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Robust Control of a Dynamic Model of an F-16 Aircraft with Improved Damping through Linear Matrix Inequalities

J. P. P. Andrade, V. A. F. Campos

***Abstract*—This work presents an application of Linear Matrix Inequalities (LMI) for the robust control of an F-16 aircraft through an algorithm ensuring the damping factor to the closed loop system. The results show that the zero and gain settings are sufficient to ensure robust performance and stability with respect to various operating points. The technique used is the pole placement, which aims to put the system in closed loop poles in a specific region of the complex plane. Test results using a dynamic model of the F-16 aircraft are presented and discussed.

***Keywords*—F-16 Aircraft, linear matrix inequalities, pole placement, robust control.

I. Introduction

THE dynamic response characteristics of an aircraft are highly non-linear. Generally, flight control systems have been designed using mathematical models of an aircraft, linearized around several operation points, the controller parameters are programmed in accordance with the flight conditions.

To the F-16 aircraft control, several techniques have been applied. In [1], a linear strategy control and adaptive control, in which the parameters are calculated by a convex multiobjective optimization, are performed and applied to the longitudinal dynamic model of the F-16 aircraft in order to ensure at the same time the evolution of the error within a minimum invariant set while the linear gain is minimized. The longitudinal model of a hypersonic flight vehicle was also used for evaluation of the implementation of a robust adaptive controller [2], the methodology of this study addresses the issue of controller design and stability analysis in relation to parametric model uncertainties and saturations entrance to the oriented model control. In [3], the adaptive control technique $\mathcal{L}_1$ is applied in closed loop longitudinal F-16 aircraft model linearized around an operating point. In order to guarantee stability and performance of the resulting gain-scheduled controllers, analytical frameworks of gain scheduling have been developed including the technique of linear-parameter-varying (LPV) control [4], [5]. An application of a conditional integrator based sliding mode control design for robust regulation of minimum-phase nonlinear systems to the control of the longitudinal flight dynamics of an F-16 aircraft is made by [6]. In [7], a reliable robust tracking controller design method is developed based on the mixed linear quadratic (LQ)/H_∞ tracking performance index and multiobjective optimization in terms of linear matrix inequalities.

Among the techniques presented for control of an F-16 aircraft, the linear matrix inequality became a possible tool in finding solutions for various optimization problems, control systems and recently identification systems. One of the great advantages of this approach is to allow the simultaneous treatment of various performance and robustness requirements. This is because of the emergence of interior point algorithms for the solution of convex optimization problems, which made it possible to numerically solve the linear matrix inequalities faster and more efficiently.

This paper presents the application of linear matrix inequalities for robust control of an F-16 aircraft. Based on the algorithm presented and developed by [8], there is the guarantee of the damping factor for the closed-loop system for various operating points by allocating system poles using a predefined controller. The flexibility of the controller structure is an important feature explored in this paper.

The paper is organized as follows: In Section II, we describe the nonlinear mathematical aircraft model and its linearization. This section is extracted mostly from [9], with the Simulink model for simulation purposes based on [10]. In Section III, the linear matrix inequalities will be presented for pole placement in a particular region of the complex plane. Section IV presents the mathematical formulation of the controller structure for the system of F-16 aircraft considering several operation points. Results of tests and simulations performed by applying the robust controller to the longitudinal dynamic model of the F-16 aircraft are presented in Section V. In the last section, we present the conclusions of this work.

TABLE I
Mass and Geometric Properties

Parameter	Symbol	Value
Weight	$W\ (kg)$	9298.64
Moment of inertia	$J_y\ (kg/m^2)$	75673.62
Wing area	$S\ (m^2)$	27.87
Mean aerodynamic chord	$\bar{c}\ (m)$	3.45
Reference CG location	x_{cg}	$0.35\bar{c}$

II. Longitudinal Dynamic Model of an F-16 Aircraft

The flat-earth, body-axis 6-Degrees of Freedom (6-DOF) nonlinear control-oriented model for the F-16 fighter aircraft

V. A. F. de Campos is with Federal University of ABC, 5001 State Avenue, Santo Andre, SP, BR.
J. P. P. Andrade is with Federal University of ABC, 5001 State Avenue, Santo Andre, SP, BR (e-mail: joao.andrade@ufabc.edu.br).

presented in [9] and [10] has been employed in the this paper. The nonlinear model is linearized around the operating points (altitude = 4,57 km; total velocity = 549 km/h), and decoupled to obtain separate longitudinal and lateral-directional linear models. The properties of F-16 aircraft considered in this work are the same in [10], with the mass and geometric properties as listed in Table I and only the longitudinal-directional, low fidelity [10] state-space model given by (1) is investigated further under the influence of thrust and elevator control inputs.

$$\begin{bmatrix} \dot{h} \\ \dot{\theta} \\ \dot{V} \\ \dot{\alpha} \\ \dot{q} \\ \dot{\delta}_t \\ \dot{\delta}_e \end{bmatrix} = \mathbf{A}.\begin{bmatrix} h \\ \theta \\ V \\ \alpha \\ q \\ \delta_t \\ \delta_e \end{bmatrix} + \mathbf{B}.\begin{bmatrix} \delta_t \\ \delta_e \end{bmatrix}$$
$$\begin{bmatrix} h \\ \theta \\ V \\ \alpha \end{bmatrix} = \mathbf{C}.\begin{bmatrix} h \\ \theta \\ V \\ \alpha \end{bmatrix} + \mathbf{D}.\begin{bmatrix} \delta_t \\ \delta_e \end{bmatrix} \quad (1)$$

where h, θ, V, α, q, δ_t and δ_e are the aircraft's altitude (km), pitch angle (degrees), total velocity (km/h), angle of attack (degrees), pitch rate (rad/s), thrust (kg) and elevator deflection (degrees) respectively. The matrix A, B, C and D can be found using the Simulink program based on [10]. The eigenvalues, the damping ζ, the natural frequency w (rad/s) and the overshoot (%) of the longitudinal model dynamic (1) are shown in Table II. As we can see, the longitudinal model has a pole on the right side of the complex plane, moreover, has poles -0.00523 $\pm$ 0.0634i, which shows that the system has insufficient damping.

TABLE II
PROPERTIES OF LONGITUDINAL F-16 AIRCRAFT DYNAMIC MODEL IN OPEN LOOP

Eigenvalues	Damping	w (rad/s)	Overshoot (%)
1.03×10^{-13}	-1.00	$1{,}03\times10^{-13}$	0
-0.00523 + 0.0634i	0.0822	0.0636	77.2
-0.00523 - 0.0634i	0.0822	0.0636	77.2
-1.00	1.00	1	0
-1.06 + 1.69.i	0.53	1.99	14
-1.06 - 1.69.i	0.53	1.99	14
-20.2	1.00	20.2	0

III. SYSTEM CLOSED LOOP STRUCTURE AND PREDEFINED CONTROLLERS

The theory presented here is based on [8]. The fundamental equations that define the physical behavior of any system linearized about an operating point has the following generic model:

$$\begin{aligned} \dot{x} &= A.x + B.u \\ y &= C.x \end{aligned} \quad (2)$$

where x is the state vector, y is the output vector (or measurements vector), and u is the input vector (or control vector) [8]. The structure of the controller to be used to control the F-16 aircraft is pre-defined, which is an important feature, considering the practical application of control systems. This restricted structure is given by the following transfer function:

$$K_{y_k \to u_l}(s) = \frac{a_{y_k \to u_l}.s^2 + b_{y_k \to u_l}.s + c_{y_k \to u_l}}{s^2 + (p_1 + p_2).s + p_1.p_2} \quad (3)$$

where the notation $y_k \to u_l$ indicates de controller of the output y of the longitudinal F16 model, with k = 1,...,r, where r is the number of system outputs, to the input u, with l = 1,...,p, where p is the number of system inputs. The poles p_1 and p_2 are pre-determined. In this scheme, we work with pre-defined poles and we have to obtain the gain and the zeros, given by the values $a_{y_k \to u_l}$, $b_{y_k \to u_l}$ and $c_{y_k \to u_l}$ of the controller, constrained to feasible values. Our control method comprises applying an output feedback for the F-16 system. The closed-loop system is given in Fig. 1.

$K(s)$ is the matrix of transfer functions of the controllers and $G(s)$ is the matrix of transfer functions of the longitudinal F-16 nominal system. The matrix of $K(s)$ controllers given by (3) can be rewritten in the form of state space as:

$$\begin{aligned} \dot{x}_c &= A_c.x_c + B_c.y \\ u &= C_c.x_c + D_c.y \end{aligned} \quad (4)$$

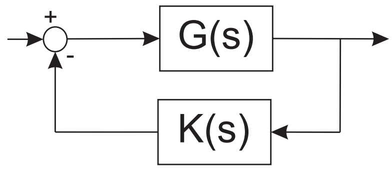


Fig. 1 Structure of the closed loop system

To define the matrices of the model above, we can use state space realizations, like those described in [8]. Then, matrices A_c and C_c are:

$$A_c = \begin{bmatrix} 0 & 1 & \dots & 0 & 0 \\ k & j & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & \dots & k & j \end{bmatrix} \quad (5)$$

$$C_c = \begin{bmatrix} 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 1 \end{bmatrix} \quad (6)$$

where $k = -p_1.p_2$ and $j = -(p_1 + p_2)$. Note that the matrices A_c and C_c are predefined matrices, since the poles are no problem variables. Applying the controller (4) to the system described by (2), we have the following description of the system in closed loop:

$$\begin{bmatrix} \dot{x} \\ \dot{x}_c \end{bmatrix} = \begin{bmatrix} A + B.D_c.C & B.C_c \\ B_c.C & A_c \end{bmatrix} . \begin{bmatrix} x \\ x_c \end{bmatrix} \quad (7)$$

Following standard procedure for the design of dynamic controllers using linear matrix inequalities, the problem (7) is rewritten as a static feedback problem output:

$$A_m = \begin{bmatrix} A & B.C_c \\ 0 & A_c \end{bmatrix}, B_m = \begin{bmatrix} B & 0 \\ 0 & I \end{bmatrix}$$
$$C_m = \begin{bmatrix} C & 0 \end{bmatrix} \tag{8}$$

Consequently, the static controller output is:

$$K_c = \begin{bmatrix} D_c \\ B_c \end{bmatrix} \tag{9}$$

With this redesign, the control problem is equivalent to the following structure:

$$\begin{array}{l} \dot{x}_m = A_m.x_m + B_m.u_m \\ y = C_m.x_m \end{array} \tag{10}$$

where $x_m = \begin{bmatrix} x & x_c \end{bmatrix}^T$ and the control law $u_m = K_c.y = K_c.C_m.x_m$. Using the state space description and the matrices of the controller A_c e C_c we can evaluate the matrices A_m, B_m e C_m. Therefore, the resulting control problem can be stated as: Calculate the static gain feedback output, so that the poles of the closed loop system (10) are located in a particular region of the complex plane.

IV. Pole Placement through Linear Matrix Inequalities

Linear matrix inequalities are mathematical tools that have various applications in control theory, especially in the robust control area. For purposes of pole placement it is important to define regions in a linear matrix inequality.

A. Regions of a Linear Matrix Inequality

A region of a linear matrix inequality is any subset of the complex plane that can be defined as [11]:

$$D = \{z \in C / L + z.R + \bar{z}.R^T < 0\} \tag{11}$$

where L and R are square real matrices with $L^T = L$ and $\bar{z}$ is the complex conjugate of z. Two important features of the regions of a linear matrix inequality are:

- A real matrix is D-stable, that is, has all of its eigenvalues in the linear matrix inequality region D if and only if a real symmetric matrix Q exists such that:

$$\begin{array}{l} L \otimes Q + R \otimes (AQ) + R^T \otimes (QA^T) < 0 \\ Q > 0 \end{array} \tag{12}$$

where $\otimes$ denotes the Kronecker product.

- Intersection regions of linear matrix inequalities are also regions of a linear matrix inequality

Two regions of a linear matrix inequality interest in control applications for pole placement are as:

- Conical sector with vertex at the origin and interior angle 2θ:

$$L = 0 \quad and \quad R = \begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix}; \tag{13}$$

- Half-plane $\Re(z) < \alpha$:

$$L = 2\alpha \quad and \quad R = 1; \tag{14}$$

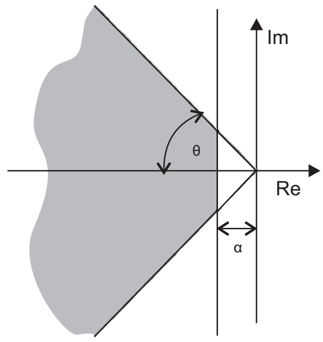


Fig. 2 Complex region plan for pole placement closed loop

The region of the formed complex plane (13) and (14) can be seen in Fig. 2. In this work, the goal is to allocate the system poles of F-16 aircraft in closed loop at the intersection between the sectors of the expressions (13) and (14), ensuring a minimum damping coefficient $\zeta = \cos\theta$, and a transient response with minimum decay rate equal to α for the closed-loop system. Thereby allocating the poles closed loop in this region guarantee adequate performance. To do this, we use the linear matrix inequality.

B. Control through Output Feedback

Applying the control law $u_m = K_c.y$ (with K_c given by (9) to the system (10), we can change the position of the system poles in closed loop, because:

$$\dot{x}_m = (A_m + B_m.K_c.C_m).x_m = A_{cl}.x_m \tag{15}$$

In order to put the closed loop poles in the region described above, we use the results presented in (12). However, the term $(A_{cl}.Q)$ is not linear, since it involves terms of two variables (K_c e Q):

$$A_{cl}.Q = A_m.Q + B_m.K_c.C_m.Q \tag{16}$$

To resolve this issue and transform (16) in a linear matrix inequality problem, it makes a transformation of variables [12]:

$$K_c.C_m.Q = N.C_m \tag{17}$$

Substituting (17) into (16) we obtain:

$$A_{cl}.Q = A_m.Q + B_m.N.C_m \tag{18}$$

Note that (18) is a linear equation. Once solved the problem in the transformed variables, the controller gain matrix is retrieved using the following reverse transformation:

$$M.C_m = C_m.Q \tag{19}$$

From the matrix M, the gain matrix K_c is calculated using the following expression:

$$K_c = N.M^{-1} \tag{20}$$

Substituting (18) into (12), we find a set of matrix inequalities that allocate the system poles (10) in the desired region of the complex plan (these inequalities are found in [8]).

$$\begin{array}{l} L \otimes Q + R \otimes (A_m.Q + B_m.N.C_m) + \\ R^T \otimes (Q)_m^T + C_m^T.N^T.B_m^T < 0 \\ Q > 0 \end{array} \quad (21)$$

Solving (21) to the resulting sector of the intersection of the two sectors defined by (13) and (14) of the complex plane, it is ensured that the closed loop poles of the aircraft F-16 belong to a desired region of the complex plane. Substituting the values of L and R in the specified regions, the following is obtained:

- For the conical sector with angle 2θ:

$$\begin{bmatrix} f.A_{cl}.Q + f.Q.A_{cl}^T & g.A_{cl}.Q - g.Q.A_{cl}^T \\ * & f.A_{cl}.Q + f.Q.A_{cl}^T \end{bmatrix} < 0 \quad (22)$$

where $*$ denotes symmetrical term, and:

$$\begin{array}{l} A_{cl}.Q = A_m.Q + B_m.N.C_m \\ Q.A_{cl}^T = Q.A_m^T + C_m^T.N^T.B_m^T \\ f = \sin\theta \\ g = \cos\theta \end{array} \quad (23)$$

- Half-plane $Re(z) < -\alpha$:

$$2.\alpha.Q + A_m.Q + B_m.N.C_m + Q.A_m^T + C_m^T.N^T.B_m^T < 0 \quad (24)$$

C. The Robust Procedure

It was described in the previous section a procedure for pole assignment of the F-16 aircraft system, in a specific region of the complex plane. The system of the aircraft is described by a set of nonlinear equations are linearized around some operating points. However, these operating points only represent the system behavior in a specific condition, and changes in operating points often occur. Thus, it is necessary to ensure that the F-16 will present good performance of the system in case of changes in operating points. To overcome this problem, we will make use of polytopic models.

To set a polytopic model, consider that only the matrix A of the system varies due to changes in operating points of the F-16 aircraft. Therefore, a polytope Ω is set [13]:

$$\Omega = \{A/A \in R^{n\times n}, A = \sum_{i=1}^{m} \lambda_i.A_i, \lambda_i \geq 0, \sum_{i=1}^{m} \lambda_i = 1\} \quad (25)$$

where n is the dimension of the matrices A_i and m the number of operating points. The matrices A_i are called polytope vertices.

To ensure that the poles of any closed loop system associated with the matrix $A \in \Omega$ are in the region of the complex plane defined by (13) and (14), it is necessary to resolve m linear matrix inequalities with the same variables Q and N, in other words [8]:

$$\begin{bmatrix} f.A_{cl,i}.Q + f.Q.A_{cl,i}^T & g.A_{cl,i}.Q - g.Q.A_{cl,i}^T \\ * & f.A_{cl,i}.Q + f.Q.A_{cl,i}^T \end{bmatrix} < 0 \quad (26)$$

for i=1,2,...,m, with:

$$A_{m,i} = \begin{bmatrix} A_i & B.C_c \\ 0 & A_c \end{bmatrix} \quad (27)$$

$$A_{cl,i}.Q = A_{m,i}.Q + B_m.N.C_m \quad (28)$$

A_i, with i = 1,2, ..., m, are the matrices in state space that define the mathematical model of the F-16 aircraft, and each of these matrices represent a model not linear linearized around a specific operating point. Solving the system of linear matrix inequalities, it ensures that the system poles in closed loop are in the region defined by the intersection of the conical sector with semi-plan for all m operating points considered.

V. Results and Discussion

The method presented in previous section was applied to the dynamic longitudinal-directional model of F-16 aircraft to the system considering three operation points, the only parameter that changed was the total velocity V to obtain the new linearized system around these points operation, the altitude navigation was kept constant at 4,57 km/h. The velocities considered are shown in Table III.

TABLE III
Operation Points

Operation Point	Velocity
1	549 km/h (Nominal)
2	658 km/h
3	768 km/h

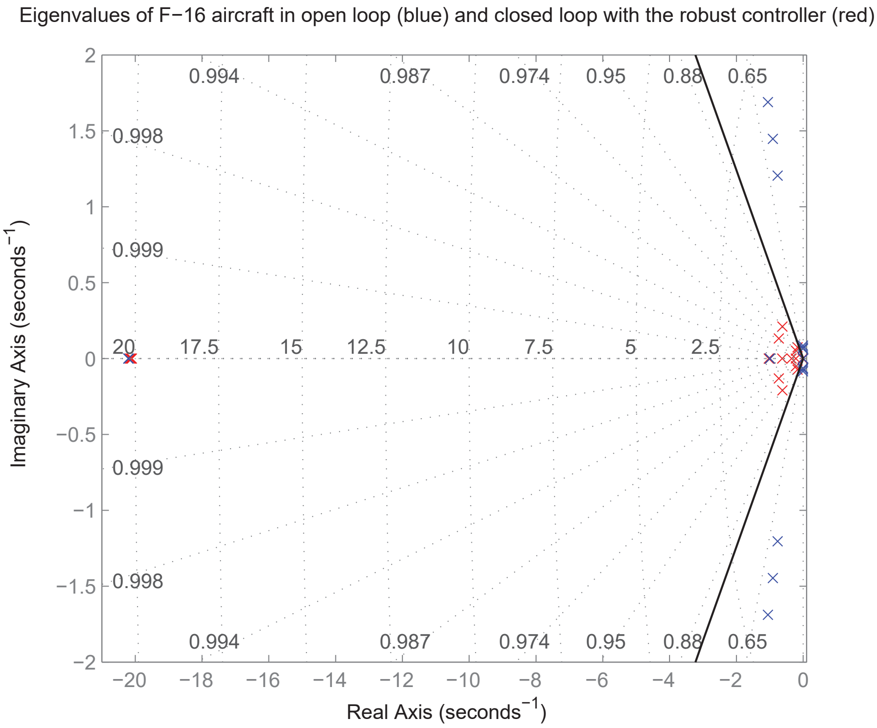
For each point of operation, new matrices were obtained for the system in the state space, through simulations using the program based on [10], however, as described in robust method, consider only the variations in matrix A of the system, the matrices B, C and D were considered to be the nominal system (operation point 1).

According to equation fixed controller (3), the poles of the controller were chosen p_1 = -5000 and p_2 = -5000, with a value of α = 0 and the following performance index (minimum damping coefficient of eigenvalues of the closed loop system):

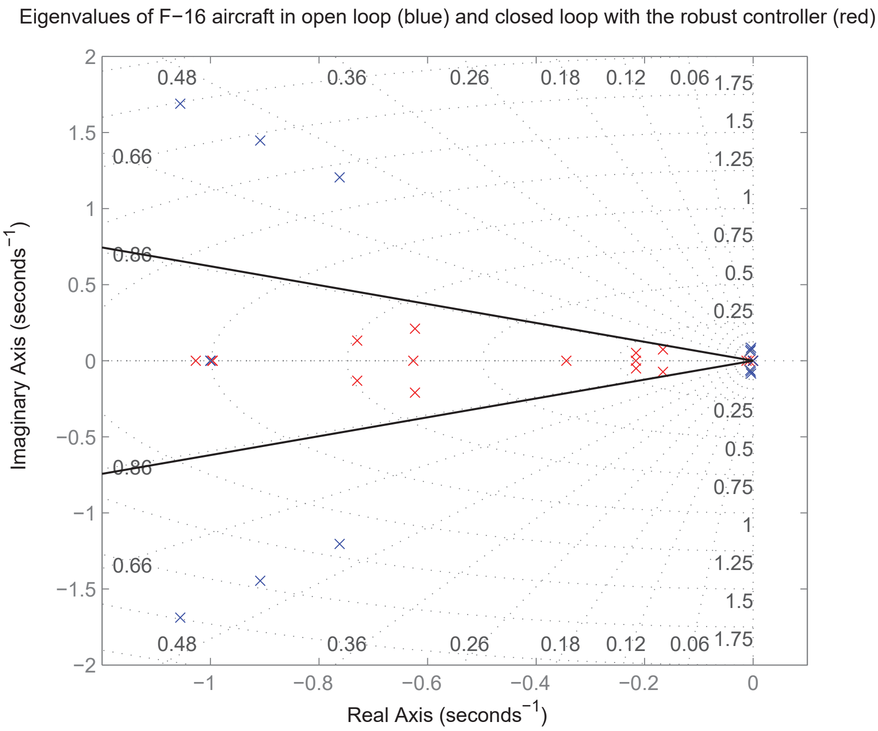
$$\zeta = \cos\theta = 0.9 \quad (29)$$

These two zeros were left free, as well as its static gain. The simulation was performed using the software $Matlab$© version R2013b with his toolbox for calculation of linear matrix inequalities. For this case the simulation lasted 3.48 s using an Intel Core i5 computer 2.20 GHz, 4GB of RAM, 64-bit. The parameters obtained for the robust controller are shown in Table IV. The controller parameters are referenced to (3).

To evaluate system performance in closed loop, we were plotted all the eigenvalues of the matrices **A** in closed loop for the three operation points, together with the eigenvalues of the original systems, as can be seen in Fig. 3. In this figure, with two expands, we just show the regions of interest. The black line is an approach to the conic section defined by the angle θ.



(a) Zoom 1



(b) Zoom 2

Fig. 3 Eigenvalues of F-16 aircraft in open loop (blue) and closed loop with the robust controller (red)

TABLE IV
CONTROLLER PARAMETERS FOUND FOR THE LONGITUDINAL F-16 CONTROL SYSTEM

y	u	$a_{y_k \to u_l}$	$b_{y_k \to u_l}$	$c_{y_k \to u_l}$
h	δ_t	-0.1277	-1277	3.191×10^{06}
	δ_e	0.0003772	4.191	8437
θ	δ_t	-1.596	-15960	$-\ 3.99 \times 10^{07}$
	δ_e	0.04666	484.2	1.06×10^{06}
V	δ_t	-2.402	-24020	$-\ 6.005 \times 10^{07}$
	δ_e	0.0002307	2.011	6039
α	δ_t	1.581	15810	3.953×10^{07}
	δ_e	-0.232	- 2321	$-\ 5.857 \times 10^{06}$

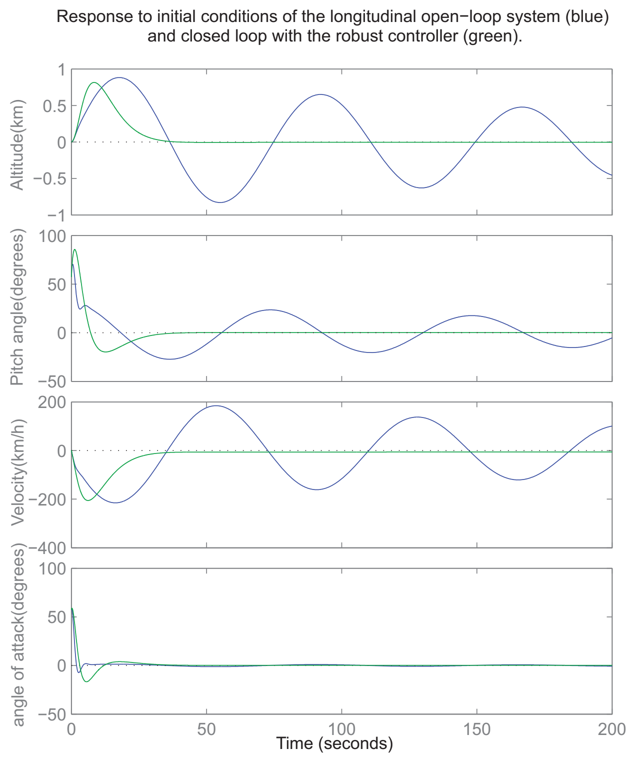


Fig. 4 Response to initial conditions of the longitudinal open-loop system (blue) and closed loop with the robust controller (green)

On the basis of information obtained by the simulation of the three operating points, the worst obtained damping factor was $\zeta = 0.914$, with an overshoot of 0.086 % and a time of accommodation signal of 3.04 s, as well as all the eigenvalues were contained within the LMI region of the complex plane specified by the intersection between the region of linear inequality matrix formed by the cone sector defined by θ angle and the half-plane defined by α. For this longitudinal system of F-16 aircraft, other α values were tested, but for a small increase in its value, the linear matrix inequality became infeasible. Another fact to be noted is that in this case performance specification, the performance obtained by application of the robust controller with this system is that the results obtained were better as the velocity of the F-16 aircraft is increased.

To check system behavior in closed loop with the robust controller, we apply the initial condition $x_0 = [\ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ ]^T$, to the operation nominal point, which showed the worst result among the three operating points, and compared with the open-loop system. Then, we obtain the graph shown in Fig. 4. As can be seen in this image, the system oscillation and settling time improved significantly compared to the system without the robust controller.

VI. CONCLUSION

In this paper, we present a methodology for pole placement of a linearized system around various operating points in a particular region of the complex plane, defined by the intersection of two regions of a linear matrix inequalities. The controller used here has a fixed structure, in which initially define the poles and control through linear matrix inequalities, obtain their gain values and their zeros.

It was applied to pole placement through the linear matrix inequalities longitudinal system F-16 aircraft. For specified performance conditions, there was an improvement in all the properties considered in this work to the closed loop system. For this case, all the eigenvalues of the system were allocated within the complex plane specified region, the value chosen for the damping coefficient was higher than that obtained for the open-loop system, yet the damping obtained for all operating points were higher than specified.

As can be seen in the responses to the initial conditions for the system, there is an evident improvement in the response rate and damping output. In addition, the applied controller is robust, which gives it advantages over any other controller, as it considers various system operating points to be controlled. One of the main advantages of the formulation presented in this work is that it generates controllers with a pre-specified structure, which can be applied to control the F-16 aircraft. This makes it easy to practical implementation to test the controllers obtained on stabilization and increased performance.

As a proposal for future work, we suggest testing with various performance specification values, varying for example the value of the angle θ of the linear matrix inequality region and checking the system behavior for all these values.

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