Experimental and Graphical Investigation on Oil Recovery by Buckley-Leveret Theory

Khwaja Naweed Seddiqi, Zabihullah Mahdi, Shigeo Honma

Abstract—Recently increasing oil production from petroleum reservoirs is one of the most important issues in the global energy sector. So, in this paper, the recovery of oil by the waterflooding technique from petroleum reservoir are considered. To investigate the aforementioned phenomena, the relative permeability of two immiscible fluids in sand is measured in the laboratory based on the steady-state method. Two sorts of oils, kerosene and heavy oil, and water are pumped simultaneously into a vertical sand column with different pumping ratio. From the change in fractional discharge measured at the outlet, a method for determining the relative permeability is developed focusing on the displacement mechanism in sand. Then, displacement mechanism of two immiscible fluids in the sand is investigated under the Buckley-Leveret frontal displacement theory and laboratory experiment. Two sorts of experiments, one is the displacement of pore water by oil, the other is the displacement of pore oil by water, are carried out. It is revealed that the relative permeability curves display tolerably different shape owing to the properties of oils, and produce different amount of residual oils and irreducible water saturation.

Keywords—Petroleum reservoir engineering, relative permeability, two-phase flow, immiscible displacement in porous media, steady-state method, waterflooding.

I. INTRODUCTION

AS oil production starts from petroleum reservoir, the pressure declines and production rate of oil may be drop. Therefore, water injecting technique is used to increase the oil production from petroleum reservoir. That is called waterflooding technique and can provide high rates of oil production and maybe pushed oil toward the outlet. Hence, the large amount of oil may be recovered by applying this method [1]. Fig. 1 demonstrates a petroleum reservoir in general condition, it seems that the gas is at top then the oil and water are accumulated in reservoir rock and confined below the cap rock [2], [3].

An untouched reservoir may be under enough pressure to drive hydrocarbons to the surface. When the fluids are produced, the pressure is going to be reduced and accordingly the production factor is minimized. Therefore, through the water injection, the reservoir pressure can be maintained at a

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sufficient level to push the fluids. The average recovery factor (i.e., the proportion of oil in place) by the combination of initial and secondary recovery methods are estimated in the range of 30-35%, which demonstrates the amount of recoverable reserves [4], [15].

While water is injected in two-phase flow condition, the oil is displaced upward. Oil and water are mutually immiscible so that this mechanism is considered to the immiscible displacement in porous media. The mechanism of immiscible displacements of two-phase fluids has been studied over the criteria of fluid flow through porous media and a simple approach to this problem was provided by [5].
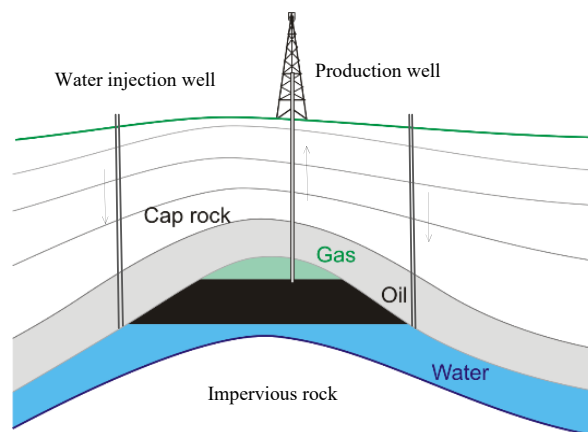

Fig. 1 Typical petroleum reservoir [3]

The Buckley-Leverett frontal displacement theory demonstrates a method to calculate the saturation profiles based on relative permeability of oil and water. This is to mention that the effect of capillary pressure and the gravity effects are assumed to be neglected. Based on this theory, the advance of saturation front of displacing fluid is affected by the permeability of oil and water related to reservoir rock, and to the viscosity ratio between the two fluids [4], [5].

In this paper, relative permeabilities of two different oils (kerosene and heavy oil) are determined first, then based on Buckley-Leverett frontal displacement theory, the displacement of oil-water in a vertical column is determined by graphical calculation and laboratory experiments.

II. RELATIVE PERMEABILITY

Relative permeability is an important conductive parameter controlling the immiscible displacement of multiphase fluids flowing in a porous medium. Petroleum reservoirs having

simple single-phase fluid system seldom exist because reservoirs are saturated with at least two immiscible fluid phases such as gas and oil or oil and water or gas, oil and water [4]. The relative permeability significantly affects flow processes when gas or water is injected into the reservoir. For example, the technique of injecting water into a reservoir is widely used in petroleum engineering to increase oil recovery that is known as the waterflooding technique [4], [14].

The relative permeability is affected by many factors including fluid saturations, saturation history, the magnitude of initial-phase saturation, wettability, the effect of rock pore structure and temperature so on. The graphs have been plotted with wetting fluid saturation (S_w), ranged from the irreducible wetting-phase saturation, S_{wi}, to the residual oil saturation, S_{or}, [7], [10].

A. Experimental Method

Experimental apparatus depicted in Fig. 2 was used in the experiment. Sand with the particle diameter of $D=0.105$-0.425 mm is packed in the experimental model. The sand is saturated by water, for water displacement and by oil for oil displacement test.
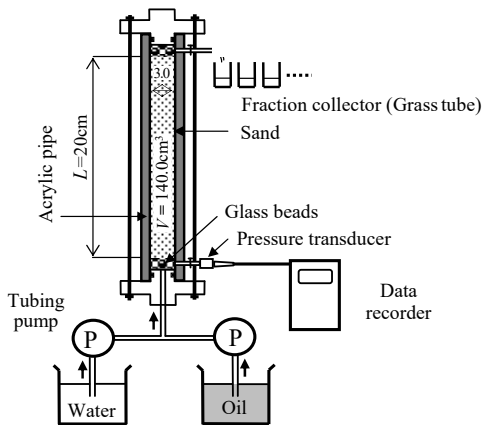

Fig. 2 Experimental apparatus

Two tubing pumps were connected at the bottom of the column by which displacing fluids (oil and water) are pumped at the same time with different pumping rates (e.g. q_w: $q_o = 4$: 6). The pore liquids displaced by pumping were collected at the top of the column by fraction collectors (2 cc test tube) until fraction ratio of oil and water reaches to the pumping ratio (steady state). The pressure gauge was installed at the bottom of the column to measure pumping pressure.

B. Method for Determining Relative Permeability

Fig. 3 shows a schematic diagram of the displacement of water by oil in a porous medium. Because there exists immobile water, i.e., absorbed water and stagnant water in soil pores, oil displaces only mobile water with velocity v_o, together with the mobile water velocity v_w, through effective porosity n_e of the soil. The void ratio e and porosity n of the soil may be calculated by soil mechanics as [6], [12];

$$e = \frac{\rho_s}{\rho_d} - 1 \qquad (1)$$

$$n = \frac{e}{1+e} \qquad (2)$$

where ρ_s is the density of soil particle, ρ_d is the dry density of soil packed. If the void volume V_v assumed as the unit pore volume, 1 V_p, fluids will flow through effective pore, the n_e, V_v, and fractional discharge of water can be calculated as:

$$f_{wd} = \frac{V_{wm}}{V_d} \qquad (3)$$

where V_{wm} is the volume of mobile water collected by the test tube and V_d is the volume of total liquid (water and oil) discharged at the outlet. When f_{wd} reaches to the pumping ratio $f_w = q_w / q_T$, a steady state is said to be attained.
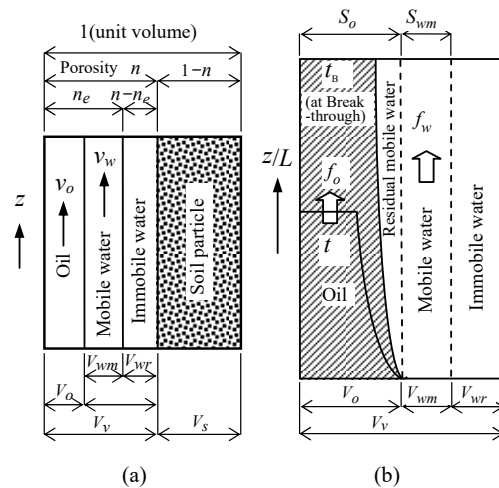

Fig. 3 Displacement mechanism of pore water by oil

Fig. 4 illustrates the change in f_{wd} and f_{od} with the change in Vp; (a) shows the displacement of water by oil only, $q_o= q_T$ and $q_w= 0$, (b) is the case of $q_o= q_w= q_T/2$ (q_T is the total pumping rate). If soil pore was initially saturated by water, the fractional discharge at the outlet f_{wd} is 1 until some pore volume, thereafter some amount of oil would be discharged. Thus, f_{wd} begins to decrease and f_{od} increases. When water discharge is ceased, only oil is discharged ($f_{od} = 1$). Therefore, the slashed area in Fig. 4 (a) indicates the mobile pore water displaced by oil, and the degree of saturation for the case is computed as the slashed area/Vp. Fig. 4 (b) illustrates the case of pumping ratio $q_o= q_w= 0.5q_T$. When fractional discharges, f_{wd} and f_{od}, reach to the fractional rate of pumping, f_w and f_o, the degree of saturation can be calculated in the same way. The slashed area in Fig. 4 (a) is always less than the total pore volume, V_v, and the remaining is the immobile water volume previously shown in Fig. 3 (a). The water saturation for this situation is mathematically expressed as;

$$S_w = 1 - \int_0^{V_{ps}} (f_{wd} - f_w) \, dV_p \qquad (4)$$

If pumping rate is $q_o = q_T$ and $q_w = 0$, the water saturation calculated by (4) represents the irreducible water saturation, S_{wi}. By changing the rate of pumping for oil and water in stages, we can obtain the degree of saturation of both phases at each rate. The relative permeability of oil and water, k_{ro} and k_{rw}, are calculated as follows. Darcy's law for oil and water flow through porous medium can be written as;

$$q_w = - \frac{k_z k_{rw} A}{\mu_w} \frac{\partial p_w}{\partial z} \qquad (5)$$

$$q_o = - \frac{k_z k_{ro} A}{\mu_o} \frac{\partial p_o}{\partial z} \qquad (6)$$

In (5) & (6), k_{ro} and k_{rw} are the relative permeability of oil and water, μ_w and μ_o are the dynamic viscosity of water and oil and the pore pressure of oil and water are Po and Pw, respectively. The intrinsic permeability of the sand were shown by k_z and A is the cross-sectional area of the reservoir. As mentioned the k_{ro} and k_{rw} is a function of water saturation (S_w). The k_{ro} and k_{rw} from (5) & (6) can give

$$k_{ro} = - \frac{q_o \mu_o}{k_z A} / \frac{\partial p_o}{\partial z} \qquad (7)$$

$$k_{rw} = - \frac{q_w \mu_w}{k_z A} / \frac{\partial p_w}{\partial z} \qquad (8)$$

C. Results

Fig. 5 shows the relative permeabilities obtained from the experiments for water displacement by kerosene and heavy oil. It is seen from the figure that the relative permeability of water, k_{rw}, decreases as saturation of water decreases, and the relative permeability of oil, k_{ro}, increases as the saturation of pore water. A considerable amount of oil is stored in sand pores, denoting the residual oil saturation, S_{or}, that is remarkable for Kerosene. It is also noticed that the amount of irreducible water saturation, S_{wi}, decreases as the viscosity of oil increases.

Subsequently, relative permeability data obtained from the experiments for oil displacements by water are presented. These data are necessary for the design of waterflooding technique to predict the amount of oil recovery by artificially injecting water into the reservoir [1]. The displacement mechanism for this situation is illustrated in Figs. 6 and 7. The degree of water saturation for this case is calculated by (9):

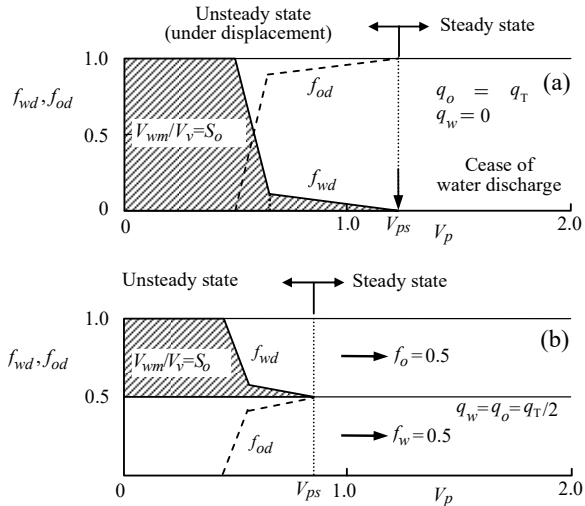$$S_w = \int_0^{V_{ps}} (f_{od} - f_o) \, dV_p = 1 - S_o \qquad (9)$$

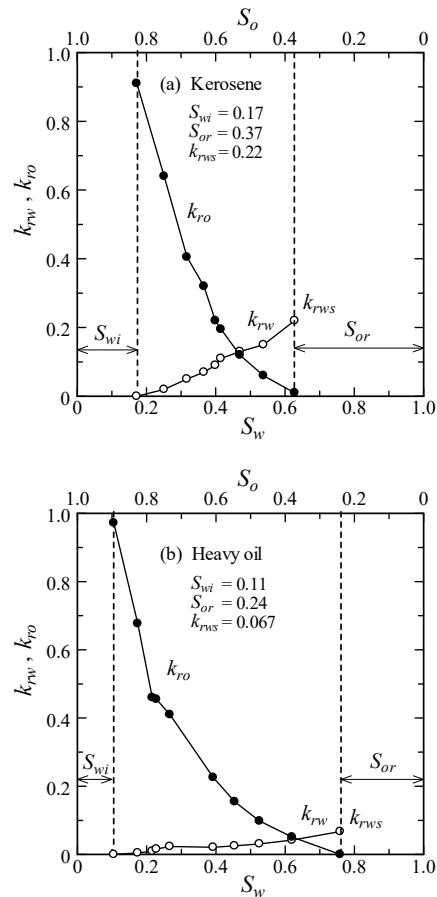Fig. 4 Fractional discharge of pore water and oil

Fig. 5 Relative permeability curves for water displacement by oils

Fig. 8 shows the relative permeabilities obtained from the tests. It is very interesting that the relative permeability curves follow the almost same rout with the change in saturation (compare Figs. 5 and 8) regardless of the displacement order.

From all of the results reported herein, typical relative

permeability curves with different residual oil and water saturations should be used for the design of waterflooding technique to evaluate the amount of oil recovery by artificially injecting water into the reservoir. The rate of advance of waterfront can be calculated based on the Buckley-Leveret frontal displacement theory using fractional flow curves that are evaluated from the relative permeability curves [7].
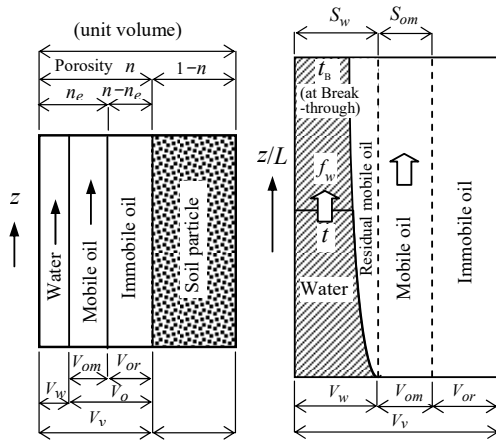

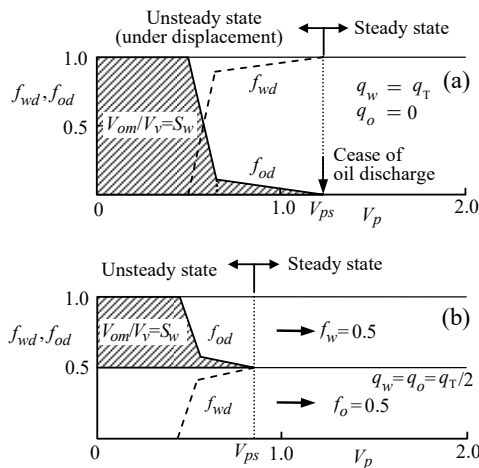Fig. 6 Displacement mechanism of pore oil by water

Fig. 7 Change in the fractional discharge of pore oil and water

III. Buckley-Leverett Theory

The Buckley-Leverett theory models the rate at which an injected water bank moves through a porous medium by using fractional flow theory. In this theory, the fluid flow is assumed to be linear and vertical. The oil and water both are incompressible. While water is injected into the petroleum reservoir, the capillary pressure between two fluids is small, so the capillary pressure in this method is assumed to be neglected [14].
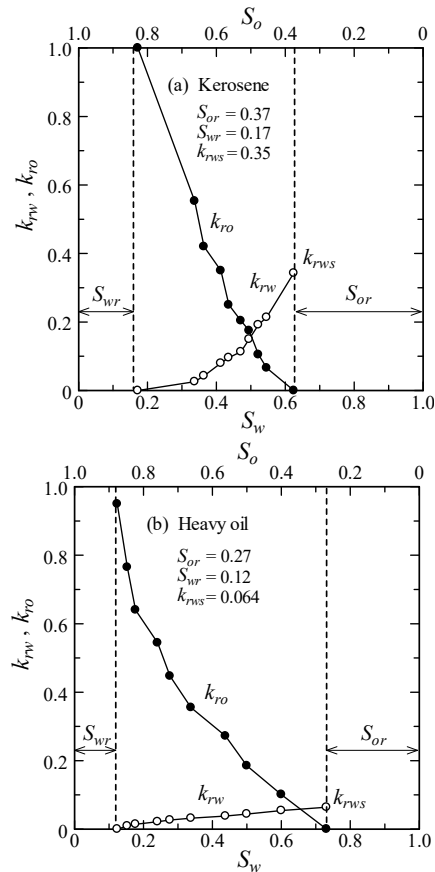

Fig. 8 Relative permeability curves for oil displacement by water

The one-dimensional vertical fluid flow through porous media in a completely saturated layer describes by Darcy's law. The Darcy's equations are explained in Section II B (5) & (6). If the water pressure is replaced by the capillary pressure between the oil and water, the Daric's equation can be rewritten as:

$$q_w = -\frac{k_z k_{rw} A}{\mu_w} \frac{\partial (p_o - p_{o/w})}{\partial z} \qquad (10)$$

After subtracting (5) from (10),

$$-\frac{1}{k_z A}(q_w \frac{\mu_w}{k_{rw}} - q_o \frac{\mu_o}{k_{ro}}) = -\frac{\partial p_{o/w}}{\partial z} \qquad (11)$$

Equation (11) is given as

$$q_T = q_w(1 + \frac{k_{ro}}{k_{rw}} \frac{\mu_w}{\mu_o}) - \frac{k_z k_{ro} A}{\mu_o} \frac{\partial p_{o/w}}{\partial z} \qquad (12)$$

From (12), the fraction flow of pore water (i.e., fractional flow rate) f_w is expressed as;

$$f_w = \frac{q_w}{q_T} = \frac{1 + \frac{k_z k_{ro} A}{q_T \mu_o} \frac{\partial p_{o/w}}{\partial z}}{1 + \frac{k_{ro}}{k_{rw}} \frac{\mu_w}{\mu_o}} \qquad (13)$$

The capillary pressure $p_{o/w}$ is actually in small amount comparing to the pressure of oil/water while oil is displacing by water in the reservoir. If the effect of capillary pressure is neglected, (13) becomes the following simple expression:

$$f_w = \frac{1}{1 + \frac{k_{ro}}{k_{rw}} \frac{\mu_w}{\mu_o}} \qquad (14)$$

Continuity equation is given by

$$-\frac{\partial q_w}{\partial z} = A\varphi \frac{\partial S_w}{\partial t} \qquad (15)$$

where φ is the porosity of the reservoir rock. Using the relations of $q_w = f_w q_T$ and $f_w(S_w)$, (15) is calculated as

$$\frac{\partial S_w}{\partial t} = -\frac{q_T}{A\varphi} \left(\frac{df_w}{dS_w} \frac{\partial S_w}{\partial z} \right) \qquad (16)$$

$$dS_w = \frac{\partial S_w}{\partial z} dz + \frac{\partial S_w}{\partial t} dt \qquad (17)$$

In (17), we follow a fluid of constant saturation during the displacement process; thus

$$0 = \frac{\partial S_w}{\partial z} dz + \frac{\partial S_w}{\partial t} dt \qquad (18)$$

Then, it follows that

$$\frac{\partial S_w}{\partial t} = -\frac{\partial S_w}{\partial z} \frac{dz}{dt} \qquad (19)$$

Substituting (19) into (16), we obtain

$$\frac{\partial z}{\partial t} = \frac{q_T}{A\varphi} \left(\frac{df_w}{dS_w} \right) \qquad (20)$$

Equation (20) is known as the Buckley-Leverett equation. It implies that the degree of advance of a plane of saturation S_w is equivalent to the composition change of the flowing stream with saturation. Equation (20) can be summarized to demonstrate the position of a particular saturation as a function of time.

$$z_{S_w} = \frac{q_T t}{A\varphi} \left(\frac{df_w}{dS_w} \right) + z_0 \qquad (21)$$

z_0 is the position of the water saturation at time $t = 0$.

Equation (21) calculates the saturation front in the petroleum reservoir at a constant rate along the z direction. The saturation front is direct proportion to $f_w' = df_w / dS_w$. The saturation front profile which is calculated by (21) is illustrated in Fig. 9 and denoted by abcd curve [7], [11]. The principle conservation of mass laws over the advance position has been applied by Morel-Seytoux [9]. Consider the conservation of mass law over the front position that A hatched area equal to the B hatched area (Fig. 9). The saturation established in the following system immediately behind the front Buckley-Leverett saturation can be calculated from tangent point c on the fractional flow curve (Fig. 11). The abrupt front in the saturation profile is given by the line cf in Fig. 11.
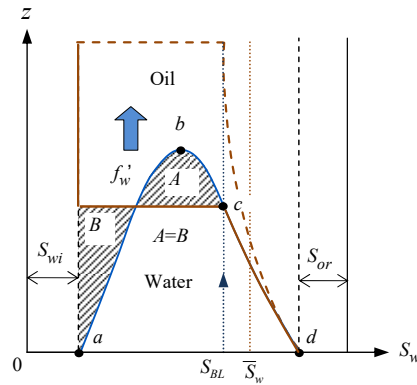

Fig. 9 Tentative saturation profile [12], [14]

IV. WATER DISPLACEMENT BY OIL

Water displacement by kerosene and heavy oil were determined in the sand column by laboratory experiment and Buckley-Leverett frontal displacement theory. The calculation is done in Experimental sand column illustrated in Fig. 2. As a result how much pore water displaced by kerosene and heavy oil, recovery factor (RF) of water and irreducible water saturation S_{wi}, were calculated successfully.

A. Experimental Investigation on Water Displacement by Oil

The sand column shown in experimental apparatus is illustrated in Fig. 2, first saturated by water. Kerosene is pumped through the saturated sand column to displaced pore water. Fig. 10 (a) shows the displacement of water by kerosene, $q_o = q_T$ and $q_w = 0$, (q_T is the total pumping rate). As illustrated in Fig. 10 (a), when soil pore was saturated by water, the fractional discharge at the outlet f_{wd} is 1 until some pore volume, and later some amount of kerosene discharged. Thus, there was only water discharged at first 10 min during the operation. After 10 min, the water fractional discharge decreased and oil discharge was increased. Finally at 16.4 min, water discharge has already ceased and only oil was

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discharged ($f_{od} = 1$). There was still some amount of water remained in the column that can refer to irreducible water saturation ($S_{wi} = 0.18$). The slashed area in Fig. 10 (a) indicates the mobile pore water displaced by oil, and the degree of saturation for the case is computed as the slashed area/Vp. In this experiment amount of water displaced by kerosene until 10 min was 32 cm^3 from total pore volume (52 cm^3). This experiment was done in 26 min and total discharge of fluids was $Q_T = 103$ cm^3.

Fig. 10 (b) illustrates water displacement by heavy oil, $q_o = q_T$ and $q_w = 0$, (q_T is the total pumping rate). There was pore water displaced by heavy oil during 16 min discharged of water. After 16 min water discharge dropped down and oil and some small amount of water started to flow out together until 18 min. Finally, water discharge has already stopped, only heavy oil discharged ($f_{od} = 1$). There was still some amount of water remained in the column that can refer to irreducible water saturation ($S_{wi} = 0.10$). The slashed area in Fig. 10 (b) indicated the mobile pore water displaced by oil, and the degree of saturation for this case is computed same as kerosene oil. In water displacement by heavy oil experiment, the total amount of displaced fluid until 15 min was 40 cm^3 from entire pore volume ($V_v = V \times n = 56.17$ cm^3). This experiment was done in 20 min and total discharge of fluids was $Q_T = 64$ cm^3.
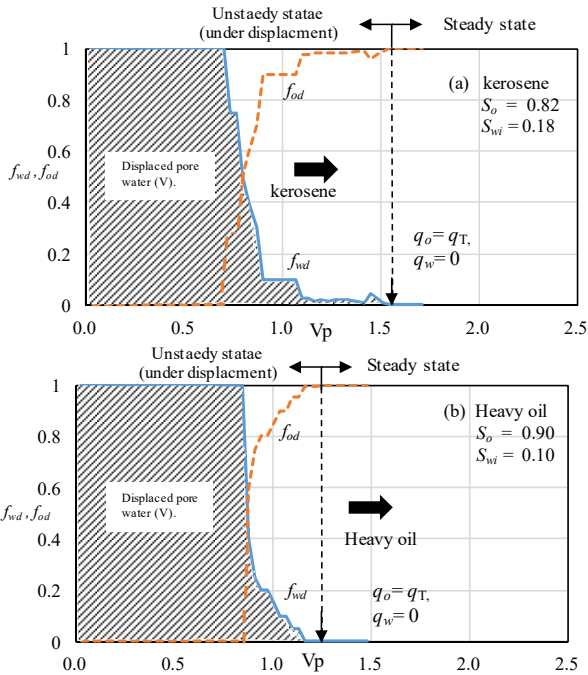

Fig. 10 Experimental result of the fractional discharge of pore water and oil curves

It has seemed from the experiment result that kerosene is displaced by water faster than heavy oil. It is also noticed that the amount of irreducible water saturation, S_{wi}, decreases as the viscosity of oil increases.

B. Buckley-Leverett Method for Water Displacement by Oil

Table I shows the property of oil, water and column area which applied for Buckley-Leverett frontal displacement analysis. Relative permeability of water displacement by kerosene was illustrated in Fig. 5 (a) were used in here. And the fractional flow of water can be calculated by (14). Relative permeability of water and oil curves is expressed as the following cubic functions with respect to effective water saturation for convenience [9].

$$k_{rw} = k_{rws} S_e^3 \qquad (22)$$

$$k_{ro} = (1 - S_e)^3 \qquad (23)$$

Here, S_e is the effective (normalized) saturation given by

$$S_e = \frac{S_w - S_{wi}}{1 - S_{wi} - S_{or}} \qquad (24)$$

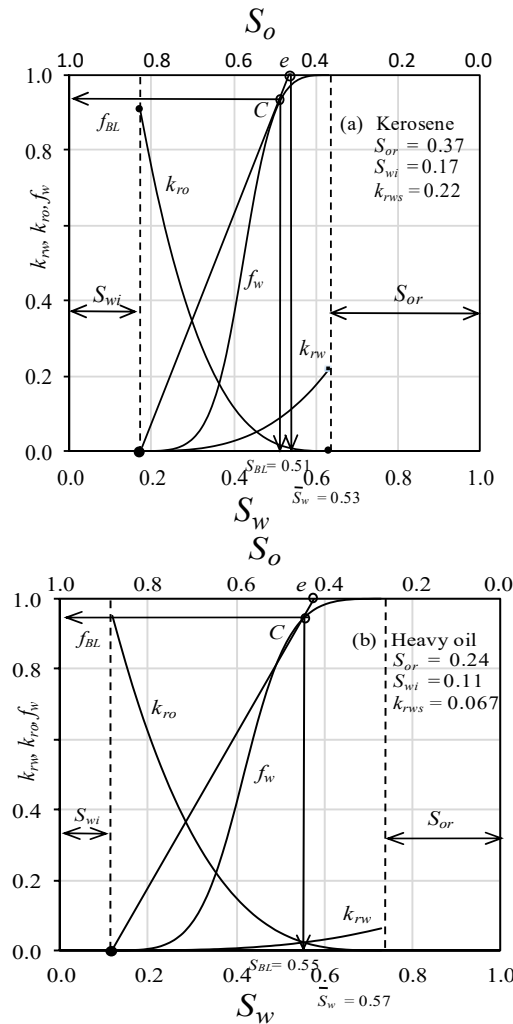
Fig. 11 Relative permeability for water displacement by oil curves

Fig. 11 (a) illustrated the relative permeability of kerosene and water k_{ro}, k_{rw} and fraction flow of water f_w. It is known from the experiment that discharge of fluids is 3.6 cm³ per min, so the water saturation at the front and the average saturation behind the fluid front are found through the graphical method to be $S_{BL} = 0.51$ and $S_w = 0.53$. As illustrated in Fig. 11 (a), the fraction flow of water is an S shape and by using intersection between the tangent line and $f_w = 1$ can calculate the S_{BL} and f_{BL}.

TABLE I
PROPERTY OF OIL, WATER AND EXPERIMENTAL COLUMN AREA APPLIED IN BUCKLEY-LEVERETT ANALYSIS

	Kerosene			Heavy oil	
H	20	(cm)	H	20	(cm)
A	7.065	(cm²)	A	7.065	(cm²)
n	0.376	-	n	0.38	-
Q_T	3	(cm³/minuet)	Q_T	2.77	(cm³/minute)
μ_w	0.01	(g/cm*s)	μ_w	0.01	(g/cm*s)
μ_o	0.0242	(g/cm*s)	μ_o	0.167	(g/cm*s)

Fig. 11 (b) illustrated the relative permeability of heavy oil and water and fraction flow of water. As explained in experiment part 7V.*A*. fluids discharged was 3.35 cm³ per min, through the graphic method to be $S_{BL} = 0.55$ and $S_w = 0.57$.

Fig. 12 (a) illustrated the results of saturation profile calculated by Buckley-Leverett analysis. It has seen the saturation front progresses upward at a constant speed, and kerosene reached at the top of the column at $t = 8$ min. As illustrated in Fig. 12 (a) the kerosene displaced water after 8 min discharged of saturated fluid. But there is still some amount of water remained in the column which may be discharged together with kerosene as point out in experiment calculation in part 7 V.*A*.

The amount of oil produced can be calculated as follows equation. For the area A and thickness B of the oil reservoir, (20) can be written in:

$$dz/B = f_w' q_T \, dt/A\phi B = f_w' q_T \, dt/V_P = f_w' dv_p \qquad (25)$$

Such, Vp = A φ B is the pore volume of the reservoir and d vp is the pore volume of water injected. Since the saturation in (25) is steady, the equation can be integrated:

$$z = B f_w' v_p \qquad (26)$$

It obtained the height of swept by water saturation concerning the pore volume of water injected v_p. When the front saturation reaches B, the water saturation at the front is S_{BL}, which make it possible to evaluate f_w' and compute v_p, the total amount of oil displaced by water in units of pore volume [8].

At breakthrough, when the water saturation front reaches the outlet face, the average saturation $\bar{S}_w$ behind the front can be calculated as:

$$\bar{S}_w = S_{wi} + \frac{1}{(df_w/dS_w)_{BL}} \qquad (27)$$

Through drawing a tangent line that starts at $S_w = S_{wi}$ and $f_w = 0$, having a point of tangency at $S_w = S_{BL}$ and $f_w = f_{BL}$, it ultimately made to intersect the line $f_w = 1$. The point e in Figs. 11 (a) & (b) represents S_w. The oil recovery factor of this condition can be calculated by

$$RF = \frac{\bar{S}_w - S_{or}}{1 - S_{or}} \qquad (28)$$

$$RFD = \frac{\bar{S}_w - S_{wi}}{1 - S_{wi} - S_{or}} \qquad (29)$$

The water recovery factor is calculated from (28) and found to be $RF = 0.43$, from which the total amount of water produced up to the breakthrough is $A\phi B \times RF = 23.04$ cm³ from total pore volume $(V_v = V \times n = 52$ cm³$)$ for the given sand column.

As a compare of experimental and Buckley-Leverett frontal displacement analysis, in experimental result the kerosene displaced water within 10 min and reach to the top of the column. But calculated results of saturation profile by Buckley-Leverett analysis in Fig. 12 (a) illustrated that oil displaced water in 8 min.

Fig. 12 (b) illustrated water displacement by heavy oil calculated by Buckley-Leverett frontal displacement analysis. It has seen the saturation front progresses upward at a constant speed, and breakthrough occurs at $t = 10$ min. As illustrated in Fig. 12 (b) oil reached to the top of the column after 10 min and 20 seconds discharged of water. But there is still some amount of water remained in the column which may be discharged together with oil as point out in experiment calculation in part 7V.*A*. The water recovery factor is calculated from (28), same as explained before and found to be $RF=0.51$, from which the total amount of water produced up to the breakthrough is $A\phi B \times RF = 27.46$ cm³ from total pore volume $(V_v = V \times n = 56.17$ cm³$)$ for the given sand column.

V. OIL DISPLACEMENT BY WATER

A. Experimental Investigation on Oil Displacement by Water

The column packed from sand illustrated in experimental apparatus in Fig. 2 part II *A*, first saturated by kerosene. Water pumped through the saturated sand column to displaced kerosene. Fig. 13 (a) shows displacement of kerosene by water only, $q_w = q_T$ and $q_o = 0$, (q_T is the total pumping rate). As illustrated in Fig. 13 (a) when soil pore was saturated by oil, the fractional discharge at the outlet f_{od} is 1 until some pore volume, and later some amount of water discharged. Thus there was only oil discharged at first 12 min during the

operation. After 12 min the oil fractional discharge decreased and water discharge was increased. Finally, after 22 min fluid flow experiment, kerosene discharge has already ceased, only water is discharged ($f_{od} = 1$) and the flow became the steady state. There was still some amount of kerosene remained in the column that can refer to residual oil saturation ($S_{or} = 0.18$).
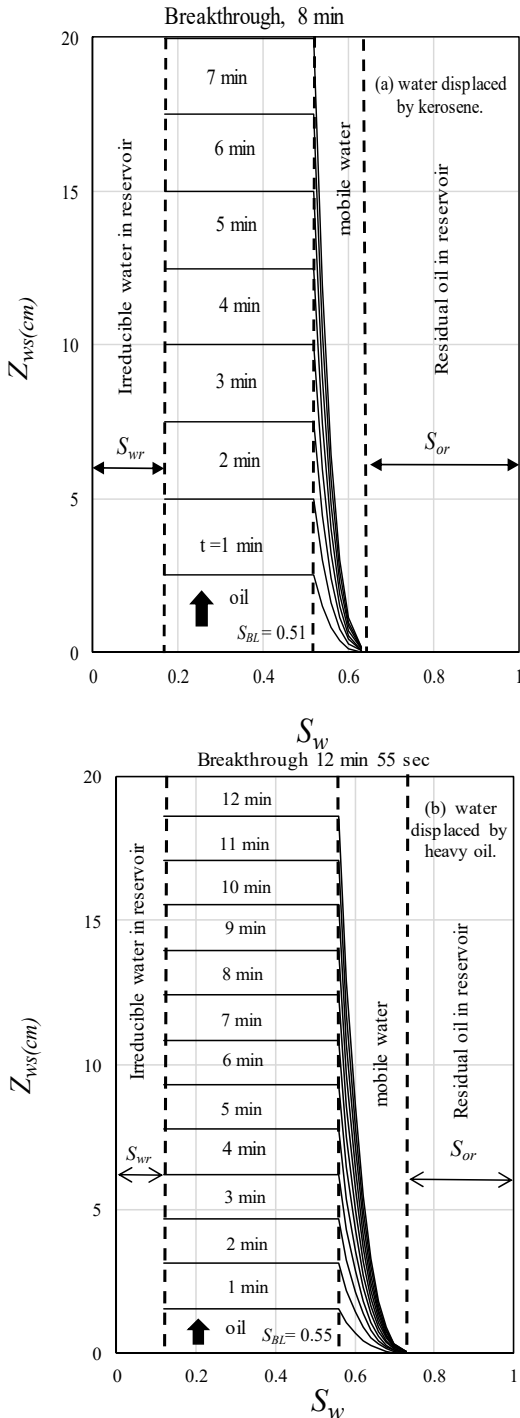

Fig. 12 Calculated results of saturation profile by Buckley-Leverett analysis for displacement of water by (a) kerosene (b) heavy oil

The slashed area in Fig. 13 (a) indicated the mobile pore oil displaced by water and the degree of saturation for the case is computed as the slashed area/Vp. In this experiment total amount of kerosene displaced by water until 12 min was 38 cm³ from total pore volume ($V_v = V \times n = 56.1 \text{cm}^3$). This experiment was done in 34 min and total discharge of fluids were $Q_T = 130$ cm³. Fig. 13 (b) illustrated the displacement of heavy oil by water only, $q_w = q_T$ and $q_o = 0$, (q_T is the total pumping rate). As illustrated in Fig. 13 (b) there was only heavy oil discharged at first 10 min during the operation. After 10 min the oil fractional discharge decreased and water discharge was increased. Finally at 31 min heavy oil discharge has already ceased, only water is discharged ($f_{wd} = 1$) and became steady state. There is still some amount of heavy oil remained in the column that can refer to residual oil saturation ($S_{or} = 0.13$). The slashed area in Fig. 13 (b) indicates the mobile pore oil displaced by water and the degree of saturation for the case is computed same as explained before. In this experiment the amount of heavy oil displaced by water until 10 min was 32 cm³ from total pore volume ($V_v = V \times n = 54.17$ cm³). This experiment done in 34 min and total discharge of fluids were $Q_T = 142$ cm³.
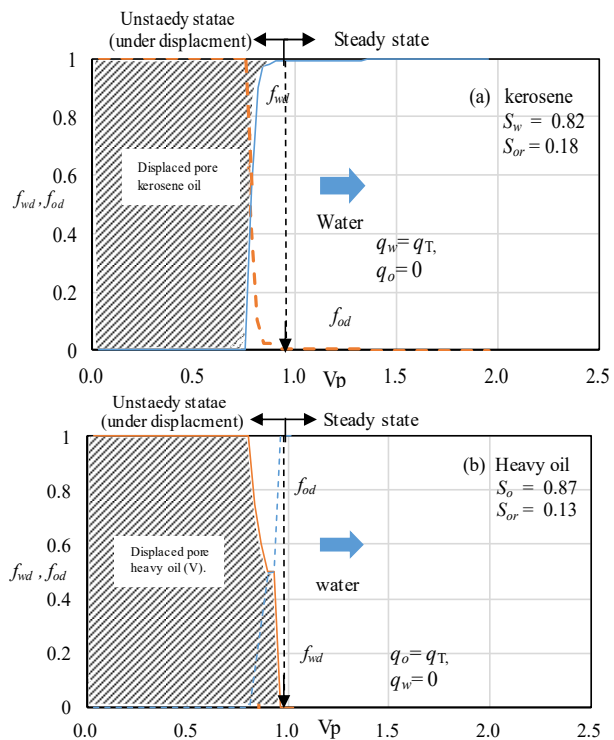

Fig. 13 Experimental result of fractional discharge of pore kerosene and heavy oil by water

B. Buckley-Leveret Method for Calculation of Oil Displacement by Water

Table II shows the property of kerosene and heavy oil, water and column area that applied at Buckley-Leveret frontal displacement analysis. Relative permeability of kerosene and

water k_{ro}, k_{rw} for displacement of kerosene by water shown in Fig. 8 was used in here. And fractional flow of water f_w can be calculated by (14).

Fig. 14 (a) illustrated the relative permeability of kerosene and water k_{ro}, k_{rw} and fraction flow of water f_w. It is known from the experiment that discharge of fluids is 3.86 cm³ per min so the water saturation at the front and the average saturation behind the front are found through the graphical method to be $S_{BL} = 0.49$ and $S_w = 0.51$. As illustrated in Fig. 14 (a), the fraction flow of water is an S shape and by using intersection between the tangent line and $f_w = 1$ can calculate the S_{BL} and f_{BL}.

Fig. 14 (b) illustrates the relative permeability of heavy oil and water and fraction flow of water. As it seems from the figure, the fraction flow of water is S shape and by using tangent point can find the S_{BL} and f_{BL}. It is known from the experiment that discharge of fluids is 4.17 cm³ per min so the $S_{BL} = 0.53$ and $S_w = 0.57$.

Fig. 15 (a) illustrated the results of kerosene displacement by water, calculated by Buckley-Leverett analysis. It has seen the saturation front progresses upward at a constant speed, and breakthrough occurs at $t = 6$ min. As illustrated in Fig. 15 (a) the water reached to the top of column after 6 min and 50 second and kerosene discharge has already ceased, there was still some amount of kerosene remained in the column which may be discharge with water together as point out in experiment investigation in part V A. The kerosene recovery factor is calculated from (28) and found to be $RF = 0.41$, from which the total amount of water produced up to the breakthrough is $A\varphi B \times RF = 23$ cm³ from total pore volume ($V_v = V \times n = 56.$ cm³) for the given sand column.

TABLE II
PROPERTIES OF OIL, WATER AND EXPERIMENTAL COLUMN AREA APPLIED IN BUCKLEY-LEVERET ANALYSIS

	Kerosene			Heavy oil	
H	20	(cm)	H	20	(cm)
A	7.065	(cm²)	A	7.065	(cm²)
n	0.386	-	n	0.385	-
Q_T	3.86	(cm³/minute)	Q_T	4.17	(cm³/minute)
μ_w	0.01	(g/cm*s)	μ_w	0.01	(g/cm*s)
μ_o	0.0242	(g/cm*s)	μ_o	0.167	(g/cm*s)

Fig. 15 (b) illustrated displacement of heavy oil by water analyzed by Buckley-Leverett frontal displacement theory. It is seen the saturation front progresses upward at a constant speed, and breakthrough occurs at $t = 6$ min. As shown in Fig. 15 (b), the water displaced heavy oil and reach to the top of column at 6 min and 50 second and breakthrough occurred, but there is still some amount of heavy oil remained in the column which may be discharged with water together as point out in experiment investigation in Part V A. The heavy oil recovery factor is also calculated from (28) same as kerosene and found to be $RF = 0.51$, from which the amount of water produced up to the breakthrough is $A\varphi B \times RF = 28$ cm³ from total pore volume ($V_v = V \times n = 54.17$ cm³) for the given experimental sand column.
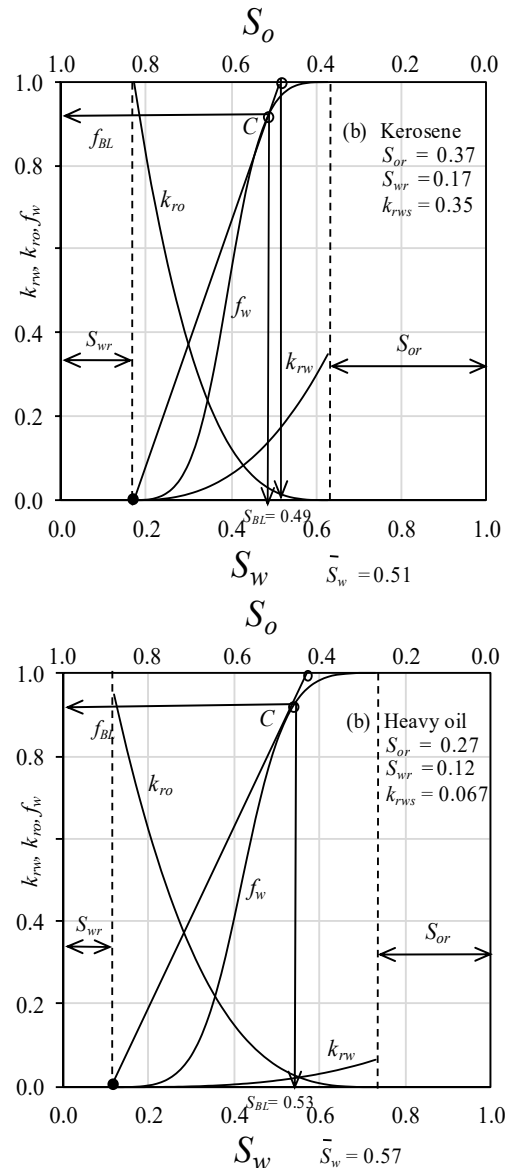

Fig. 14 Calculated results of saturation profile by Buckley-Leverett analysis for displacement of oil by water

VI. CONCLUSION

The conclusion of this investigation can be as:

1) Relative permeability is an important conductive parameter controlling immiscible displacement of multiphase fluids flow in a porous medium.

2) From relative permeability curves of oil and water with different residual oil and water saturations can be used for the design of waterflooding technique to evaluate the amount of oil recovery by artificially injecting water into the reservoir.

3) The rate of advance of waterfront in a vertical flow can be calculated based on the Buckley-Leveret frontal displacement theory by using fractional flow curves.

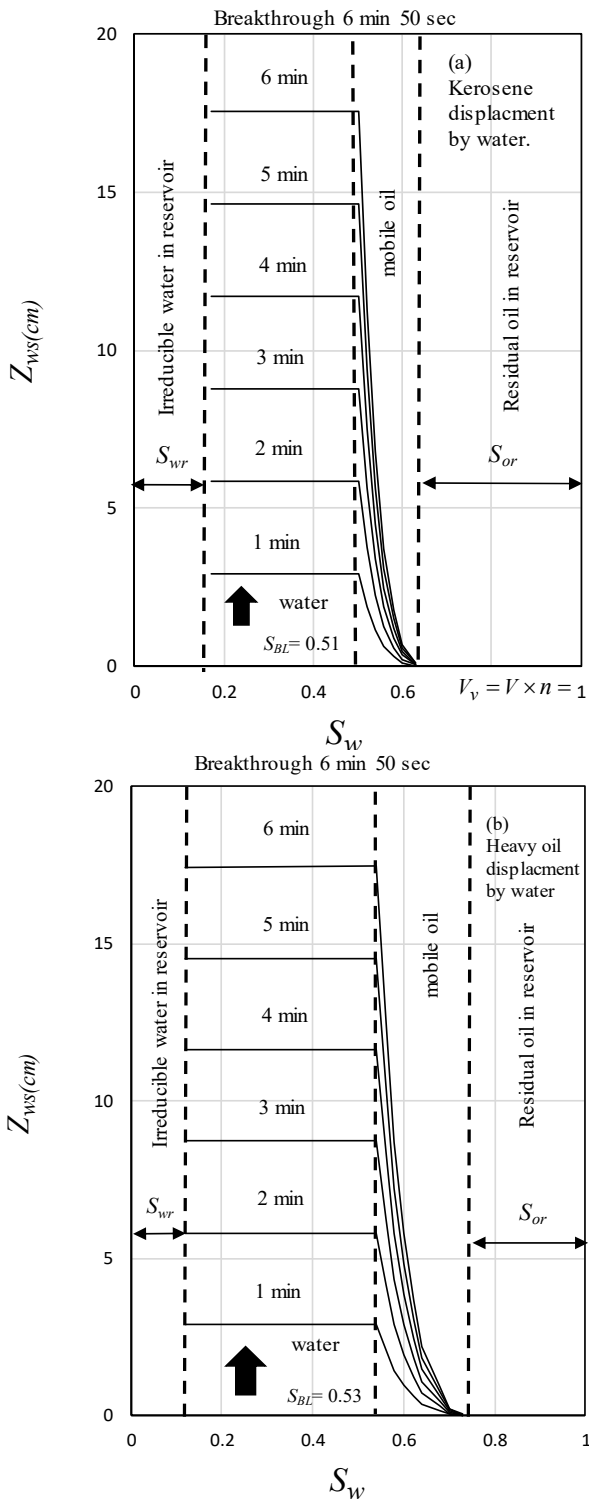4) The continuity equation may be presented to take the conservation of fluid mass into consideration.

Fig. 15 Calculated results of saturation profile by Buckley-Leverett analysis for displacement of heavy oil by water.

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