

# A Wall Law for Two-Phase Turbulent Boundary Layers

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**Abstract**—The presence of bubbles in the boundary layer introduces corrections into the log law, which must be taken into account. In this work, a logarithmic wall law was presented for bubbly two phase flows. The wall law presented in this work was based on the postulation of additional turbulent viscosity associated with bubble wakes in the boundary layer. The presented wall law contained empirical constant accounting both for shear induced turbulence interaction and for non-linearity of bubble. This constant was deduced from experimental data. The wall friction prediction achieved with the wall law was compared to the experimental data, in the case of a turbulent boundary layer developing on a vertical flat plate in the presence of millimetric bubbles. A very good agreement between experimental and numerical wall friction prediction was verified. The agreement was especially noticeable for the low void fraction when bubble induced turbulence plays a significant role.

**Keywords**—Bubbly flows, log law, boundary layer.

## I. INTRODUCTION

IN a wide variety of engineering systems, the turbulent bubbly two phase flows play an essential role in many domains such as heat exchangers, petroleum transportation systems and nuclear reactors. Therein, accurate predictions of the flow characteristics are essentially required for the design, process optimization and safety control. With the development of the experimental techniques and computational fluid dynamics (CFD), numerous researches on the turbulent bubbly flow have been carried out [8], [15]-[20], on the basis of the improvement of understanding and modeling the turbulent bubbly flow. However, the existence of the multi-deformable and moving interfaces therein could induce the significant discontinuities of the fluid properties and the complex flow field near the interface, To understand the physical process and develop the model of the turbulent bubbly flows, the detailed flow information such as the drag resistance, the temporal and spatial evolutions of velocities and turbulence in two phases and the detailed characteristic of bubbles such as the bubble concentration, the bubble size, the bubble shape and the bubble motion are necessary.

In the case of boundary layer flow multiple efforts for the modeling and understanding of the flow characteristics and physical process have been performed, the experimental results on the boundary layer development on a vertical flat plate indicate that the near wall average velocity profiles in two-phase bubbly flows has a logarithmic behaviour. These experiences also show that the constants of the logarithmic profiles are sensibly modified in bubbly flows and the

experimental results indicate that these constants depend on the amplitude of the wall void fraction peaking but most of the models employed  $k-\epsilon$  closure to model Reynolds stress in the liquid phase. Over the last few years, there have been serious efforts to understand the near-wall region of gas-liquid, bubbly turbulent flows and to propose wall-functions specifically designed for these flows [16], [17].

Reference [1] developed the measurements in an upward turbulent bubbly boundary layer along a vertical flat plate based on LDV. Therein, the authors focused on the void fraction distribution, the wall shear stress, and the mean liquid velocity profiles. According to their study, the lateral bubble migration toward to the wall occurs depending on the bubble mean diameter and the void fraction similar to the duct flow. In addition, the wall skin friction coefficient was observed to increase because of the presence of the bubbles, which modifies the universal logarithmic law near the wall. This supports the idea of [2]. The authors studied a turbulent boundary layer developing on a vertical flat plate in the presence of millimetric bubbles and showed that the slope of the logarithmic law tends to decrease when the peak of void fraction is located in the logarithmic region. Reference [3] developed a near wall function for isothermal bubbly flows based on the asymptotic methodology and they proposed two approaches: The first one based on the void fraction distribution with an assumed constant and the other one based on a model to predict the wall peaking effect. Mikielewicz tested his approaches against the experimental data of [2]. Reference [4] performed experiment of a turbulent boundary layer for an air-water dispersed bubbly flow in a 20 mm×100 mm vertical rectangular channel having a void fraction smaller than 3%. The authors obtained the acceleration of the liquid velocity in the vicinity of the wall when liquid flow rate is reduced. Recently, [5] used an innovative measuring techniques PTV (Particle Tracking Velocimetry) that can provide whole-field and multi-scale measurement of two phase flow turbulence parameters. Measurements of the liquid parameters such as the velocity, RMS of the liquid velocity, and Reynolds stress were provided. More recently, [6] proposed a new two-fluid model averaging in the near-wall region; this approach is validated with the experimental data boundary layer, laminar flow and turbulent flow in pipes. The comparisons between the numerical results with the experimental data are in good agreements All models relied on a single-phase logarithmic law of the wall as a boundary condition. However, single-phase wall law is not valid for turbulent bubbly boundary layer.

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The purpose of this work is to obtain a direct formulation of two-phase logarithmic wall law. The presented wall law contained empirical constant. This constant was deduced from experimental data. In this first part, we presented the wall law for two-phase turbulent boundary layers. Finally we compare our results to the experimental data obtained by [2] in the case of a turbulent boundary layer developing on a vertical flat plate in the presence of millimetric bubbles.

II. TWO PHASE WALL LAW

In this section, we use the experimental results obtained by [2] in the case of a turbulent boundary layer developing on a vertical flat plate in the presence of millimetric bubbles. The logarithmic plot of the velocity in terms of the inner variable,  $y^+ = \frac{u_* y}{\nu}$ , shows that the three zones are usually encountered in a single-phase boundary layer are preserved: viscous sublayer, logarithmic zone, and the wake region (Fig. 1).

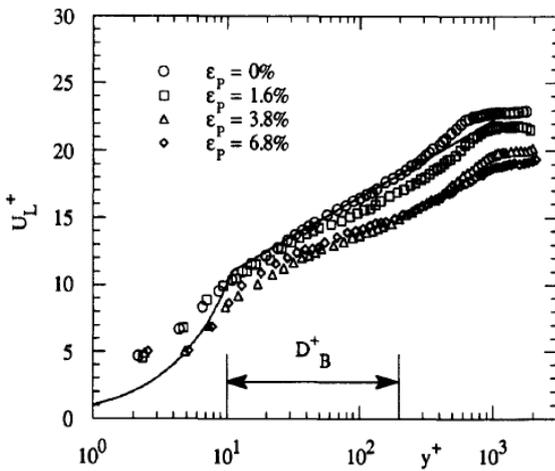


Fig. 1 Velocity profile plotted in inner variables [5]

In the logarithmic zone,  $30 \leq y^+ \leq 200$ , [1] shows that the velocity profile can be described by a logarithmic law, whose constants  $\kappa$  and  $C$  differ from the single-phase flow values  $\kappa^{SP}$  and  $C^{SP}$  and functions of the peak void fraction and the mean liquid velocity. By attaching the turbulent friction in two-phase flow to the one in the single-phase flow, we will determine the constants  $\kappa^{TP}$  and  $C^{TP}$  of the logarithmic law in two-phase flow:

$$\frac{\bar{U}}{U_w^{TP}} = \frac{1}{\kappa^{TP}} \ln(y_+) + C^{TP} \tag{1}$$

In single-phase flow, the logarithmic law is given by:

$$\frac{\bar{U}}{U_w^{SP}} = \frac{1}{\kappa^{SP}} \ln(y_+) + C^{SP} \tag{2}$$

where

$$\frac{\bar{U}}{U_w^{TP}} \tag{3}$$

is the normalized liquid velocity parallel to the wall.

$$y_+ = \frac{y U_w^{TP}}{\nu_c} \tag{4}$$

is the normalized distance normal to the wall.

$$U_w^{TP} \tag{5}$$

is the two-phase frictional velocity defined as,

$$U_w^{TP} = \sqrt{\frac{\tau_w^{TP}}{\rho}} \tag{6}$$

$\tau_w^{TP}$  is the two-phase wall shear stress exerted on liquid.  $\kappa^{TP}$  and  $C^{TP}$  are the two-phase von Karman and additive.

It is assumed that the liquid turbulent stress in the log region can be represented as the sum of two components. The first component accounts for shear induced turbulence. The second component is associated with the wakes of bubbles present in the inner layer. Such superposition is supported by experimental data of [7]. For low void fraction in the boundary layer, the velocity gradient in a two-phase boundary layer can be written as:

$$\frac{dy}{\kappa^{OP} y} = \frac{dU_c}{\chi U_w^{TP}} \tag{7}$$

where

$$\chi = \left[ (1 - \alpha_g) \left[ 1 + \frac{\alpha_p U_r K_L}{\kappa^{SP} U_w^{TP}} - \frac{\alpha_E U_r K_L}{\kappa^{SP} U_w^{TP}} \right] \right]^{-1} \tag{8}$$

is the correction coefficient.

Empirical correction factor  $K_L$ , is introduced to account for the non-linear interaction between bubble and shear induced turbulence fields. A new frictional velocity is introduced

$$U_w^{(TP)^c} = \chi U_w^{TP} \tag{9}$$

Then, a solution of (7) will be the logarithmic law:

$$U_+^{(TP)^c} = \frac{1}{\kappa^{TP}} \ln(y_+^c) + C^{(TP)^c} \tag{10}$$

where all wall variables are calculated using new velocity scale:

$$U_+^{(TP)^c} = \chi U_+^{TP} \tag{11}$$

$$\kappa^{TP} = \kappa^{OP} \chi^{-1} \tag{12}$$

Local slip velocity in (8) was evaluated using the distorted bubble expression [14]:

$$U_r = 0.934 (1 + 1.41 \alpha) j + 1.53 \left( \frac{\sigma g}{\rho_l} \right)^{1/4} \tag{13}$$

where  $\sigma$ : the surface tension,  $g$ : gravitational acceleration,  $J$ : superficial velocity,  $\rho$ : Density of liquid.  $\kappa_L$  can be approximated by:

$$\kappa_l = -0.0071 e^{1.4970 U_r^{TP}} \tag{14}$$

On the other hand and despite some scatter in the data, [1] showed that the non-dimensional thickness,  $s_0$ , of the viscous sublayer (defined here, as the ordinate of the intersection of the logarithmic and linear parts of the velocity profile) is approximately constant. So, we can determine the constant  $C$  as:

$$C^{TP} = 10 (1 - \chi^{-1}) + 4.9 \chi \tag{15}$$

### III. RESULTS AND DISCUSSIONS

#### A. CFD Simulation set-up

Reference [1] developed the measurements in an upward turbulent bubbly boundary layer along a vertical flat plate based on LDV. The hydrodynamic tunnel is a closed loop, with a 50 m<sup>3</sup> tap water tank and a 2.5 m long vertical square channel, whose cross section is 400 × 400 mm<sup>2</sup>, Fig. 2. It is operated in the upward direction at atmospheric pressure, ambient temperature and at liquid velocities  $U_L$  which do not exceed 1.5 m/s. Air is blown uniformly into the water, Therein, they focused on the void fraction distribution, the wall shear stress, and the mean liquid velocity profiles. According to their study, the lateral bubble migration toward to the wall occurs depending on the bubble mean diameter and the void fraction similar to the duct flow.

The computations were performed using (CFX 15.0), the model of [8] was implemented in CFX. In the present study, we use the drag force and the lift force proposed respectively by [9] and [10], the turbulent dispersion force obtained by [11], the wall force of [12] and the eddy viscosity proposed by [13]. Geometric modeling and meshing was done using the meshing software Pointwise 16.0. Convergence was tested by requiring the sum of the absolute residual values to be less than 10<sup>-6</sup>. The residual value is calculated for each solved variable and it is equal to the absolute difference between left- and right-hand sides of the different equations are solved at each node point. Relaxation does not alter the final solution, but affects only the way in which it is achieved. The final mesh is shown in Fig. 2.

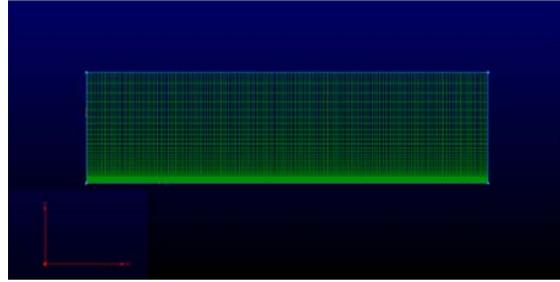


Fig. 2 The Computational Domain and Typical CFX Mesh

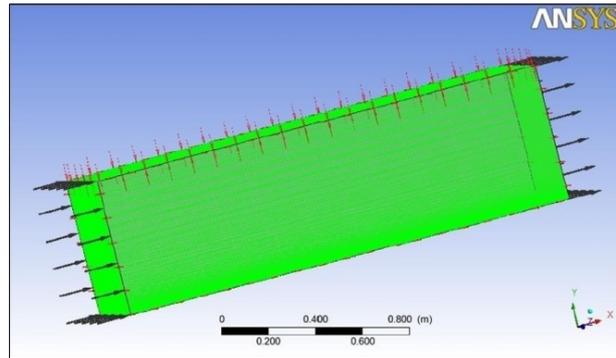


Fig. 3 Computational Domain

#### B. Single-Phase Boundary Layer

In this section, the numerical results obtained for an air-water up-flow in a vertical duct are compared with experimental data. Figs. 4, 5 represent respectively the liquid velocity, the logarithmic profile of velocity. Fig. 4 shows that the velocity profile was well predicted by the model. The profile of experimental velocity represents a boundary layer located at section  $x=22$  mm from the wall; this result has been found by the numerical simulation in Fig. 5. The agreement between our model and data is shown by comparison with the single-phase wall-bounded flow theory where the three zones usually encountered in a single-phase boundary layer are preserved: Viscous sub layer, logarithmic zone, and the wake region. The concordance is quite good except near the wall.

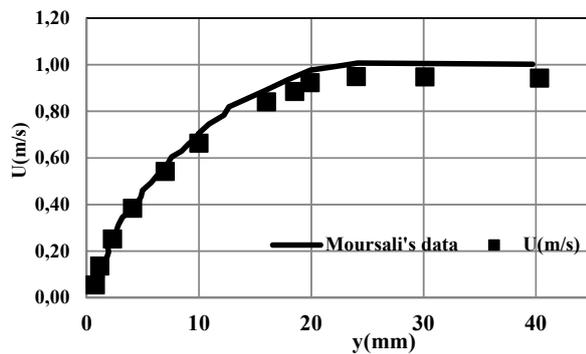


Fig. 4 Comparison of the liquid velocity profile with experimental results single phase flow

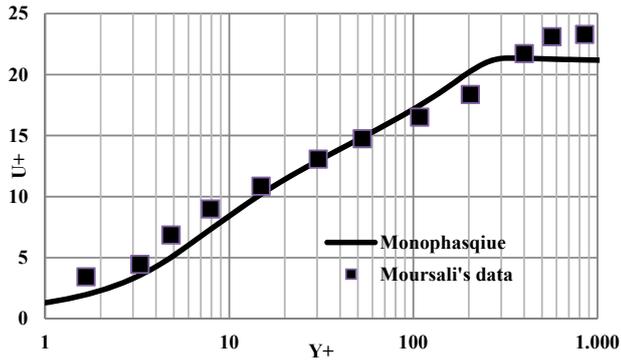


Fig. 5 Mean velocity predictions Comparison with experimental results

C. Two Phase Boundary Layer

The applications of the wall law postulated in wall bounded bubbly flows confirm the pertinence of the improvements proposed to ameliorate the predetermination of the turbulence structure. Figs. 6-8 show a satisfactory concordance agreement between the numerical results and experimental data of [2]: These results have led to the adjustment of the constants  $K_L$ . The values of the constant that allow a good prediction of experimental results are:

$$\kappa_l = -0.0071 e^{1.4970 U_w^{TP}} \tag{16}$$

This result is confirmed by the good concordance between the numerical results and the experimental data. Remember that these experiences indicate a logarithmic behaviour of velocity profiles in bubbly flows near the wall; however the logarithmic law applies to these profiles with constants different from those of the single-phase flow.

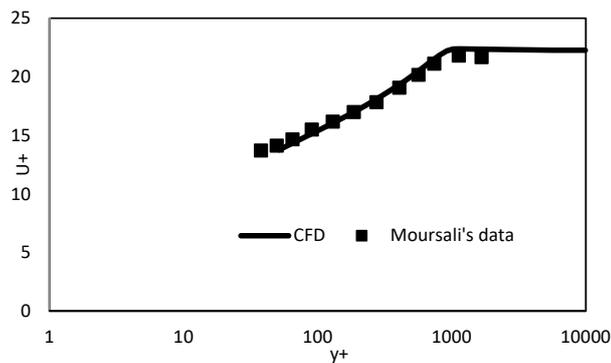


Fig. 6 Velocity profile plotted using the model (UL=1m/s; void fraction=0.5%): Comparison with experimental results

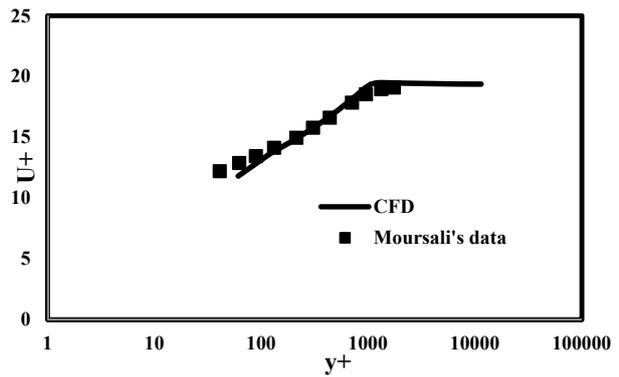


Fig. 7 Velocity profile plotted using the model (UL=1m/s; void fraction=0.2%): Comparison with experimental results.

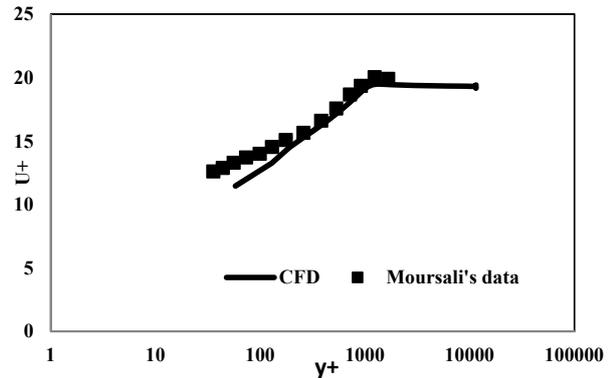


Fig. 8 Velocity profile plotted using the model (UL=1m/s; void fraction=1.5%): Comparison with experimental results.

IV. CONCLUSIONS

The experimental results on the boundary layer development on a vertical flat plate indicate that the near wall average velocity profiles in two-phase bubbly flows has a logarithmic behaviour. These experiences also show that the constants of the logarithmic profiles are sensibly modified in bubbly flows and the experimental results indicate that these constants depend on the amplitude of the wall void fraction peaking. A wall law was presented where mixing velocity scale is a function of local two-phase parameters. The law was validated against experimental data. A good concordance between the profiles from the logarithmic phase flow model and the experiments was achieved.

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