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H_∞ Takagi-Sugeno Fuzzy State-Derivative Feedback Control Design for Nonlinear Dynamic Systems

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***Abstract*—**This paper considers an H_∞ TS fuzzy state-derivative feedback controller for a class of nonlinear dynamical systems. A Takagi-Sugeno (TS) fuzzy model is used to approximate a class of nonlinear dynamical systems. Then, based on a linear matrix inequality (LMI) approach, we design an H_∞ TS fuzzy state-derivative feedback control law which guarantees L_2-gain of the mapping from the exogenous input noise to the regulated output to be less or equal to a prescribed value. We derive a sufficient condition such that the system with the fuzzy controller is asymptotically stable and H_∞ performance is satisfied. Finally, we provide and simulate a numerical example is provided to illustrate the stability and the effectiveness of the proposed controller.

***Keywords*—**H_∞ fuzzy control, LMI, Takagi-Sugano (TS) fuzzy model, nonlinear dynamic systems, state-derivative feedback.

I. Introduction

RECENTLY, H_∞ fuzzy control systems have been extensively studied by many researchers. Most studies have considered the design of a H_∞ control of the fuzzy system, which can be represented by Takagi-Sugeno (TS) fuzzy model [1]. Based on this fuzzy model, the overall model of the system is attained by mixing these linear models via nonlinear membership functions. For example, a H_∞ fuzzy output feedback controller based on an LMI approach has designed in [2], [3]. A H_∞ fuzzy state-feedback controller for nonlinear singularly perturbed systems with pole placement constraints has proposed in [4]-[6]. In [7]-[9] proposed H_∞ fuzzy design for Markovian jump nonlinear systems based on an LMI approach and [10] developed the H_∞ fuzzy technique with D-stability constraints which guarantee the L_2 gain of the mapping from the exogenous input noise to the regulated output to be less than some prescribed value. A H_∞ fuzzy state feedback controller based on LMIs has been widely presented in [11]-[14]. Although, there are many studies that address robustness in the sense of stability and satisfactory performance of the closed-loop system as shown in [15]-[17]. However, in practice, the state-derivative signals are readily obtained when compared with the common state signal, i.e., mechanical systems. Nevertheless, the state-derivative controller design for nonlinear systems is more complicated [18]-[20]. It is still very difficult to find a global solution either analytically or numerically. Therefore, the aim of this paper is to design the design of a H_∞ TS fuzzy state-derivative controller to stabilize outcome for a class of nonlinear dynamical systems. First, we approximate a class of nonlinear dynamical systems by a TS fuzzy model. Then, we design technique for a H_∞ TS fuzzy state-derivative controller, which guarantees L_2-gain of the mapping from the exogenous input noise to the regulated output to be less or equal to a prescribed value. We derive a sufficient condition such that the system with the fuzzy controller is asymptotically stable and H_∞ performance is satisfied. Finally, we present the example by applying the proposed controller with nonlinear dynamical models for a dc/dc converter of the photovoltaic (PV) systems to regulate its power generation

The rest of the paper is managed as follows. Section II describes the systems and presents definitions. Section III introduces main results based on an LMI approach. We develop a technique for synthesizing a H_∞ TS fuzzy state-derivative feedback controller, which guarantees L_2-gain of the mapping from the exogenous input noise to the regulated output to be less or equal to a prescribed value. Section IV illustrates the step designing of the proposed controller for a dynamic model of a photovoltaic (PV) system. Finally, in Section V, we provide our conclusions.

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II. The System Description

A. TS Fuzzy Model

A fuzzy dynamic model proposed as by Takagi- Sugeno (TS) is explained by IF-THEN rules. A nonlinear system can be approximated by blending these linear models via nonlinear membership functions. The ith rule of a TS fuzzy model can be expressed as follows:

Plant Rule i:

IF $x_{k1}(t)$ is F_{1i} and $x_{kj}(t)$ is F_{ji} THEN

$$\dot{x}(t) = A_i x(t) + B_i u(t) + B_w w(t) \quad (1)$$

$$z(t) = C_i x(t) \qquad i = 1,\ 2, \ldots, r \quad (2)$$

where F_{ji} are the fuzzy sets, r is the number of IF-THEN rules, $x_{kj}(t)$ are the premise variables, $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the input, $w(t) \in R^p$ is the disturbance, $z(t) \in R^s$ is the regulated output. The matrices A_i, B_i, B_w, and C_i, are suitable matrices of the system and t indicates the time.

This paper uses $x_k(t)$ to denote the vector containing all the individual elements. For any specified state vector and the control input $u(t)$, the TS fuzzy model is expressed as

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(x_k(t))(A_i x(t) + B_i u(t) + B_w w(t) \quad (3)$$

$$z(t) = \sum_{i=1}^{r} h_i(x_k(t)) C_i x(t) \quad (4)$$

where $x_k(t) = \left[x_{k1}(t)\ \ x_{k2}(t) \ldots x_{kj}(t)\right]$ $h_i(x_k(t)) = \varpi_i(x_k(t)) / \sum_{i=1}^{r} \varpi_i(x_k(t))$ with $\varpi_i(x_k(t)) = \prod_{j=1}^{v} F_{ji}(x_k(t))$ for all t. The term $F_{ji}(x_k(t))$ is the grade of membership of $x_k(t)$ in F_{ji}. It is assumed in this research that $\sum_{i=1}^{r} \varpi_i(x_k(t)) > 0$, $\varpi_i(x_k(t)) \geq 0$, i = 1, 2,…, r. We have $\sum_{i=1}^{r} h_i(x_k(t)) = 1$ and $h_i(x_k(t) \geq 0$, i = 1, 2,…, r for all t. Now, we recall the following definition:

Definition: Suppose γ is a specified positive real number. A system of the forms specified by (3) and (4) is said to have L_2-gain less than or equal to γ if [6]:

$$\int_0^{t_f} z^T(t) z(t)\, dt \leq \gamma^2 \left[\int_0^{t_f} w^T(t) w(t)\, dt\right] \quad (5)$$

for all $t_f \geq 0$ and $w \in L_2\left[0,\ t_f\right]$.

III. Main Results

We develop the synthesis of an H_∞ TS fuzzy state-derivative feedback controller in this section. An LMI approach is utilized to derive fuzzy controller gains, which stabilize the system as described by (3) and (4). When (3) and (4) are feasible, they can be easily solved using available software, such as an LMI solver. Suppose that there exists a fuzzy controller of the term:

Controller Rule j:

IF $x_{k1}(t)$ is F_{1i} and … and $x_{kj}(t)$ is F_{ji} THEN

$$u(t) = -K_j \dot{x}(t), \quad \forall_j = 1, 2, ..., r. \quad (6)$$

The fuzzy controller can be stated as

$$u(t) = -\sum_{j=1}^{r} h_j(x_k(t)) K_j \dot{x}(t) \quad (7)$$

The system (3) can be rewritten as

$$\begin{aligned}\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(x_k(t)) h_j(x_k(t)) \left[A_i x(t) - B_i K_j \dot{x}(t)\right] \\ + B_w w(t)\end{aligned} \quad (8)$$

The following result deals with the system (8):

Theorem: Given the system (3) and (4) with the fuzzy controller (7), a scalar $\gamma > 0$ exists, and the inequality (5) hold if there exists a positive definite matrix $P_\infty = P_\infty^T$ and matrices Y_j, $j = 1, 2, ..., r$ satisfying the following conditions

$$P > 0 \quad (9)$$

$$\begin{bmatrix} A_i P_\infty + P_\infty A_i^T + B_i Y_j A_i^T + A_i Y_j^T B_i^T & B_w & P_\infty C_i^T + B_i Y_j C_i^T \\ B_w^T & -\gamma^2 I & 0 \\ C_i P_\infty + C_i Y_j^T B_i^T & 0 & -I \end{bmatrix} < 0 \quad (10)$$

$$\forall i, j = 1, 2, ..., r$$

where

$$K_j = Y_j P_\infty^{-1} \quad (11)$$

Proof: The system (8) and (4) with (7) yields

$$\begin{aligned}&\left(I + \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(x_k(t)) h_j(x_k(t)) \left[B_i K_j\right]\right) \dot{x}(t) \\ &= \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(x_k(t)) h_j(x_k(t)) \left(A_i x(t)\right) + B_w w(t)\end{aligned} \quad (12)$$

The issue is to obtain state-derivative gains $K_j\,(j = 1, 2, ..., r)$, such that the following conditions hold:

1) Matrices $\left[I + B_i K_j\right]$, $(i, j = 1, 2, ..., r)$ have a full rank.
2) Based on the sufficient condition, (12) with the fuzzy state-derivative feedback controller (7) is asymptotically stable and the H_∞ performance is satisfied.

From these conditions, we define $E_{ij} = \left(I + B_i K_j\right)^{-1}$, then (12) can be rewritten as:

$$\begin{aligned}\dot{x}(t) &= \Sigma_{i=1}^{r} \Sigma_{j=1}^{r} h_i(x_k(t)) h_j(x_k(t)) E_{ij} \left(A_i x(t)\right) \\ &+ E_{ij} B_w w(t)\end{aligned} \quad (13)$$

Let us consider the quadratic *Lyapunov* function:

$$V(x(t)) = x^T(t) Q x(t) \quad (14)$$

where Q=Q^T= P_∞^{-1} is a symmetric and positive-definite matrix. Differentiating $V(x(t))$ along the system (13) with (7) yields:

$$\begin{aligned}\dot{V}(x(t)) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(x_k(t)) h_j(x_k(t)) x^T(t) A_i^T E_{ij}^{\ T} Q x(t) \\ &+ \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(x_k(t)) h_j(x_k(t)) x^T(t) Q E_{ij} A_i x(t) \\ &+ x^T(t) Q E_{ij} B_w w(t) + w^T(t) B_w^T E_{ij}^{\ T} Q x(t)\end{aligned} \quad (15)$$

Adding and subtracting $-z^T(t) z(t) + \gamma^2 w^T(t) w(t)$ to (15) yields

$$\dot{V}(x(t)) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(x_k(t))h_j(x_k(t))\begin{bmatrix} x^T(t) & w^T(t)\end{bmatrix} \times \begin{bmatrix} A_i^T E_{ij}{}^T Q + QE_{ij}{}^T A_i + C_i^T C_i & QE_{ij}B_w \\ B_w^T E_{ij}{}^T Q & -\gamma^2 I \end{bmatrix}\begin{bmatrix} x(t) \\ w(t)\end{bmatrix} - z^T(t)z(t) + \gamma^2 w^T(t)w(t) \quad (16)$$

Let us consider (10), using (11), then the system (10) can be rewritten by:

$$\begin{bmatrix} \left(I + B_iK_j\right)P_\infty A_i^T + A_iP_\infty\left(I + B_iK_j\right)^T & B_w & \left(I + B_iK_j\right)P_\infty C_i^T \\ B_w^T & -\gamma^2 I & 0 \\ C_iP_\infty\left(I + B_iK_j\right)^T & 0 & -I \end{bmatrix} < 0 \quad (17)$$

Pre-multiplication by $\begin{pmatrix} \left(I + B_iK_j\right)^{-1} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix}$, post-multiplication by $\begin{pmatrix} \left(I + B_iK_j\right)^{-T} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix}$ in (17), with $E_{ij} = \left(I + B_iK_j\right)^{-1}$. Then the system (17) can be rewritten as;

$$\begin{bmatrix} P_\infty A_i^T E_{ij}{}^T + E_{ij}A_iP_\infty & E_{ij}B_w & P_\infty C_i^T \\ B_w^T E_{ij}{}^T & -\gamma^2 I & 0 \\ C_iP_\infty & 0 & -I \end{bmatrix} < 0 \quad (18)$$

Multiplying both sides of (18) by $diag(Q,\ I,\ I)$

$$\begin{bmatrix} Q & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}\begin{bmatrix} P_\infty A_i^T E_{ij}{}^T + E_{ij}A_iP_\infty & E_{ij}B_w & P_\infty C_i^T \\ B_w^T E_{ij}{}^T & -\gamma^2 I & 0 \\ C_iP_\infty & 0 & -I \end{bmatrix}\begin{bmatrix} Q & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} < 0 \quad (19)$$

where $Q = P_\infty^{-1}$. Then, (19) becomes;

$$\begin{bmatrix} A_i^T E_{ij}{}^T Q + QE_{ij}A_i & QE_{ij}B & C_i^T \\ B_w^T E_{ij}{}^T Q & -\gamma^2 I & 0 \\ C_i & 0 & -I \end{bmatrix} < 0 \quad (20)$$

Let us consider (20), now using *Schur* complement. The equation above is equivalent to:

$$\begin{bmatrix} A_i^T E_{ij}{}^T Q + QE_{ij}A_i & QE_{ij}B_w \\ B_w^T E_{ij}{}^T Q & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} C_i^T \\ 0 \end{bmatrix}\begin{bmatrix} C_i & 0 \end{bmatrix} < 0 \quad (21)$$

or in more compact form as:

$$\begin{bmatrix} A_i^T E_{ij}{}^T Q + QE_{ij}{}^T A_i + C_i^T C_i & QE_{ij}B_w \\ B_w^T E_{ij}{}^T Q & -\gamma^2 I \end{bmatrix} < 0 \quad (22)$$

Since (22) is less than zero and given the fact that $h_i(x_k(t)) \geq 0$ and $\sum_{i=1}^{r} h_i(x_k(t)) = 1$, (16) becomes:

$$\dot{V}(x(t)) \leq -z^T(t)z(t) + \gamma^2 w^T(t)w(t) \quad (23)$$

Integrate both sides of (23) yields:

$$V(x(t)) + V(x(0)) \leq \int_0^{t_f}\left[-z^T(t)z(t) + \gamma^2 w^T(t)w(t)\right]dt \quad (24)$$

Defining that initial condition $x(0) = 0$, we have:

$$V(x(t)) \leq \int_0^{t_f}\left[-z^T(t)z(t) + \gamma^2 w^T(t)w(t)\right]dt \quad (25)$$

Since $V(x(t)) > 0$, this implies:

$$0 \leq -\int_0^{t_f} z^T(t)z(t)dt + \gamma^2\int_0^{t_f}\left[w^T(t)w(t)\right]dt \quad (26)$$

Hence, the inequality (19) holds.

IV. Example

Consider the following problem of a photovoltaic (PV) system which is described by the following state equations [10].

$$\dot{x}_1(t) = \dot{V}_{pv} = \frac{1}{C_{pv}}(i_{pv} - i_L d) \quad (27)$$

$$\dot{x}_2(t) = \dot{i}_L = -\frac{V_b}{L} + \frac{V_{pv}}{L}d \quad (28)$$

$$\dot{x}_3(t) = \dot{v}_c = \frac{1}{C_b}\left(i_L - i_o\right) \quad (29)$$

$$z(t) = i_{pv} - \frac{n_p k_{pv}}{n_s} I_{rs} V_{pv} e^{q(V_{pv})/n_s AKT} \quad (30)$$

where V_{pv} and i_{pv} are the output voltage and current of the PV array respectively. The factor A considers the cell deviation from the ideal p-n junction characteristic, varying between 1 to 5, q is the charge of an electron, K is Boltzmann's constant, I_{rs} denotes the reverse saturation current, and T is the cell temperature. i_L and V_b are the current and voltage on the output terminals of the dc/dc converter, v_c is voltage on C_b, where $V_b = E_b + v_c + (i_L - i_o) R_b$, i_o is measurable current, and d is the duty ratio of the pulse-width-modulated signal to control the switching IGBT. Taking (30) as the regulated output $z(t)$.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{C_{pv}}\frac{i_{pv}}{V_{pv}} & 0 & 0 \\ -\frac{E_b}{L}\frac{1}{V_{pv}} & -\left(1 - \frac{i_o}{i_L}\right)\frac{R_b}{L} & -\frac{1}{L} \\ 0 & \frac{1}{C_b}\left(1 - \frac{i_o}{i_L}\right) & 0 \end{bmatrix}\begin{bmatrix} V_{pv} \\ i_L \\ v_c \end{bmatrix} + \begin{bmatrix} -\frac{i_L d}{C_{pv}} & \frac{V_{pv} d}{L} & 0 \end{bmatrix}^T u(t) + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} \quad (31)$$

where $x(t)=\begin{bmatrix}V_{pv} & i_L & v_c\end{bmatrix}^T$ is the state vector of the system, $u(t)$ is the control input, $\gamma_z = q/n_s AKT$, and $w(t)=[w_1 \quad w_2 \quad w_3]^T$ represents some disturbances during abnormal conditions such as the electrical noise, the disturbance in grid voltage and the loss of grid. The matrices $A(x)$, $B(x)$, and $C(x)$ have to be exactly represented by TS fuzzy rules. We have five fuzzy premise variables $x_{k1}=V_{pv}$, $x_{k2}=i_{pv}$, $x_{k3}=i_o$, $x_{k4}=i_L$, and $x_{k5}=n_p\gamma_z I_{rs}e^{\gamma_z V_{pv}}$.

$$\dot{x}(t) = A(x)x(t) + B(x)u(t) + B_w w(t) \qquad (32)$$

$$z(t) = C(x)x(t) \qquad (33)$$

Then, the membership function of the fuzzy premise variables $x_{k1}, x_{k2}, ..., x_{k5}$ should be chosen such that $A(x)=\sum_{i=1}^{r} h_i A_i$, $B(x)=\sum_{i=1}^{r} h_i B_i$, and $C(x)=\sum_{i=1}^{r} h_i C_i$. For simplicity and without loss of generality the following form of the membership function can be expressed as:

$$E_{aj} = \frac{x_{kj}-m_j}{M_j - m_j}, \qquad E_{bj} = \frac{m_j - x_{kj}}{M_j - m_j} \qquad (34)$$

where $E_{aj}+E_{bj}=1$, $M_j = \max_{\bar{x}} x_{kj}$, and $m_j = \min_{\bar{x}} x_{kj}$. Assuming that $\bar{x}$ is a workspace with in [min, max] where $\left\{\bar{x}=\begin{bmatrix}V_{pv} & i_{pv} & i_o & i_L & n_p\gamma_z I_{rs}e^{\gamma_z V_{pv}}\end{bmatrix} \,\middle|\, \bar{x}_i \in [\min \ \max]\right\}$ for i = 1 to 5. The local system matrices are as:

$$A_i = \begin{bmatrix} \frac{1}{C_{pv}}\frac{v_{2i}}{v_{1i}} & 0 & 0 \\ -\frac{E_b}{L}\frac{1}{v_{1i}} & -\left(1-\frac{v_{3i}}{v_{4i}}\right)\frac{R_b}{L} & -\frac{1}{L} \\ 0 & \frac{1}{C_b}\left(1-\frac{v_{3i}}{v_{4i}}\right) & 0 \end{bmatrix}, \ B_i = \begin{bmatrix} -\frac{1}{C_{pv}}v_{4i} \\ \frac{1}{L}v_{1i} \\ 0 \end{bmatrix} \qquad (35)$$
$$C_i = \begin{bmatrix}(v_{2i}/v_{1i}) - v_{5i} & 0 & 0\end{bmatrix}$$

The dynamic model of the PV system is represented by TS fuzzy model (1) and (2), which is composed of 16 rules and local system matrices A_i, B_i, and C_i, respectively. The parameters v_{ji} corresponds with each rule is addressed in Table I. The fuzzy premise variables x_{kj} rely on the state equations. The lower and upper bound parameters (M_j, m_j) can be obtained from the specification of the PV system.

We used computer simulation to evaluate the performance of an H_∞ fuzzy state-derivative feedback controller for the photovoltaic system. Firstly, we conduct the TS fuzzy control of the PV system and use the parameters of the PV system as shown in [21]. The PWM generator produces a pulse to fire the IGBT switch of a dc/dc buck converter. The duty ratio determines the percentage of the pulse period at d = 0.8 and specifies the switching frequency f_{sw}= 10 KHz. According to the fuzzy model (1) and (2), defined the workspace to be [$\bar{x}$ = (V_{pv}, i_{pv}, i_o, i_L, $n_p\gamma_z I_{rs}e^{\gamma_z V_{pv}}$)| 0.1 < x_{k1} < 1, 0.1 < x_{k2} < 1, 0.01 < x_{k3} < 0.02, 42 < x_{k4} < 80 and -0.1 < x_{k5} < 0.1] where x_{k1} = V_{pv}, x_{k2} = i_{pv}, x_{k3} = i_o, x_{k4} = i_L, and x_{k5} = $n_p\gamma_z I_{rs}e^{\gamma_z V_{pv}}$ are given as the fuzzy premise variables. Based on athe membership functions (34), the fuzzy parameters in Table I were chosen as M_1 = 1, m_1 = 0.1, M_2 = 1, m_2 = 0.1, M_3 = 0.02, m_3 = 0.01, M_4 = 80, m_4 = 42, M_5 =0.1, and m_5 = -0.1. Thus, the subsystem matrices A_i, B_i, and C_i in the consequences part (35) were obtained. Using an LMI approach, γ = 0.1, and following *Theorem 1*, we obtain the positive definite symmetric matrix P_∞ given as

TABLE I
FUZZY RULES

Rules	*Fuzzy-Sets*	*Then-Part*
i	$(F_{1i}, F_{2i}, F_{3i}, F_{4i}, F_{5i})$	$(v_{1i}, v_{2i}, v_{3i}, v_{4i}, v_{5i})$
1	(E_{a1}, E_{a2}, E_{a3}, E_{a4}, E_{b5})	(M_1, M_2, M_3, M_4, m_5)
2	(E_{a1}, E_{a2}, E_{a3}, E_{b4}, E_{b5})	(M_1, M_2, M_3, m_4, m_5)
3	(E_{a1}, E_{a2}, E_{b3}, E_{a4}, E_{b5})	(M_1, M_2, m_3, M_4, m_5)
4	(E_{a1}, E_{a2}, E_{b3}, E_{b4}, E_{b5})	(M_1, M_2, m_3, m_4, m_5)
5	(E_{a1}, E_{b2}, E_{a3}, E_{a4}, E_{b5})	(M_1, m_2, M_3, M_4, m_5)
6	(E_{a1}, E_{b2}, E_{a3}, E_{b4}, E_{b5})	(M_1, m_2, M_3, m_4, m_5)
7	(E_{a1}, E_{b2}, E_{b3}, E_{a4}, E_{b5})	(M_1, m_2, m_3, M_4, m_5)
8	(E_{a1}, E_{b2}, E_{b3}, E_{b4}, E_{b5})	(M_1, m_2, m_3, m_4, m_5)
9	(E_{b1}, E_{a2}, E_{a3}, E_{a4}, E_{a5})	(m_1, M_2, M_3, M_4, M_5)
10	(E_{b1}, E_{a2}, E_{a3}, E_{b4}, E_{a5})	(m_1, M_2, M_3, m_4, M_5)
11	(E_{b1}, E_{a2}, E_{b3}, E_{a4}, E_{a5})	(m_1, M_2, m_3, M_4, M_5)
12	(E_{b1}, E_{a2}, E_{b3}, E_{b4}, E_{a5})	(m_1, M_2, m_3, m_4, M_5)
13	(E_{b1}, E_{b2}, E_{a3}, E_{a4}, E_{a5})	(m_1, m_2, M_3, M_4, M_5)
14	(E_{b1}, E_{b2}, E_{a3}, E_{b4}, E_{a5})	(m_1, m_2, M_3, m_4, M_5)
15	(E_{b1}, E_{b2}, E_{b3}, E_{a4}, E_{a5})	(m_1, m_2, m_3, M_4, M_5)
16	(E_{b1}, E_{b2}, E_{b3}, E_{b4}, E_{a5})	(m_1, m_2, m_3, m_4, M_5)

TABLE II
MATRICES OF Y_j

Y_J	MATRICES
Y_1, Y_3	$\begin{bmatrix}1.6147\times10^{-4} & -0.0028 & -6.989\times10^{-4}\end{bmatrix}$
Y_2, Y_4	$\begin{bmatrix}3.0756\times10^{-4} & 0.03326 & -0.00203\end{bmatrix}$
Y_5, Y_7	$\begin{bmatrix}3.0161\times10^{-4} & 3.1607\times10^{-4} & -0.0075\end{bmatrix}$
Y_6, Y_8	$\begin{bmatrix}5.7451\times10^{-4} & 0.0392 & -0.0149\end{bmatrix}$
Y_9, Y_{11}	$\begin{bmatrix}1.4747\times10^{-4} & -0.0347 & -1.2785\times10^{4}\end{bmatrix}$
Y_{10}, Y_{12}	$\begin{bmatrix}2.8088\times10^{-4} & -0.0624 & -3.1255\times10^{4}\end{bmatrix}$
Y_{13}, Y_{15}	$\begin{bmatrix}1.6147\times10^{-4} & -0.0035 & -0.0074\end{bmatrix}$
Y_{14}, Y_{16}	$\begin{bmatrix}3.0756\times10^{-4} & -0.003 & -0.0142\end{bmatrix}$

$$P_\infty = \begin{bmatrix} 7.7812 & \text{-0.0018} & -1.0262\times10^{6} \\ -0.0018 & 53565.0776 & -961.4049 \\ -1.0262\times10^{6} & -961.4049 & 17.3024 \end{bmatrix}$$

The matrices Y_1, Y_2,…,Y_{16} and the controller gains K_1,K_2,…,K_{16} are shown in Tables II and III, respectively. Using the simulation, we determine the disturbance $w(t)$, the insolation λ, and the measurable current i_o as shown in Figs. 1-3. The disturbances are defined to be a random wave due to loss of grid, system fault, and disturbance in grid voltage. While the power output with the regulated control input is indicated in Fig. 4, the dynamic responses of PV-voltage, and

PV-current are illustrated in Figs. 5 and 6, respectively. Furthermore, the dynamic responses of the power output show that the state-feedback controller has a high transient response occurred immediately at $0 < t < 0.02$s, while the proposed controller has a smooth transient response as shown in Fig. 7. It is clear that the proposed controller provides the effectiveness for regulating the power generation of the PV system better than the state-feedback controller. The resulting TS fuzzy controller is:

$$u(t) = -\sum_{j=1}^{16} h_j(x_k(t)) K_j \dot{x}(t) \tag{36}$$

$$K_j = Y_j P^{-1} \tag{37}$$

TABLE III
MATRICES OF K_j

K_J	MATRICES
K_1, K_3	$\begin{bmatrix} 2.0682\times10^{-5} & -2.8731\times10^{-4} & -0.016 \end{bmatrix}$
K_2, K_4	$\begin{bmatrix} 3.9395\times10^{-5} & -5.4868\times10^{-4} & -0.0306 \end{bmatrix}$
K_5, K_7	$\begin{bmatrix} 3.8075\times10^{-5} & -0.0028 & -0.1597 \end{bmatrix}$
K_6, K_8	$\begin{bmatrix} 7.2523\times10^{-5} & -0.00546 & -0.3042 \end{bmatrix}$
K_9, K_{11}	$\begin{bmatrix} 1.8882\times10^{-5} & -2.8835\times10^{-4} & -0.016 \end{bmatrix}$
K_{10}, K_{12}	$\begin{bmatrix} 3.5965\times10^{-5} & -5.5057\times10^{-4} & -0.0306 \end{bmatrix}$
K_{13}, K_{15}	$\begin{bmatrix} 2.0064\times10^{-5} & -0.0029 & -0.1597 \end{bmatrix}$
K_{14}, K_{16}	$\begin{bmatrix} 3.8218\times10^{-5} & -0.0054 & -0.3041 \end{bmatrix}$

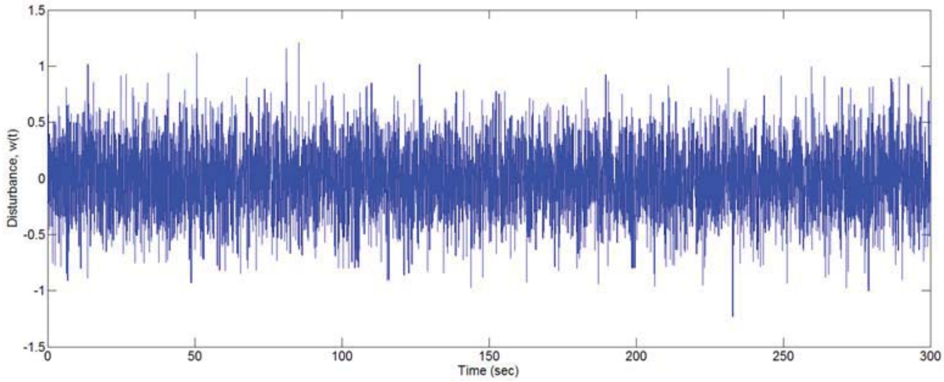

Fig. 1 Disturbance

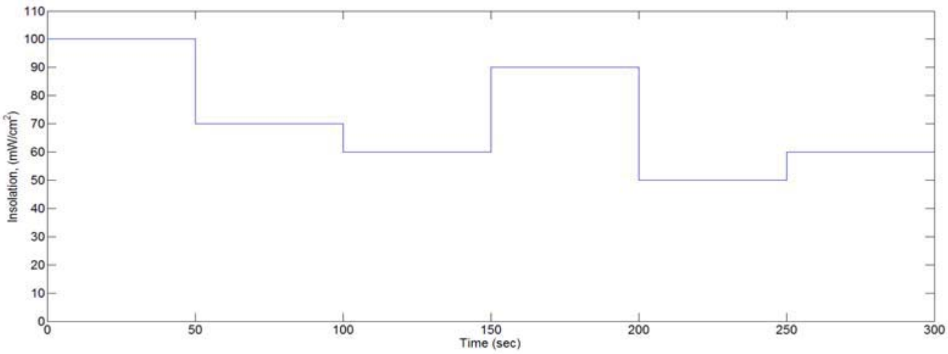

Fig. 2 Insolation

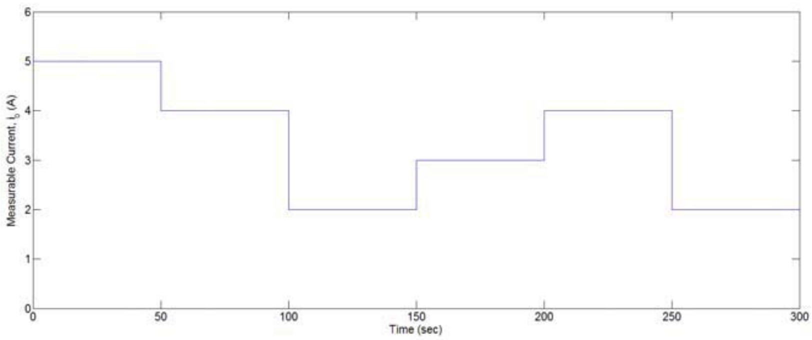

Fig. 3 Measurable current

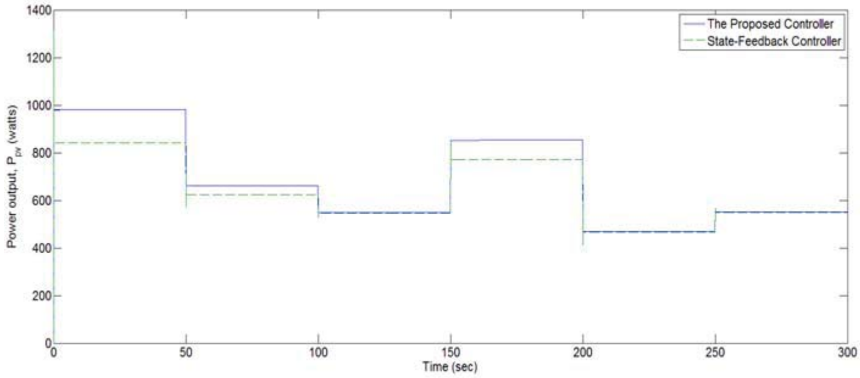

Fig. 4 Power output responses

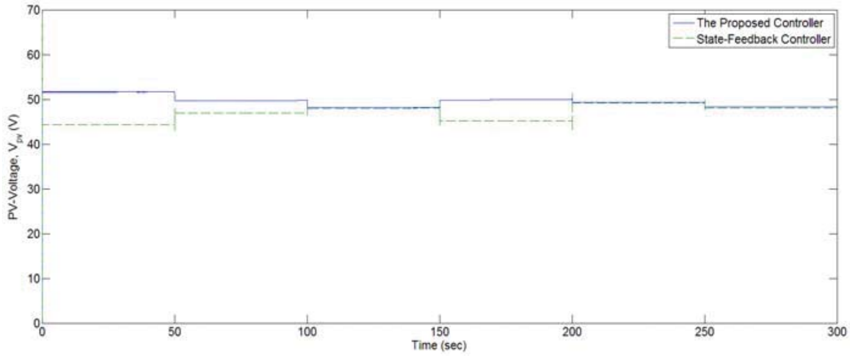

Fig. 5 PV-Voltage responses

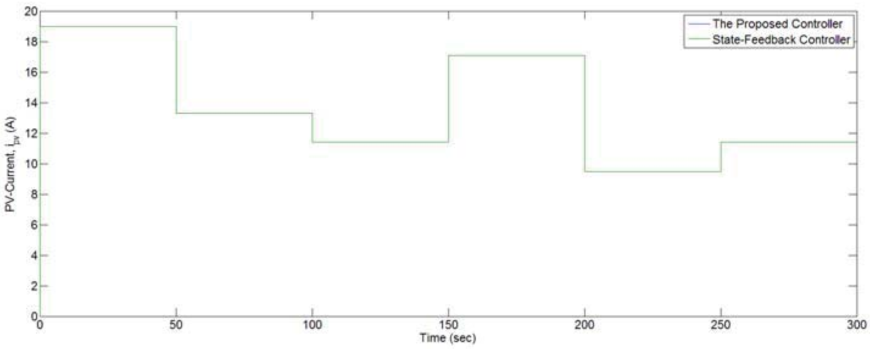

Fig. 6 PV-Current responses

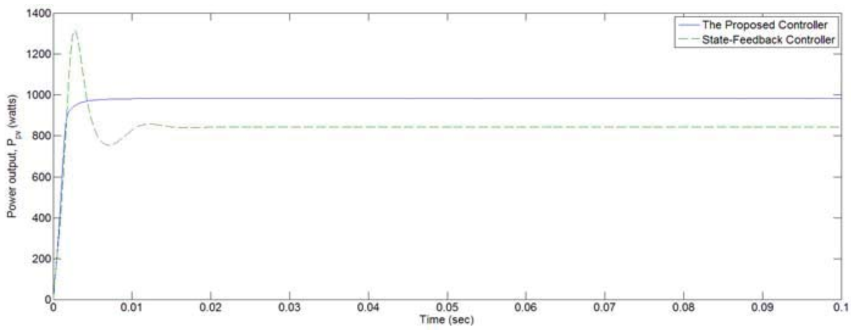

Fig. 7 Power output responses

V. Conclusions

This paper presents an H_∞ TS fuzzy state-derivative feedback control design for nonlinear dynamical systems based on an LMI approach that guarantees the L_2 gain of the mapping from the exogenous input noise to regulated output to be less than some prescribed value. According to numerically experimental results, the proposed method can achieve quickly steady-state performance. Although the varying insolation of the sunlight is considered, the performance of the proposed controller can still be continuously achieved a good performance. Thus, the proposed control system can provide very good dynamic response and can guarantee the stability of a class of nonlinear dynamical systems. Future work, we will consider the problem of an H_∞ fuzzy control design by combining state feedback controller and state-derivative controller.

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References

[1] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Trans. on Syst. Man, and Cybern.*, vol. SMC-15, 1985, pp. 116-132.

[2] S. K. Nguang and P. Shi, " H_∞ fuzzy output feedback control design for nonlinear systems: An LMI approach," *IEEE Trans. on Fuzzy Syst.*, vol. 11, 2003, pp. 331-340.

[3] W. Assawinchaichote, "A non-fragile H_∞ output feedback controller for uncertain fuzzy dynamical systems with multiple time-scales," *Int. J. Computers, Communications & Control*, vol. 7, 2012, pp. 8-19.

[4] W. Assawinchaichote and S. K. Nguang, "Fuzzy control design for singularly perturbed systems: An LMI approach," *Proc. ICAIET*, (Kota Kinabalu, Malaysia), 2002, pp. 146-151.

[5] W. Assawinchaichote and S. K. Nguang, " H_∞ fuzzy control design for nonlinear singularly perturbed systems with pole placement constraints: An LMI approach," *IEEE Trans. on Syst. Man, and Cybern.*, vol. 34, 2004, pp. 579-588.

[6] W. Assawinchaichote, S. K. Nguang, P. Shi, and M. Mizumoto, "Robust H_∞ control design for fuzzy singularly perturbed systems with Markovian jumps: an LMI approach," *43rd IEEE Conference on Decision and Control*, 2004, pp. 803-808.

[7] S. K. Nguang, W. Assawinchaichote, P. Shi, and Y. Shi, "H_∞ fuzzy filter design for uncertain nonlinear systems with Markovian jumps: an LMI approach," *Proceedings of the American Control Conference*, 2005, pp. 1799-1804.

[8] S. K. Nguang, W. Assawinchaichote, and P. Shi, "H_∞ filter for uncertain Markovian jump nonlinear systems: An LMI approach," *Int. J. Circuits Syst. Signal Process*, vol. 26, 2007, pp. 853-874.

[9] W. Assawinchaichote, "A new robust H_∞ fuzzy state-feedback control design for nonlinear Markovian jump systems with time-varying delay," *Control and Cybernetics*, vol. 43, no. 2, 2014, pp. 227-248.

[10] W. Assawinchaichote, S. K. Nguang, and P. Shi, "H_∞ fuzzy state feedback control design for nonlinear systems with *D*-stability constraints: An LMI approach," *ScienceDirect Maths. and Com. Sim.*, vol. 78, 2008, pp. 514-531.

[11] S. H. Kim and P. Park, "H_∞ state-feedback-control design for discrete-time fuzzy systems using relaxation technique for parameterized LMI," *IEEE Trans. Fuzzy Syst.* vol.18, no. 5, 2010, pp. 985-993.

[12] W. Assawinchaichote, "Synthesis of a robust H_∞ fuzzy controller for uncertain non-linear dynamical systems," in *Fuzzy Controllers Theory and Applications* ed. T. L.Grigorie Intech Press, Croatia, 2011, pp. 111-132.

[13] N. Kaewpraek and W. Assawinchaichote , "Control of PMSG wind energy conversion system with TS fuzzy state-feedback controller," *Applied Mechanics and Materials*, vol. 446-447, 2014, pp. 728-732.

[14] W. Assawinchaichote, "Further results on robust fuzzy dynamic systems with D-stability constraints," *Int. J. Applied Mathematics and Computer Science*, vol. 24, no. 4, 2014, pp. 785-794.

[15] H. Gassara, A. EI Hajjaji and M. Chaabane, "Observer-based robust $H\infty$ reliable control for uncertain T-S fuzzy systems with state time delay," *IEEE Trans. Fuzzy Syst.* vol.18, no.6, 2010 pp. 1027-1040.

[16] J. Luo, Y. Liu, Y. Dong and X. Min, "State-derivative-dependent robust H_∞ controller design for T-S fuzzy time-delay systems," *IEEE Conference. Chinese cont. and Decision.*,2010, pp. 311-316.

[17] H. Ghorbel, A. EI Hajjaji, M. Souissi and M. Chaabane, "Robust tracking control for Takagi-Sugeno fuzzy systems with unmeasurable premise variables: Application to tank system," *ASME Trans. Dyna. Syst., Measu., and Cont.* vol.136, 2014, pp. 1-8.

[18] G. R. Duan and X. Zhang, "Regularizability of linear descriptor systems via output plus partial state derivative feedback", *Asian J. Control.* Vol. 5, no. 3, 2003, pp. 334-340.

[19] T. H. S. Abdelaziz and M.Valas'ek, "Pole-placement for SISO linear systems by state-derivative feedback", *IEEE Proc. Contr. Theory Appl.* vol. 151, no. 4, 2004, pp. 377-358.

[20] R. Cardim, M. C. M. Teixeira, E. Assuncao and F. A. Faria, "Control designs for linear systems using state-derivative feedback", in *Systems Structure and Control* ed. P. Husek (InTech, Available form: http://www.cdn.intechopen.com/pdfs/5339.pdf), 2008, pp. 1-28.

[21] F. Valenciaga, P. F. Puleston, and P. E. Battaiotto, "Power control of photovoltaic array in a hybrid electric generation system using sliding mode techniques," *IEE Proc. on Cont. Theory Appl.*, vol. 148, 2001, pp. 448-455.