefore, the proposed algorithm provides a reliable and flexible tool to stacking sequence optimization of composite patches for the cracked pipes. The validity of the proposed algorithm has been studied by comparing with the results given by existing literature.

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