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Sampled-Data Model Predictive Tracking Control for Mobile Robot

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***Abstract*—In this paper, a sampled-data model predictive tracking control method is presented for mobile robots which is modeled as constrained continuous-time linear parameter varying (LPV) systems. The presented sampled-data predictive controller is designed by linear matrix inequality approach. Based on the input delay approach, a controller design condition is derived by constructing a new Lyapunov function. Finally, a numerical example is given to demonstrate the effectiveness of the presented method.

***Keywords*—Model predictive control, sampled-data control, linear parameter varying systems, LPV.

I. Introduction

MOBILE robots nowadays move autonomously by recognizing external environment and determining the situation through the remote control. With the development of network communication, implementation employing wireless & wired network is widespread [1]. Though control through network is advantageous in maintenance, installation, flexibility and cost, it has to be carefully designed in reality. It may cause instability and performance degradation without considering network induced delay or data packet losses. Therefore, the design of control scheme should consider with aspects and performances of whole systems.

Model predictive control (MPC) scheme is very useful since it provides good tracking performance and the MPC tuning parameters are explicitly related to the key characteristics safety, comfort, and fuel economy. But if the model is not accurate, the control technique does not guarantee the stability and performance [2]. Also, an important issue in the implementation of MPC algorithm is the discretization. A continuous-time model is much more natural and accurate in terms of describing the behavior of a system, Also, in network control systems, choosing proper sampling interval is very important for designing suitable controllers. It is clear that a longer sampling period will lead to lower communication channel occupation, few actuation of the controller, and less signal transmission. Thus, it is very important to consider the stabilizing control design problem under a bigger sampling period [5]. For sampled-data systems, the input delay approach has been widely used [4], which is based on the representation of the sampled-data system as a continuous-time system with a delayed control input. Then, the Lyapunov Krasovskii functional (LKF) method can be used to establish the stability conditions. Recently, based on the input delay approach, the sampled-data control problem of dynamical systems with time-varying delay has been investigated in [3], [4].

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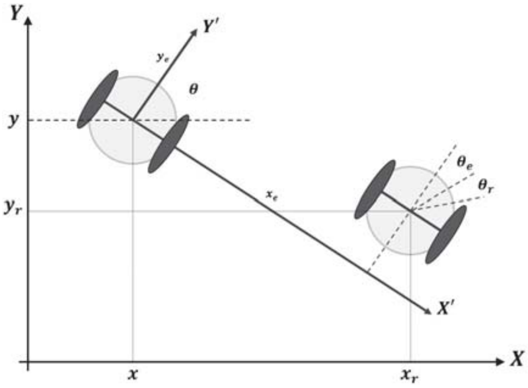

Fig. 1 Mobile robot in X-Y coordination

In this paper, we consider a continuous-time LPV model to handle mobile robot systems and present a model predictive control method for the systems with sampled-data. To the best of authors' knowledge, there are no approaches considering sampled-data MPC for mobile robots. The presented synthesis condition is formulated by construction of a suitable Lyapunov-Krasovskii's functional and control inputs are obtained by minimizing the upper bound of the cost function satisfying the cost monotonicity. Finally, we demonstrate the effectiveness of the proposed approach via numerical simulation.

II. Description of Mobile Robot

The dynamics of mobile robot with a rigid body and wheels can be described as follows [1]

$$\begin{cases} \dot{x} & = vcos(\theta(t)), \\ \dot{y} & = vsin(\theta(t)), \\ \dot{\theta} & = w(t) \end{cases} \tag{1}$$

where $[x, y, \theta]$ denotes the position and orientation of the center with respect to a global frame, v is the translational velocity, and w is the angular velocity.

For the given mobile robot, the reference trajectory is set to

$$\begin{cases} \dot{x}_r(t) & = v_r(t)cos(\theta_r(t)), \\ \dot{y}_r(t) & = v_r(t)cos(\theta_r(t)), \\ \dot{\theta}_r(t) & = \omega_r(t), \end{cases} \tag{2}$$

where x_r, y_r, θ_r are references in Cartesian coordination, v_r is the reference translational velocity, and ω_r is the reference angular velocity. Considering local coordinate frame, define

$$\begin{bmatrix} x_e(t) \\ y_e(t) \\ \theta_e(t) \end{bmatrix} \begin{bmatrix} cos(\theta)(t) & sin(\theta)(t) & 0 \\ -sin(\theta)(t) & cos(\theta)(t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r(t) - x(t) \\ y_r(t) - y(t) \\ \theta_e(t) - \theta(t) \end{bmatrix}. \quad (3)$$

From (1)-(3), the error dynamics is obtained as

$$\begin{cases} \dot{x}_e(t) &= \omega(t) y_e(t) + v_r cos(\theta_e(t)) - v(t), \\ \dot{y}_e(t) &= -\omega(t) x_e(t) + v_r(t) sin(\theta_e(t)), \\ \dot{\theta}_e(t) &= \omega_r(t) - \omega(t), \end{cases} \quad (4)$$

In general, systems represented by nonlinear systems can be transformed into Linear Parameter Varying (LPV) systems

$$\dot{X}(t) \quad = \quad A(\bar{v}(t), \bar{\omega}(t), v_r(t))X(t) + BU(t),$$

where $A(\cdot)$ is system matrices containing a time varying parameter vector $\bar{v}(t), \bar{\omega}(t), v_r(t)$, $X = [x_e, y_e, \theta_e] - [\bar{x}_e, \bar{y}_e, \bar{\theta}_e]$, and $U = [v - \bar{v}, \omega - \bar{\omega}]$. By computing Jacobian matrix, the system matrices are given as

$$A(\bar{v}(t), \bar{\omega}(t), v_r(t)) = \begin{bmatrix} 0 & \bar{\omega}(t) & 0 \\ -\bar{\omega}(t) & 0 & v_r(t) \\ 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix}.$$

For a given sampling rates, the matrix $A(\bar{v}(t), \bar{\omega}(t), v_r(t))$ is subject to a polytope set Ω.

$$A(\bar{v}(t), \bar{\omega}(t), v_r(t)) = \sum_{i=0}^{L} \lambda_i A_i, \quad A_i \in \Omega \quad (5)$$

where $\Omega = \{A_1, A_2, \ldots, A_L\}$ is the convex hull.

In the typical system architecture, control signals are conveyed through network communication. In network environments, the control signals pass through zero-order-hold (ZOH) which generate functions with a sequence of hold times $0 \leq t_0 < t_1 < \cdots < t_k \cdots < \lim_{k \to \infty} t_k = +\infty$. Taking consideration of ZOH, the control input is

$$U(t) = KX(t_k), \quad t \in [t_k, t_{k+1}). \quad (6)$$

where K is the control gain matrix. Without loss of generality, it is assumed that the sampled time interval is bounded by

$$h(t) \leq h_M$$

where $h(t) = t_{k+1} - t_k$, and h_M is the maximum sampled delay. Using sampled signals, the systems are expressed as delayed LPV systems

$$\dot{X}(t) = A_i X(t) + BU(t - h(t)). \quad (7)$$

Lemma 1. [5] For given matrices $\Lambda_1, \Lambda_2, \Psi$, and a scalar $0 \leq \tau(t) \leq \tau_M$, if the following conditions hold

$$\tau(t)\Lambda_1 + (\tau_M - \tau(t))\Lambda_2 + \Phi < 0, \quad (8)$$

then, it is equivalent to

$$\tau_M \Lambda_1 + \Phi < 0, \tau_M \Lambda_2 + \Phi < 0. \quad (9)$$

Lemma 2. [6] For given matrices $H, N, R > 0$ and a continuously differentiable function $x(t)$ in $[a, b] \in \mathbb{R}^n$, the following inequality is ensured.

$$- \int_a^b \dot{x}^T(\alpha) R \dot{x}(s) ds \leq \quad (10)$$

$$Sym\{\epsilon_1^T H \epsilon_2 + \epsilon_1^T N \epsilon_3\} \quad (11)$$

$$+ (b - a)\epsilon_1^T (\frac{3HR^{-1}H + NR^{-1}N}{3})\epsilon_1,$$

where ϵ_1 is any vector, $\epsilon_2 = x(b) - x(a)$, and $\epsilon_3 = x(b) + x(a) - \frac{2}{b-a}\int_a^b x(s)ds$.

III. Main Results

The main purpose of this paper is to design a proper sampled-data model predictive controller. Model Predictive Control is used to approximately obtain optimal trajectories. Therefore, choosing the following performance index is reasonable:

$$J = \int_0^{\infty} X^T(t) Q X(t) + U^T(t) R U(t) dt \quad (12)$$

where Q, R are coefficients. For the given performance index, if the following condition is satisfied

$$\dot{V}(t) + \| X(t) \|_Q^2 + \| U(t) \|_R^2 < 0. \quad (13)$$

where $\| \cdot \|$ denotes 2-norm, then the upper bound of the performance index can be derived instead of directly minimizing performance index. By integrating (13) from $i = 1$ to $i = \infty$, one can notice the upper bound of the performance index is less than the Lyapunov function.

Before presenting main results, we employed the following representations for simplicity. The matrices $e_i = R^{4n \times n}$ for $i = 1, 2, \ldots, 4$ are matrices composed of nth zero elements with ith identity matrix. (For example, $e_1 = [I \quad 0 \quad 0 \quad 0]$ and $e_3 = [0 \quad 0 \quad I \quad 0]$).

$$\begin{aligned} D_1 &= \begin{bmatrix} I & -I & 0 & 0 \end{bmatrix}, \\ D_2 &= \begin{bmatrix} I & I & -2I & 0 \end{bmatrix}, \\ D_3 &= \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}, \\ \bar{E}_i &= \begin{bmatrix} A_i G & BY & 0 - G \end{bmatrix}, \\ E_i &= \begin{bmatrix} A_i & BK & 0 & -I \end{bmatrix}, \\ \zeta(t_k) &= \begin{bmatrix} X(t) & X(t_k) & \frac{1}{t - t_k}\int_{t_k}^{t} X(s)ds & \dot{X}(t) \end{bmatrix}. \end{aligned}$$

With predefined Lemmas and notations, we present design methodology of model predictive control for delayed LPV systems by deriving a set of linear matrix inequality conditions.

Theorem 1. For a given parameter h_M and a vector $X(t_k)$, if there exist positive matrices $G, \bar{P}_1 > 0, \bar{P}_2 > 0, \begin{bmatrix} \bar{U}_1 & \bar{U}_2 \\ * & \bar{U}_3 \end{bmatrix} > 0, \bar{V} > 0, Y, \bar{Z}_1, \bar{Z}_2$, satisfying the following LMI conditions, the control input at time instant t_k guarantees the performance index (12) with γ .

$$\min \gamma \tag{14}$$
$$\begin{bmatrix} 1 & X^T(t_k) \\ * & G^T + G - \bar{P}_1 \end{bmatrix} \geq 0, \tag{15}$$
$$\Sigma_1^i < 0, \quad \text{for} \quad i = 1, 2, \ldots, L, \tag{16}$$
$$\Sigma_2^i < 0, \quad \text{for} \quad i = 1, 2, \ldots, L, \tag{17}$$
$$\begin{bmatrix} G^T + G - \bar{P}_1 & Y \\ * & u_{max}^2 \end{bmatrix} \geq 0 \tag{18}$$

where

$$\Sigma_1^i = \begin{bmatrix} \Sigma_{11}^i & \Sigma_{12} & h_M \cdot D_1 \bar{Z}_1 & h_M \cdot D_1 \bar{Z}_2 \\ * & \Sigma_{22} & 0 & 0 \\ * & * & -h_M \cdot \bar{U}_1 & 0 \\ * & * & * & -3h_M \cdot \bar{U}_1 \end{bmatrix},$$
$$\Sigma_2^i = \begin{bmatrix} \Sigma_{11}^{2i} & \Sigma_{12} \\ * & \Sigma_{22} \end{bmatrix},$$
$$\Sigma_{11}^{1i} = \Sigma_1 + h_M \Sigma_2,$$
$$\Sigma_{11}^{2i} = \Sigma_1 + h_M \Sigma_3,$$
$$\Sigma_{12} = diag[G, Y],$$
$$\Sigma_{22} = diag[-\gamma Q^{-1}, -\gamma R^{-1}],$$

with

$$\begin{aligned}\Sigma_1 = {} & e_1^T \bar{P}_1 e_3 + e_3^T \bar{P}_1 e_1 - e_2^T \bar{U}_2 (e_1 - e_2) - (e_2^T \bar{U}_2 \\ & (e_1 - e_2))^T + sym\{\bar{Z}_1 D_1 + \bar{Z}_1 D_2\} \\ & + \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}^T \bar{V} \begin{bmatrix} e_1 & e_2 \end{bmatrix} + (e_1 + \alpha e_4)^T \bar{E}_i \\ & + ((e_1 + \alpha e_4)^T \bar{E}_i)^T, \\ \Sigma_2 = {} & e_3^T \bar{P}_2 e_1 + (e_3^T \bar{P}_2 e_1)^T - e_2^T \bar{U}_3 e_2 \\ & + \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}^T \bar{V} \begin{bmatrix} e_4 \\ D_3 \end{bmatrix} + \begin{bmatrix} e_4 \\ D_3 \end{bmatrix}^T \bar{V} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}, \\ \Sigma_3 = {} & \begin{bmatrix} e_4 \\ e_2 \end{bmatrix}^T \bar{U} \begin{bmatrix} e_4 \\ e_2 \end{bmatrix}.\end{aligned}$$

then, the state feedback gains are given as $K = YG^{-1}$.

Proof. Choosing the following Lyapunov-Krasovskii functional (LKF) for $t \in [t_k, t_{k+1})$ yields

$$V(x_t) = V_1(t) + V_2(t) + V_3(t) \tag{19}$$

where

$$V_1(t) = \begin{bmatrix} X(t) \\ \int_{t_k}^t X(s) ds \end{bmatrix}^T \begin{bmatrix} G^{-T} P_1 G^{-1} & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} X(t) \\ \int_{t_k}^t X(s) ds \end{bmatrix},$$
$$V_2(t) = (h_M - h(t)) \int_{t_k}^t \begin{bmatrix} \dot{X}(s) \\ X(t_k) \end{bmatrix}^T U \begin{bmatrix} \dot{X}(s) \\ X(t_k) \end{bmatrix} ds,$$
$$V_3(t) = h(t) \begin{bmatrix} X(t) \\ X(t_k) \end{bmatrix}^T V \begin{bmatrix} X(t) \\ X(t_k) \end{bmatrix}.$$

Differentiate the LKF

$$\begin{aligned}\dot{V}_1 = {} & 2X^T(t) G^{-T} P_1 G^{-1} \dot{X}(t) \\ & + 2 \int_{t_k}^t X(s) ds P_2 X(t),\end{aligned} \tag{20}$$
$$\begin{aligned}\dot{V}_2 = {} & - \int_{t_k}^t \dot{X}^T(s) U_1 \dot{X}(s) ds - 2X^T(t_k) U_2 \\ & (X(t) - X(t_k)) - h(t) X^T(t_k) U_3 X(t_k) \\ & + (h_M - h(t)) \begin{bmatrix} \dot{X}(s) \\ X(t_k) \end{bmatrix}^T U \begin{bmatrix} \dot{X}(s) \\ X(t_k) \end{bmatrix},\end{aligned} \tag{21}$$
$$\begin{aligned}\dot{V}_3 = {} & \begin{bmatrix} X(t) \\ X(t_k) \end{bmatrix}^T V \begin{bmatrix} X(t) \\ X(t_k) \end{bmatrix} \\ & + 2h(t) \begin{bmatrix} X(t) \\ X(t_k) \end{bmatrix} V \begin{bmatrix} \dot{X}(t) \\ 0 \end{bmatrix}.\end{aligned} \tag{22}$$

From Lemma 2, the following holds

$$\begin{aligned} & - \int_{t_k}^t \dot{X}^T(\alpha) U_1 \dot{X}(s) ds \\ & \leq Sym\{\epsilon_1^T Z_1 \epsilon_2 + \epsilon_1^T Z_2 \epsilon_3\} \\ & + h(t) \epsilon_1^T (\frac{3Z_1 U_1^{-1} Z_1 + Z_2 U_1^{-1} Z_2}{3}) \epsilon_1\end{aligned} \tag{23}$$

where Z_1, Z_2 are auxiliary variables. Taking into account system dynamics (7),

$$\begin{aligned} & 2[X^T(t) G^{-1} + \alpha \dot{X}^T(t) G^{-1}][-\dot{X}(t) + \\ & A_i X(t) + BKX(t_k)].\end{aligned} \tag{24}$$

Summing up from (20) to (24) leads to

$$\dot{V} + X^T(t) Q X(t) + U^T(t) R U(t) \leq \zeta(t_k) \bar{\Sigma} \zeta(t_k) \tag{25}$$

where

$$\bar{\Sigma} = \bar{\Sigma}_1 + h(t) \bar{\Sigma}_2 + (h_M - h(t)) \bar{\Sigma}_3,$$

$$\begin{aligned}\bar{\Sigma}_1 = {} & e_1^T P_1 e_3 + e_3^T P_1 e_1 - e_2^T U_2 (e_1 - e_2) - (e_2^T U_2 \\ & (e_1 - e_2))^T + sym\{Z_1 D_1 + Z_1 D_2\} \\ & + \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}^T V \begin{bmatrix} e_1 & e_2 \end{bmatrix} + (e_1 + \alpha e_4)^T E_i \\ & + ((e_1 + \alpha e_4)^T E_i)^T \\ & + e_1^T Q e_1 + e_2 K^T R K e_2, \\ \bar{\Sigma}_2 = {} & e_3^T P_2 e_1 + (e_3^T P_2 e_1)^T - e_2^T U_3 e_2 \\ & + \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}^T V \begin{bmatrix} e_4 \\ D_3 \end{bmatrix} + \begin{bmatrix} e_4 \\ D_3 \end{bmatrix}^T V \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \\ & + Z_1^T U_1^{-1} Z_1 + \frac{1}{3} Z_2^T U_1^{-1} Z_2, \\ \bar{\Sigma}_3 = {} & \begin{bmatrix} e_4 \\ e_2 \end{bmatrix}^T U \begin{bmatrix} e_4 \\ e_2 \end{bmatrix}.\end{aligned}$$

Pre-and post-multiplying with a matrix $\gamma^{1/2} \times diag\{G, G, G, G\}$, the followings are satisfied with Lemma 1.

$$\begin{aligned} & \Sigma_1 + h_M \Sigma_2 + h_M \bar{Z}_1^T \bar{U}_1^{-1} \bar{Z}_1 \\ & \qquad + \frac{1}{3} h_M \bar{Z}_2^T \bar{U}_1^{-1} \bar{Z}_2) < 0,\end{aligned} \tag{26}$$
$$\Sigma_1 + h_M \Sigma_3 < 0 \tag{27}$$

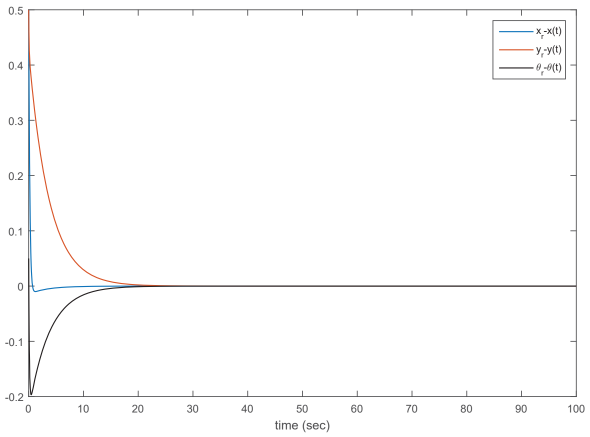

Fig. 2 error response of the system in Example 1

where $\bar{U} = GUG$, $\bar{V} = GVG$, $\bar{Z}_1 = GZ_1G$, $\bar{Z}_2 = GZ_2G$, and $K = YG^{-1}$. Using Schur complement, The equations in (25) and (26) are equivalent to those of (16) and (17). For every sampling instance, V_2 and V_3 vanish. Then, the upper bound of LKF is expressed in terms of V_1.

$$X^T(t_k)G\bar{P}_1GX(t_k) \leq \gamma, \tag{28}$$

where γ denotes the bound of optimal performance index. The effect of input saturation is considered similar to the method in [7]. This ends the proof. □

IV. Numerical Example

Example 1 This example considered the dynamical equations of the system represented from error dynamics.

$$\dot{X}(t) = A_iX(t) + BU(t - h(t)) \tag{29}$$

where

$$A_1 = \begin{bmatrix} 0 & \omega_r - 0.05 & 0 \\ \omega - 0.05 & 0 & v_r(t) \\ 0 & 0 & 0 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0 & \omega_r + 0.05 & 0 \\ \omega + 0.05 & 0 & v_r(t) \\ 0 & 0 & 0 \end{bmatrix},$$
$$B = \begin{bmatrix} -1 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix}.$$

The model parameters are calculated with a sampling time 0.1s. The sampling time $h(t)$ is less than 0.1 s. Along the reference trajectory, the input is constrained to $-0.1 \leq u(1) \leq 0.1$ and $-0.05 \leq u(2) \leq 0.05$.

The corresponding controller gain matrix is

$$K = \begin{bmatrix} -0.5777 & 0.3442 & 2.6729 \\ -0.2079 & 0.1255 & 0.9746 \end{bmatrix}$$

Fig. 2 shows the simulation result which is obtained with the above controller gain, taking $Q = I,\ R = I, \alpha = 0.1$.

V. Conclusion

The sampled-data MPC method for mobile robot systems have been investigated by considering constrained polytopic LPV model. Based on the input delay model, sufficient conditions for the sampled-data MPC controller design are obtained by constructing a new Lyapunov functional. The effectiveness of the presented method has been verified by illustrating numerical simulation.

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