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Relaxing Convergence Constraints in Local Priority Hysteresis Switching Logic

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Abstract—This paper addresses certain inherent limitations of local priority hysteresis switching logic. Our main result establishes that under persistent excitation assumption, it is possible to relax constraints requiring strict positivity of local priority and hysteresis switching constants. Relaxing these constraints allows the adaptive system to reach optimality which implies the performance improvement. The unconstrained local priority hysteresis switching logic is examined and conditions for global convergence are derived.

Keywords—Adaptive control, convergence, hysteresis constant, hysteresis switching.

I. Introduction

ADAPTIVE control is usually used to control imprecisely known plants. The main goal of adaptive control is to achieve improved performance by choosing a controller k from given finite/infinite set of candidate controllers using real-time data and prior information. Two distinct methodologies have been used to achive this goal: multiple model adaptive control ([21], [24], [11]) and unfalsified adaptive control ([20], [6], [22]). In both cases, process is orchestrated by a smart unit called a supervisor, which is responsible for making a decision, at each instant of time, about when to switch and which controller should be used next, based on the available plant input/output data and a well-defined performance criterion. The general architecture of an adaptive control system is shown in Fig. 1.

One challenge facing switching adaptive systems is the type of instability called chattering in which the supervisor cycles endlessly among two or more of the candidate controllers without converging, even when there is no change in the plant. Convergence analysis becomes more complicated when the controller or the unknown process parameters vary over a continuum.

Fundamental contributions to the solution to the parameter convergence problem for this type of set were made by Hespanha et al. ([13], [12]) and Stefanovic et al. [23]. Results in these studies overcame the above difficulties of proving convergence for a continuum of parameters. In these studies, adaptive control convergence for the case of a continuum set of parameters is ensured by adding constraints to the switching logic requiring strictly positive local priority and hysteresis constants. The origin of these ideas is the hysteresis switching algorithm which introduced by Morse et al. [17]. Alhajri et al. [1] was able to proved that the requirement that the hysteresis constant h be strictly positive can be relaxed if the transfer function is equi-quasi-positive definite (EQPD).

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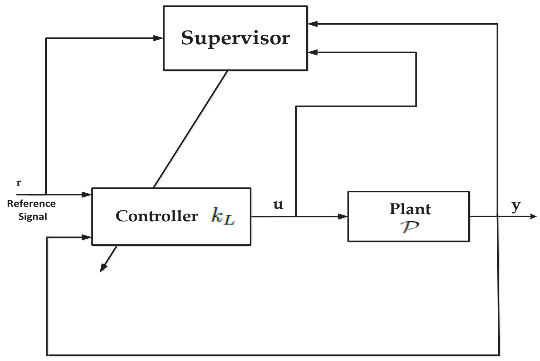

Fig. 1 Adaptive control system

Unfortunately, requiring a strictly positive hysteresis constant or imposing strict local priority constraints on switching scheme may prevent the adaptive control system from achieving optimality. In present paper we reexamine the adaptive control convergence in the context of the local priority hysteresis switching logic [13] and determine circumstances where the strict positivity constraints become unnecessary. Relaxing the strict positivity constraints become critical issue if highly performance is sought.

The paper is organized as follows. Section II collects the required and necessary definitions and notations and briefly reviews the local priority hysteresis switching logic. In Section III preliminary facts are given. Section IV contains the main result. Relation between adaptive controller convergence and the system performance is shown in Section V. Conclusions follow in Section VI.

II. Local Priority Hysteresis Switching Logic

In this section we outline local priority hysteresis switching logic for ease of reference. The contribution of the local priority hysteresis switching logic in the context of supervisory control is to introduce a new switching logic that has the ability to deal with the case when the unknown parameters of system belong to a continuum set. Using continuum set of candidate controllers instead of finite set will ensure more flexibility for the adaptive control system to deal with highly uncertainties plant ([13], [12], [23], [2]).

The main reason for introducing the supervisory control approach ([15], [16]) is to ensure a satisfactory performance (e.g., regulation and tracking problem) of a closed-loop system

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by switching among a given set of candidate controllers. The basic idea behind the controller selection strategy is to determine which nominal process model is associated with the smallest monitoring signals "$\mu_p(t)$", and then select the corresponding candidate controller. The supervisor consists of three subsections Multi-estimator "$\Sigma_{\mathbb{E}}$", Monitoring signal generator "$\Sigma_{\mathbb{M}}$" and Switching logic "$\Sigma_{\mathbb{S}}$", as shown in Fig. 2.

Now, suppose the uncertain process $\mathcal{P}$ shown in Fig. 2 admits the model of a SISO finite dimensional stabilizable and observable linear system whose control input and measured output signals are u and y respectively, u and y are the inputs of multi-estimator $\Sigma_{\mathbb{E}}$ and its output are the signals y_p, $p \in \mathbb{P}$, $\mathbb{P}$ is a compact subset of a finite-dimensional normed linear vector space. Each y_p would converge to y if the transfer function of $\mathcal{P}$ was equal to the nominal process model transfer function ϑ_p in the absence of disturbances, unmodeled dynamics and noises where disturbance input and noise signal are represented by d and n respectively. Inputs of the monitoring signal generator "$\Sigma_{\mathbb{M}}$" are the estimation errors

$$e_p = y_p - y, p \in \mathbb{P} \tag{1}$$

and its output are the monitoring signals $\mu_p, p \in \mathbb{P}$, where μ_p are defined to be the integral norms of the estimation errors. Switching logic "$\Sigma_{\mathbb{S}}$" is a system whose inputs are the monitoring signals μ_p and whose output are parameters that optimize the performance criterion $\hat{p}$, which defined as follow

$$\hat{p}(t) = \underset{p \in \mathbb{P}}{\operatorname{argmin}} \{\mu_p(t)\} \tag{2}$$

$\hat{p}(t)$ is taking its values in $\mathbb{P}$ and used to select the associated controller parameter.

Assumed that the transfer function of $\mathcal{P}$ from u, output of multi-controller "$\mathbb{K}$", to y belongs to a family of admissible process model transfer functions

$$F = \bigcup_{p \in \mathbb{P}} f(p) \tag{3}$$

for each p, $f(p)$ denotes a family of transfer functions 'centered' around some known nominal process model transfer function ϑ_p where p is a parameter taking values in some index set $\mathbb{P}$, $\mathbb{P}$ is typically a continuum. In the absence of noises, unmodeled dynamics and disturbances equation 3 will be equivalent to

$$V = \bigcup_{p \in \mathbb{P}} \vartheta_p \tag{4}$$

State-space equations for the three subsystems are described in detail in [13], recall that, the multi-estimator $\Sigma_{\mathbb{E}}$ has the following realization:

$$\dot{x}_{\mathbb{E}} = A_{\mathbb{E}} x_{\mathbb{E}} + b_{\mathbb{E}} y + d_{\mathbb{E}} u$$

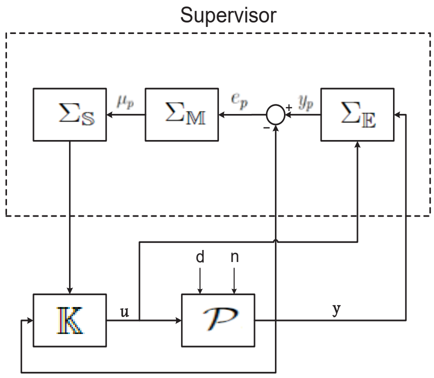

Fig. 2 Supervisory control block diagram

$$y_p = c_p x_{\mathbb{E}}, \qquad p \in \mathbb{P}$$

where $x_{\mathbb{E}}$ is estimated state and its assumed to be available for the controller in all time and $A_{\mathbb{E}}$ is a stable matrix.

The matrices c_p is design in such way for each $p \in \mathbb{P}$, c_p exists and unique (See [15] Section IV). Moreover, for the case of $\mathbb{P}$ to be continuum c_p assumed to depend linearly on p to ensure tractability property (See [15] Section XI). So the matrix c_p can be represented in the form:

$$c_p = p^T A + b \tag{5}$$

For SISO system, A is $n \times n$ nonzero matrix, p is $n \times 1$ unknown process parameters and b is $1 \times n$ vector.

In [13], the candidate controllers set $= \{k_p : p \in \mathbb{P}\}$ is chosen in such a way that for each $p \in \mathbb{P}$; k_p a controller that stabilizes all the process models in $f(p)$, where $\mathcal{P}$ is any element of F. It has been assumed that there is a controller in the candidate controller set that able to stabilize each unknown process $\mathcal{P}$.

The inputs of the local priority hysteresis switching logic are continuous signals, μ_p, and it is assumed to be strictly positive nondecreasing function. A set D_γ is define as follow

$$D_\gamma(q) := \{p \in \mathbb{P} : |q - p| \leq \gamma\} \tag{6}$$

where γ is a proper positive constant and $|\cdot|$ is a norm function in $\mathbb{P}$. The output of the switching logic, at each instant of time, is parameter that optimize the performance criterion "$\hat{p}(t)$". Pick a hysteresis constant $h > 0$ and set $\hat{p}(0) = \underset{p \in \mathbb{P}}{\operatorname{argmin}} \{\mu_p(0)\}$. Suppose that at time t_i, $\hat{p}(t_i)$ has just switched to some $q \in \mathbb{P}$ and kept fixed until a time $t_{i+1} > t_i$ such that the following inequality is satisfied:

$$(1 + h) \min_{p \in \mathbb{P}} \{\mu_p(t_{i+1})\} \leq \min_{p \in D_\gamma(q)} \{\mu_p(t_{i+1})\}$$

At this time, we set $\hat{p}(t_{i+1}) = \underset{p \in \mathbb{P}}{\operatorname{argmin}} \{\mu_p(t_i + 1)\}$. By repeating these steps we can generate a sequence of switching signal which will converge as time increase.

If $\hat{k}(t)$ is the controller parameter associated with the process parameter $\hat{p}(t)$. At each switching time t_i, the active controller in the feedback loop $k_L(t)$ is changed to

$$k_L(t_i) = \hat{k}(t_i). \tag{7}$$

A key result is the local priority hysteresis switching convergence lemma, which may be stated as follows. [Convergence Lemma [13]] Suppose that both of the following hold:

1) Monotonicity: For all p it holds that
$$\mu_p(t) \geq \mu_p(\tau) \text{ for all } t > \tau$$
2) Feasibility: There exists a $p^\star \in \mathbb{P}$ for which the monitoring signal is uniformly bounded
$$\sup_t \mu_{p^*}(t) < \infty.$$

Then, if *hysteresis constant* h and constant γ are strictly positive, the local priority hysteresis switching logic converges after at most finitely many controller switches.

A concern with the strictly positive constant γ required by Lemma II is that, the adaptive system does not switch to a new parameter $\hat{p}(t)$ that minimizes the monitoring signal $\mu_p(t)$ if this parameter happen to be in the set D_γ (i.e. $\hat{p}(t) \in D_\gamma$) . The other noticeable with this lemma is the strictly positive hysteresis constant h inherently tends to slow supervisor's adaptive response and it limits the accuracy with which the supervisor is able to minimize the monitoring signal $\mu_p(t)$ to $\pm h$. Using a smaller γ and h can partially address these concerns, but as these constants decreased toward zero the number of parameter switches usually tends to increase and chattering instability may sometimes occur in the limit as $\{\gamma, h\} \to 0$ — *though not always*.

The main contribution of this work is to reexamine the adaptive control convergence in the context of the local priority hysteresis switching logic when relaxing the constraints (i.e. $h > 0$ and $\gamma > 0$) on the switching scheme. Relaxing these constraints allowing supervisor to respond instantaneously and continuously using the zero-hysteresis optimal adaptive law

$$k_L(t) = \hat{k}(t) \tag{8}$$

III. Preliminaries

Suppose that $f : R^n \to R$ is twice differentiable on $X \subset R^n$ and that, for some $\alpha > 0$,

$$\nabla^2 f(x) \geq \alpha I \ \forall x \in X. \tag{9}$$

Then, we say f is *strongly convex (or uniformly convex)* on X.

One implication of uniform convexity is that if $f(x)$ is uniformly convex on a connected set $X \subset R^n$, then for every $\alpha > 0$ satisfying (9) we have [8, Prop. A.23]

$$f(y) - f(x) = (\nabla f(x))^T (y-x) + \int_0^1 \int_0^1 (y-x) \nabla^2 f(x+\tau y)(y-x) d\tau dt \geq (\nabla f(x))^T (y-x) + \frac{\alpha}{2}\|y-x\|^2 \tag{10}$$

for any $\alpha > 0$ satisfying (9).

(second-order Taylor-theorem expansion) Let $C \subseteq \mathbb{R}^n$ and let $f : \mathbb{R}^n \mapsto \mathbb{R}$ be twice continuously differentiable over C then,

$$f(x) = f(a) + \nabla f(a)(x-a) + \nabla^2 f(\xi)$$
$$a \leq \xi \leq x \quad \text{or} \quad \xi = \alpha a + (1-\alpha)x \quad \text{for} \quad \alpha \in [0,1]$$

where the gradient $\nabla f(x)$ of the function $f(x)$ is a row vector of size n, i.e.,

$$\nabla f(x) = \left(\frac{\partial f}{\partial x_1}(x), \quad \frac{\partial f}{\partial x_2}(x), \quad \cdots, \quad \frac{\partial f}{\partial x_n}(x) \right)$$

the Hessian $\nabla^2 f(x)$ is an $n \times n$ matrix;

$$\nabla^2 f(x) = \begin{pmatrix} \frac{\partial^2 f}{\partial x_1^2}(x) & \frac{\partial^2 f}{\partial x_1 \partial x_2}(x) & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n}(x) \\ \frac{\partial^2 f}{\partial x_2 \partial x_1}(x) & \frac{\partial^2 f}{\partial x_2^2}(x) & \cdots & \frac{\partial^2 f}{\partial x_2 \partial x_n}(x) \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_n \partial x_1}(x) & \frac{\partial^2 f}{\partial x_n \partial x_2}(x) & \cdots & \frac{\partial^2 f}{\partial x_n^2}(x) \end{pmatrix}$$

and

$$x - a = \begin{pmatrix} x_1 - a_1 \\ x_2 - a_2 \\ \vdots \\ x_n - a_n \end{pmatrix}$$

Reference [7] (Weierstrass theorem) Let $\mathbb{P}$ be a non empty subset of $\mathbb{R}^n$ and let $\mu : \mathbb{P} \mapsto \mathbb{R}$ be lower semicontinuous at all points of $\mathbb{P}$. If $\mathbb{P}$ is compact, then $\hat{p}(t) = \underset{p \,\in\, \mathbb{P}}{\operatorname{argmin}}\ \mu_p(t)$ exists.

Recall that, the authors in [13] used the integral norms of estimation errors as monitoring signal:

$$\mu_p(\tau) = \int_0^\tau \|e_p(t)\|^2 dt \tag{11}$$

where $e_p = y_p - y$ and $y_p = c_p x_{\mathbb{E}}$ so, $\mu_p(t)$ can be written as

$$\mu_p(\tau) = \int_0^\tau \|(p^T A + b)x_{\mathbb{E}}(t) - y(t)\|^2 dt \tag{12}$$

Then,

$$\nabla_p^2(\mu_p(\tau)) = 2\int_0^\tau A x_{\mathbb{E}}(t) x_{\mathbb{E}}^T(t) A^T dt \tag{13}$$

We say that the system is *persistently excited* if, for all sufficiently large $\tau > 0$ and all p it holds that $\nabla_p^2(\mu_p(\tau)) \geq \alpha I$ for some $\alpha > 0$.

Under the persistent excitation assumption, the function $\mu_p(\tau)$ is uniformly convex function in p for sufficiently large time t.

Therefore, whenever the system is persistently excited, this monitoring signal has a property of being uniformly convex after some time. The persistent excitation (PE) property

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defined here is crucial in many adaptive schemes where parameter convergence is one of the objectives and is closely related to convergence conditions of ([3]-[5], [9], [10], [19]).

The following lemmas will be used in proving our main result.

Let $\mu_p(t)$ be monotonically increasing in t for all p and suppose a minimizing value $\hat{p}(t) = \underset{p}{\operatorname{argmin}}\{\mu_p(t)\}$ exists for all t. Then,

$$\mu_{\hat{p}(t_m)}(t_m) \geq \mu_{\hat{p}(t_n)}(t_n) \text{ for all } t_m \geq t_n.$$

Proof
By monotonicity

$$\mu_p(t_m) \geq \mu_p(t_n) \ \forall t_m \geq t_n \tag{14}$$

Also, since $\hat{p}(t)$ minimizes $\mu_p(t)$

$$\mu_{\hat{p}(t)}(t) \leq \mu_p(t) \ \forall p \in \mathbb{P}. \tag{15}$$

From (14) $\mu_{\hat{p}(t_m)}(t_m) \geq \mu_{\hat{p}(t_m)}(t_n)$ and from (15) $\mu_{\hat{p}(t_m)}(t_n) \geq \mu_{\hat{p}(t_n)}(t_n)$. Hence,

$$\mu_{\hat{p}(t_m)}(t_m) \geq \mu_{\hat{p}(t_n)}(t_n) \ \forall \ t_m \geq t_n.$$

Let $\mu_p(t)$ be monotonically increasing in t for all p and suppose a minimizing value $\hat{p}(t) = \underset{p}{\operatorname{argmin}}\{\mu_p(t)\}$ exist for all t. If the system is persistently excited (Def. III) then, $\mu_{\hat{p}(t_m)}(t_m) - \mu_{\hat{p}(t_n)}(t_n) \geq \phi(||\hat{p}(t_m) - \hat{p}(t_n)||) \quad \forall \ t_m \geq t_n$.

Proof
Using second-order Taylor-theorem expansion the monitoring signal $\mu_p(t)$ can be written as:

$$\mu_p(t) = \mu_{\hat{p}(t)}(t) + (p - \hat{p}(t))^T \ \nabla_p(\mu_{\hat{p}(t)}(t)) + \tfrac{1}{2}(p - \hat{p}(t))^T \nabla_p^2(\mu_{\xi(t)}(t)) \ (p - \hat{p}(t)) \tag{16}$$

where $\xi(t)$ can be written as $\quad \alpha p + (1-\alpha)\hat{p}(t) \ ; \quad \alpha \in [0, 1]$

Since $\hat{p}(t)$ minimizes $\mu_p(t)$, we have

$$\nabla_p(\mu_{\hat{p}(t)}(t)) = 0 \tag{17}$$

Also, since the system is persistently excited, then

$$\nabla_p^2(\mu_p(t)) \geq \alpha > 0 \tag{18}$$

From (17), and (18) equation (16) can be written as

$$\mu_p(t) - \mu_{\hat{p}(t)}(t) \ \geq \frac{\alpha}{2}\|p - \hat{p}(t)\|^2 \tag{19}$$

or, equivalently,

$$\mu_{\hat{p}(t_m)}(t_n) - \mu_{\hat{p}(t_n)}(t_n) \ \geq \frac{\alpha}{2}\|\hat{p}(t_m) - \hat{p}(t_n)\|^2 \tag{20}$$

By monotonicity

$$\mu_{\hat{p}(t_m)}(t_m) \geq \ \mu_{\hat{p}(t_m)}(t_n) \quad \forall t_m \geq t_n \tag{21}$$

Therefore,

$$\mu_{\hat{p}(t_m)}(t_m) - \mu_{\hat{p}(t_n)}(t_n) \geq \mu_{\hat{p}(t_m)}(t_n) - \mu_{\hat{p}(t_n)}(t_n)$$
$$\geq \tfrac{\alpha}{2}\|\hat{p}(t_m) - \hat{p}(t_n)\|^2 \ \forall \ t_m \geq t_n$$

and hence for all $t_m \geq t_n$

$$\mu_{\hat{p}(t_m)}(t_m) - \mu_{\hat{p}(t_n)}(t_n) \geq \ \frac{\alpha}{2}\|\hat{p}(t_m) - \hat{p}(t_n)\|^2.$$

IV. Main Result

The following theorem establishes that under persistent excitation assumption if one relax the requirement that the local priority and hysteresis constants be strictly positive (i.e., that $\gamma > 0$ and $h > 0$) in the local priority hysteresis switching logic convergence lemma, one still obtains convergence of the optimal process parameter $\hat{p}(t)$, defined in (2), as $t \to \infty$ under the same conditions in [13] and by using the same monitoring signal. Relaxing these strict positivity requirements overcomes the accuracy limitation mentioned above and allows convergence to optimality.

Consider the Supervisory control system in Fig. 2.
Suppose that both of the following hold:

1) Monotonicity: For all p it holds that

$$\mu_p(t) \geq \mu_p(\tau) \text{ for all } t > \tau$$

2) Feasibility: There exists a $p^\star \in \mathbb{P}$ for which the monitoring signal is uniformly bounded

$$\sup_t \mu_{p^*}(t) < \infty.$$

If the systems is persistently excited (Def. III), then the optimal process parameter $\hat{p}(t)$ converges as t increases to infinity to a point in the closure of the set $\mathbb{P}$.

Proof:
By feasibility $\mu_{m^*} = \sup_t \mu_{\hat{p}(t)}(t)$ exists and, by Lemma III, $\mu_{\hat{p}(t)}(t)$ is monotonic in t and, by feasibility, it is bounded above. Hence,

$$\mu_{m^*} \ = \ \lim_{t\to\infty} \mu_{\hat{p}(t)}(t) \tag{22}$$
$$\geq \ \mu_{\hat{p}(t)}(t) \quad \forall t \tag{23}$$

Since the systems is persistently excited, it follows from Lemma III that for all $t_m \geq t_n$

$$\mu_{\hat{p}(t_m)}(t_m) \ - \ \mu_{\hat{p}(t_n)}(t_n) \ \geq \ \frac{\alpha}{2}\|\hat{p}(t_m) - \hat{p}(t_n)\|^2 \tag{24}$$

So, for all $t_m \geq t_n$ it holds that

$$\begin{aligned} \mu_{m^*} - \mu_{\hat{p}(t_n)}(t_n) \ &\geq \ \mu_{\hat{p}(t_m)}(t_m) - \mu_{\hat{p}(t_n)}(t_n) \\ &\geq \ \frac{\alpha}{2}\|\hat{p}(t_m) - \hat{p}(t_n)\|^2 \end{aligned}$$

Thus, for every $\epsilon > 0$ there exists t_ϵ such that for all $t_n, t_m \geq t_\epsilon$

$$\epsilon \geq \mu_{\hat{p}(t_m)}(t_m) - \ \mu_{\hat{p}(t_n)}(t_n) \geq \underbrace{\frac{\alpha}{2}\|\hat{p}(t_m) - \hat{p}(t_n)\|^2}_{\phi}.$$

and hence $\frac{\alpha}{2}\|\hat{p}(t_m) - \hat{p}(t_n)\|^2 \to 0$ as $t \to \infty$. Since, $\alpha > 0$ then, ϕ is nondecreasing continuous function satisfies $\phi(0) = 0$ and $\phi(x) > 0$ for $x > 0$, it follow that for every $\delta > 0$, there exists a t_δ such that $\|\hat{p}(t_m) - \hat{p}(t_n)\|^2 < \delta$ for all $t_n, t_m \geq t_\delta$. Therefore, the sequence $\{\hat{p}(t)\}_{t=0}^{\infty}$ is Cauchy. Since every Cauchy sequence converges [18], it follows that $\hat{p}(t)$ converges as $t \to \infty$ to a point in the closure of the set $\mathbb{P}$.

V. Performance Improvement

According to the certainty equivalence concept [14]:

> *"The nominal process model with the smallest performance criterion signal "best" approximates the actual process, and therefore the candidate controller associated with that model can be expected to do the best job of controlling the process."*

The basic idea behind the controller selection strategy is to determine which nominal process model is associated with the smallest monitoring signals, and then select the corresponding candidate controller.

As shown in theorem IV, the idea introduced in this manuscript (which relies on relaxing the local priority hysteresis switching logic constraints) improves adaptive controller convergence. By certainty equivalence concept [14], this idea improves the adaptive control performance.

VI. Conclusion

In this paper we discussed recent progress in the design and analysis of the hysteresis switching algorithm for the case of infinite parametric uncertainty (ranging over a continuum). We have examined the adaptive control convergence in the context of the local priority hysteresis switching logic; our main result establishes that when the convergence lemma conditions (i.e. monotonicity and feasibility) hold, then assuming persistent excitation assumption in the local priority hysteresis switching logic study is sufficient to ensure convergence without adding constraints on switching scheme requiring strict positivity of the hysteresis or local priority constants (i.e. $h > 0$ and $\gamma > 0$). Relaxing the strict positivity constraints overcomes the accuracy limitations associated with the local priority hysteresis switching algorithm by allowing the switching scheme to pick the parameter $\hat{p}(t)$ that minimizes the monitoring signal $\mu_p(t)$ at each time t.

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