Speeding up Nonlinear Time History Analysis of Base-Isolated Structures Using a Nonlinear Exponential Model

Nicolò Vaiana, Giorgio Serino

***Abstract*—**The nonlinear time history analysis of seismically base-isolated structures can require a significant computational effort when the behavior of each seismic isolator is predicted by adopting the widely used differential equation Bouc-Wen model. In this paper, a nonlinear exponential model, able to simulate the response of seismic isolation bearings within a relatively large displacements range, is described and adopted in order to reduce the numerical computations and speed up the nonlinear dynamic analysis. Compared to the Bouc-Wen model, the proposed one does not require the numerical solution of a nonlinear differential equation for each time step of the analysis. The seismic response of a 3d base-isolated structure with a lead rubber bearing system subjected to harmonic earthquake excitation is simulated by modeling each isolator using the proposed analytical model. The comparison of the numerical results and computational time with those obtained by modeling the lead rubber bearings using the Bouc-Wen model demonstrates the good accuracy of the proposed model and its capability to reduce significantly the computational effort of the analysis.

***Keywords*—**Base isolation, computational efficiency, nonlinear exponential model, nonlinear time history analysis.

I. INTRODUCTION

SEISMIC isolation bearings are special devices able to provide flexibility and energy dissipation capacity in horizontal directions, sufficient vertical stiffness to resist service loading, rigidity under low levels of later loads due to wind or minor earthquakes, and recentering capability [1], [2].

A mathematical model is required to predict the dynamic behavior of seismic isolators. Within a relatively large displacements range, generally reached under the design dynamic loading, models with bilinear characteristics can represent the dynamic behavior of elastomeric bearings, such as high damping rubber bearings and lead rubber bearings, whereas models with rigid-plastic characteristics can be adopted to simulate the dynamic response of sliding bearings, such as flat sliding bearings and friction pendulum bearings [3]. The widely used differential equation Bouc-Wen Model (BWM), developed by [4], and then adopted by [5], [6] for the study of the random vibration of hysteretic systems, has been adapted for modeling the uniaxial behavior of both elastomeric and sliding bearings [7], [3] and has been implemented in many computer programs such as 3D-BASIS, SAP2000, and ETABS. Since the BWM requires the numerical solution of a first order ordinary nonlinear differential equation for each time step of a nonlinear time history analysis, the use of such conventional model can increase the computational effort very significantly.

The main aim of this work is to reduce numerical computations in the nonlinear dynamic analysis of seismically base-isolated structures by using a Nonlinear Exponential Model (NEM), proposed by [8], which is able to reproduce the dynamic behavior of seismic isolators within the relatively large displacements range without requiring the solution of a nonlinear differential equation for each time step of the analysis.

In order to show the decrease in the required computational effort when the proposed mathematical model is adopted, a three-dimensional (3D) base-isolated structure with a lead rubber bearing system subjected to harmonic earthquake excitation is analyzed. The nonlinear dynamic equilibrium equations are solved using the implicit unconditionally stable Newmark's constant average acceleration method used in conjunction with the pseudo-force iterative procedure. This conventional monolithic solution approach has been proposed by [3] specifically for the nonlinear dynamic analysis of base-isolated structures. The numerical results obtained by modeling the seismic isolators using the proposed NEM are compared with those obtained by adopting the conventional BWM in order to show the significant reduction of the total computational time when the former is employed.

II. EQUATIONS OF MOTION

In this section, the equations of motion of a typical seismically base-isolated structure are formulated. The discrete structural model of such a structure can be decomposed into two substructures: the *n*-story superstructure and the base isolation system. The latter consists of seismic isolation bearings and a full diaphragm above the seismic isolators.

In this work, the superstructure is considered to remain elastic during the earthquake excitation because the introduction of a flexible base isolation system generally reduces the earthquake response in such a way that the former deforms within the elastic range. In addition, the elastic superstructure is assumed to be a shear building, thus the 3d

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discrete structural model has three Degrees of Freedom (DOFs) per floor, floor diaphragms are considered to be rigid in its own plane, the beams are considered to be axially inextensible and flexurally rigid, and the columns are considered to be axially inextensible.

As far as the base isolation system is concerned, the diaphragm is assumed to be infinitely rigid in its own plane, the seismic isolators are considered rigid in vertical direction, and torque resistance of individual bearing is neglected.

A global coordinate system, denoted with upper case letters *X*, *Y*, and *Z*, is attached to the mass center of the base isolation system.

As a result of the above-described structural idealization, the total number of DOFs of the 3D structural model of a base-isolated structure is equal to $3n + 3$. The *i*-th floor diaphragm has three DOFs defined at the diaphragm reference point o_i, which is vertically aligned to the global coordinate system origin *O*. The DOFs for the *i*-th floor are the translation u_{ix} along the *X*-axis, the translation u_{iy} along the *Y*-axis, and the rotation $u_{i\vartheta}$ about the vertical axis *Z*; u_{ix} and u_{iy} are defined relative to the ground. The $3n$ superstructure DOFs are listed in the displacement vector $\boldsymbol{u}_s$, whereas the three DOFs of the base isolation system are listed in the displacement vector $\boldsymbol{u}_b$. The *i*-th diaphragm mass is lumped in its mass center (MC_i) which is also the geometric center.

The earthquake excitation is defined by the horizontal ground acceleration $\ddot{u}_g(t)$ whose line of action is given by the angle α_g that the epicentral direction forms with the *X*-axis.

The equations of motion of the 3D discrete structural model of a base-isolated structure are:

$$\begin{bmatrix} \boldsymbol{m}_b & \boldsymbol{0}^T \\ \boldsymbol{0} & \boldsymbol{m}_s \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{u}}_b \\ \ddot{\boldsymbol{u}}_s \end{Bmatrix} + \begin{bmatrix} \boldsymbol{c}_b + \boldsymbol{c}_1 & \boldsymbol{c}^T \\ \boldsymbol{c} & \boldsymbol{c}_s \end{bmatrix} \begin{Bmatrix} \dot{\boldsymbol{u}}_b \\ \dot{\boldsymbol{u}}_s \end{Bmatrix} + \begin{bmatrix} \boldsymbol{k}_b + \boldsymbol{k}_1 & \boldsymbol{k}^T \\ \boldsymbol{k} & \boldsymbol{k}_s \end{bmatrix} \begin{Bmatrix} \boldsymbol{u}_b \\ \boldsymbol{u}_s \end{Bmatrix} + \begin{Bmatrix} \boldsymbol{f}_n \\ \boldsymbol{0} \end{Bmatrix} = - \begin{bmatrix} \boldsymbol{m}_b & \boldsymbol{0}^T \\ \boldsymbol{0} & \boldsymbol{m}_s \end{bmatrix} \begin{Bmatrix} \boldsymbol{r}_b \\ \boldsymbol{r}_s \end{Bmatrix} \ddot{\boldsymbol{u}}_g , \tag{1}$$

with

$$\boldsymbol{c} = \left[-\boldsymbol{c}_1 \quad \boldsymbol{0} \right]^T , \tag{2}$$

$$\boldsymbol{k} = \left[-\boldsymbol{k}_1 \quad \boldsymbol{0} \right]^T , \tag{3}$$

$$\ddot{\boldsymbol{u}}_g = \left\{ \ddot{u}_g \cos(\alpha_g) \quad \ddot{u}_g \sin(\alpha_g) \quad 0 \right\}^T , \tag{4}$$

where $\boldsymbol{m}_s$, $\boldsymbol{c}_s$, and $\boldsymbol{k}_s$ are the superstructure mass, damping and stiffness matrices, respectively. Taking into account that the base isolation system can include linear and nonlinear isolation elements, $\boldsymbol{m}_b$ is the isolation system mass matrix, $\boldsymbol{c}_b$ is the damping matrix of linear viscous isolation elements, $\boldsymbol{k}_b$ is the stiffness matrix of linear elastic isolation elements, and $\boldsymbol{f}_n$ is the resultant nonlinear forces vector of nonlinear elements. In addition, $\boldsymbol{c}_1$ and $\boldsymbol{k}_1$ are the viscous damping and stiffness matrices of the superstructure first story, $\boldsymbol{r}_s$ and $\boldsymbol{r}_b$ are the superstructure and base isolation system influence matrices, respectively, and $\ddot{\boldsymbol{u}}_g$ is the ground acceleration vector.

III. Bouc-Wen Model (BWM)

In the following, the widely used differential equation BWM, developed by [4], and then adopted by [5], [6] to study the random vibration of systems having hysteretic behavior, is described. According to this model, the restoring force of a hysteretic system is given by:

$$f(u) = f_e(u) + f_h(u) , \tag{5}$$

where $f_e(u)$ is a linear elastic force evaluated as:

$$f_e(u) = \alpha \, k \, u , \tag{6}$$

and $f_h(u)$ is a nonlinear hysteretic force defined as:

$$f_h(u) = (1-\alpha) \, k \, z , \tag{7}$$

in which, α is a dimensionless parameter, k is a stiffness parameter, *u* is the displacement of the hysteretic system, and *z* is a hysteretic quantity, having the unit of displacement, obtained by solving the following first order ordinary nonlinear differential equation:

$$\dot{z} = -\gamma \left|\dot{u}\right| z^n - \beta \, \dot{u} \, \left|z^n\right| + A \, \dot{u} \text{ ,for } n \text{ odd} \tag{8}$$

$$\dot{z} = -\gamma \left|\dot{u}\right| z^{n-1} \left|z\right| - \beta \, \dot{u} \, z^n + A \, \dot{u} \text{ ,for } n \text{ even} \tag{9}$$

where *n* is a positive integer number, and *A*, β, and γ are real constants. Equations (8) and (9) can be written in a more compacted form as:

$$\dot{z} = A \, \dot{u} - \gamma \left|\dot{u}\right| z \left|z\right|^{n-1} - \beta \, \dot{u} \left|z\right|^n , \tag{10}$$

valid for *n*, odd or even.

It is important to note that *A*, α, β, γ, and *n* are dimensionless quantities that control the shape of the force-displacement loop. Furthermore, Constantinou and Adnane [9] have shown that, for $A = 1$ and $\beta + \gamma = 1$, the model collapses to a model of viscoplasticity that was proposed by [10].

Dividing (10) by $\dot{u}$ gives:

$$\frac{dz}{du} = A - \gamma \frac{\left|\dot{u}\right|}{\dot{u}} z \left|z\right|^{n-1} - \beta \left|z\right|^n . \tag{11}$$

For systems with softening behavior, z attains a maximum value, obtained by setting (11) equal to zero, which for positive $\dot{u}$ and z is given by:

$$z_{max} = \left[\frac{A}{\beta + \gamma} \right]^{1/n}. \tag{12}$$

Thus, the yield level f_y is expressed as:

$$f_y = (1-\alpha)\, k\, z_{max}. \tag{13}$$

The pre-yield stiffness k_i and the post-yield stiffness k_f are defined as:

$$k_i = \alpha\, k + (1-\alpha)\, k\, A, \tag{14}$$

$$k_f = \alpha\, k. \tag{15}$$

It is important to observe that the ratio of post-yield to pre-yield stiffness reduces to the value α when A = 1. Finally, the sharpness of the transition from the linear to nonlinear range is governed by the parameter n, with the hysteresis approaching bilinear behavior as n approaches ∞.

Nagarajaiah et al. [3] have adapted the above described uniaxial differential equation model for simulating the dynamic behavior of elastomeric bearings, such as high damping rubber bearings and lead rubber bearings, whereas Constantinou et al. [7] have adapted the BWM for modeling sliding bearings, such as flat sliding bearings and friction pendulum bearings. Thus, for an elastomeric bearing, the nonlinear restoring force can be evaluated by using the following equation:

$$f(u) = \alpha \frac{f_y}{u_y} u + (1-\alpha) \frac{f_y}{u_y} z, \tag{16}$$

where α is the post-yield to pre-yield stiffness ratio, f_y is the yield force, and u_y is the yield displacement.

For a friction pendulum bearing, the nonlinear restoring force can be obtained as:

$$f(u) = \frac{N}{R} u + \frac{\mu N}{u_y} z, \tag{17}$$

in which N is the vertical load carried by the seismic isolator, R is the radius of curvature of the spherical concave surface of the bearing, μ is the sliding friction coefficient, which depends on the value of bearing pressure and the instantaneous velocity of sliding $\dot{u}$. For a flat sliding bearing, (17) becomes:

$$f(u) = \frac{\mu N}{u_y} z. \tag{18}$$

Since the explicit expressions for z are possible only for n = 1 or 2, the unconditionally stable semi-implicit Runge-Kutta method [11] has been proposed by [3] to solve (10).

IV. Proposed Analytical Model

In this section, the NEM proposed by [8] is described. The presented model is able to predict the dynamic behavior of seismic isolation devices displaying a continuously decreasing tangent stiffness with increasing displacement within a relatively large displacements range, such as elastomeric and sliding bearings. Fig. 1 shows a typical normalized symmetric softening force-displacement hysteresis loop.

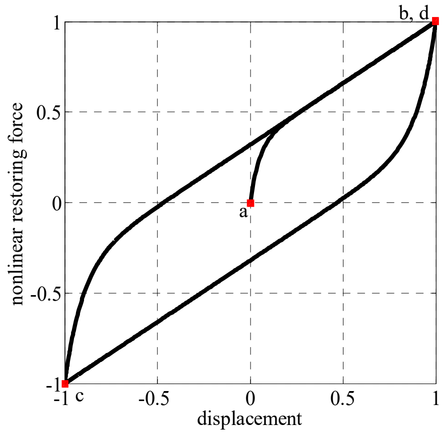


Fig. 1 Typical normalized symmetric softening force-displacement loop

The force-displacement hysteresis loop shown in Fig. 1 can be decomposed into three curves: the first loading curve (portion a-b), namely, virgin curve, the unloading curve (portion b-c), and the loading curve (portion c-d). Plotting the tangent stiffness of the loading curve as a function of the displacement (Fig. 2), it can be observed that the former exponentially decreases with increasing displacement.

The tangent stiffness $k_t(u)$ can be expressed by the following two mathematical expressions, valid for a loading and an unloading curve, respectively:

$$k_t(u) = k_2 + (k_1 - k_2)\, e^{-a\,(u - u_{min})}, \quad (\dot{u} > 0) \tag{19}$$

$$k_t(u) = k_2 + (k_1 - k_2)\, e^{-a\,(u_{max} - u)}, \quad (\dot{u} < 0) \tag{20}$$

where k_1 and k_2 are the initial and the asymptotic values of the tangent stiffness, u_{max} and u_{min} are the displacement values at the most recent point of unloading and loading, respectively, and a is a parameter that defines the transition from k_1 to k_2.

Integration of (19) and (20) gives the following nonlinear hysteretic restoring force:

$$f(u) = f(u_{min}) + k_2\,(u - u_{min}) - \frac{(k_1 - k_2)}{a}\left[e^{-a\,(u - u_{min})} - 1\right], \quad (\dot{u} > 0) \tag{21}$$

$$f(u) = f(u_{max}) - k_2\,(u_{max} - u) + \frac{(k_1 - k_2)}{a}\left[e^{-a\,(u_{max} - u)} - 1\right]. \quad (\dot{u} < 0) \tag{22}$$

According to Masing's rule, the virgin curve can be obtained applying a similitude transformation of ratio 0.5 to the generic loading or unloading curve of the nonlinear restoring force $f(u)$. Thus, if the initial displacement and velocity of the bearing are equal to zero, the virgin curve can be evaluated using the following expressions, valid for a first loading and unloading curve, respectively:

$$f(u) = k_2\,u - \frac{(k_1 - k_2)}{2\,a}\left[e^{-2\,a\,u} - 1\right], \quad (\dot{u} > 0) \tag{23}$$

$$f(u) = k_2\,u + \frac{(k_1 - k_2)}{2\,a}\left[e^{2\,a\,u} - 1\right]. \quad (\dot{u} < 0) \tag{24}$$

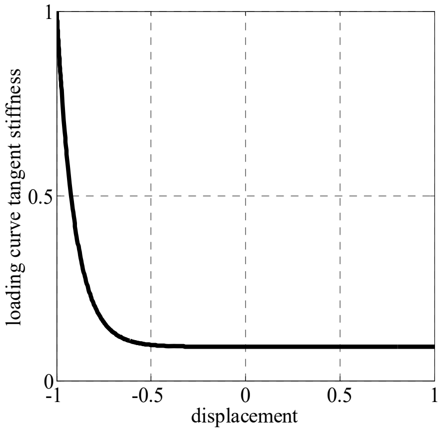


Fig. 2 Tangent stiffness variation of the loading curve

It is important to note that the proposed analytical model does not require the numerical solution of a nonlinear differential equation, as in the case of the differential equation model presented in Section II. Furthermore, the proposed NEM requires the evaluation of only three parameters, i.e. k_1, k_2, and *a*, whereas in the BWM the number of parameters to be identified is equal to 7 for an elastomeric and a flat sliding bearing and 8 for a friction pendulum bearing.

V. Numerical Application

In the following, the nonlinear dynamic response of a 3D base-isolated structure with a lead rubber bearing system subjected to harmonic earthquake excitation is simulated by modeling each seismic device with the presented NEM. In order to demonstrate the accuracy of the proposed nonlinear analytical model and its capability to decrease significantly the computational effort of the analysis, the numerical results and the computational time are compared to those obtained modeling the seismic devices with the widely used BWM. The dynamic equilibrium equations are numerically solved by using a conventional monolithic solution approach, that is, the implicit unconditionally stable Newmark's constant acceleration method employed in conjunction with the iterative pseudo-force procedure. In this paper, for brevity, the latter solution algorithm, proposed by [3] specifically for the analysis of seismically base-isolated structures, is referred to as the Pseudo-Force Method (PFM).

A. Analyzed 3d Base-Isolated Structure

The superstructure is a four-story reinforced concrete structure with plan dimensions 19 m x 11 m, and story height *h* = 3.5 m. The weight of the superstructure is 9921.24 kN and the first three natural periods are 0.33 s, 0.33 s, and 0.26 s, respectively. Each superstructure diaphragm mass includes the contributions of the dead load and live load on the floor diaphragm and the contributions of the structural elements and of the nonstructural elements between floors.

The base isolation system, having a total weight of 3006.44 kN, consists of an orthogonal mesh of foundation beams having rectangular cross section with dimensions 60 cm x 75 cm, and 24 identical Lead Rubber Bearings (LRBs), positioned centrically under all columns.

As a result of the kinematic constraints assumed in Section II, the total number of DOFs, defined relative to the ground, is equal to 15. The typical floor plan and a section of the analyzed 3D base-isolated structure are shown in Fig. 3.

The base isolation system has been designed in order to provide an effective isolation period T_{eff} = 2.50 s and an effective viscous damping υ_{eff} = 0.15 at the design displacement d_d = 0.50 m. Each elastomeric bearing has a yield force f_y = 45400.3 N, a yield displacement u_y = 0.017 m, and a post-yield to pre-yield stiffness ratio α = 0.10.

B. Analytical Models Parameters

Table I gives the parameters of the two analytical models, that is, the BWM and the proposed NEM, adopted to simulate the dynamic behavior of each LRB.

TABLE I
Analytical Models Parameters

BWM	f_y [N]	u_y [m]	α	A	β	γ	n
	45400	0.017	0.10	1	0.5	0.5	2
NEM	k_1 [N/m]	k_2 [N/m]	a				
	4513478	265499	50				

Fig. 4 shows the simulated force-displacement hysteresis loops produced by use of the BWM and NEM. They are obtained, as done in experimental tests, by applying a sinusoidal harmonic displacement having amplitude equal to 0.5 m and frequency of 0.40 Hz. It can be observed that the two analytical models adopting the parameters listed in Table I can reproduce hysteresis loops having the same area and effective stiffness.

TABLE II
NLTHAs Results

	tct[s]	$tctp$	$u_x^{(MC_b)}$ [m] max	$u_x^{(MC_b)}$ [m] min	$u_y^{(MC_b)}$ [m] max	$u_y^{(MC_b)}$ [m] min	$\ddot{u}_x^{(MC4)}$ [g] max	$\ddot{u}_x^{(MC4)}$ [g] min	$\ddot{u}_y^{(MC4)}$ [g] max	$\ddot{u}_y^{(MC4)}$ [g] min
PFM-BWM	511	-	0.071	-0.065	0.099	-0.146	0.329	-0.342	0.591	-0.562
PFM-NEM	4.51	0.88%	0.073	-0.060	0.095	-0.140	0.323	-0.334	0.510	-0.537

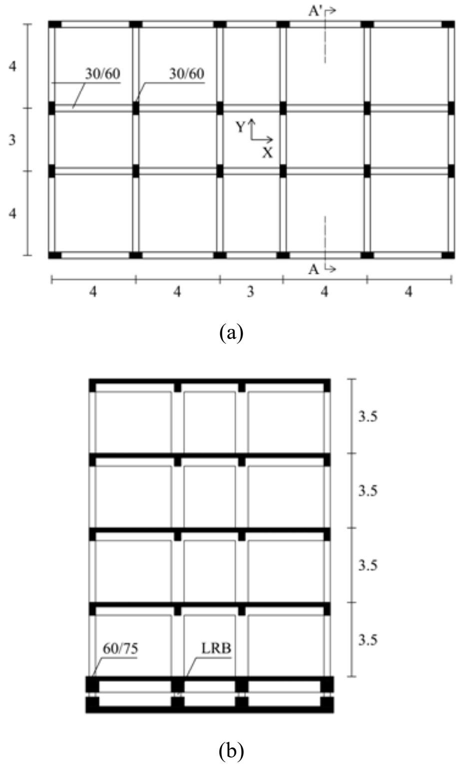


Fig. 3 Four-story reinforced concrete base-isolated structure: (a) typical floor plan; (b) section A-A'

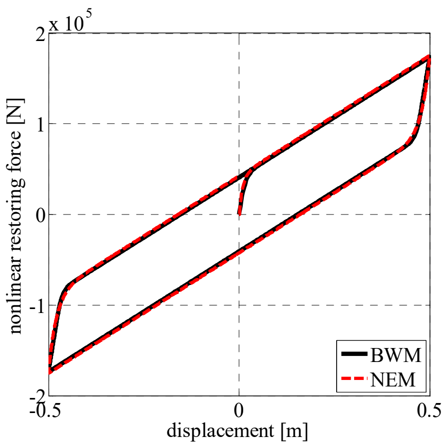


Fig. 4 Simulated force-displacement hysteresis loops

C. Numerical Results

Harmonic ground motion, having amplitude $\ddot{u}_{g0}$ = 2.5 m/s^2, frequency $\omega_g = 2\pi$ rad/s, and time duration t_d = 20 s, is imposed with an angle $\alpha_g = \pi / 3$ and time step of 0.005 s.

Table II shows the Nonlinear Time History Analyses (NLTHAs) results obtained adopting the PFM [3] to solve numerically the nonlinear dynamic equilibrium equations of the analyzed base-isolated structure. More specifically, the first row of Table II gives the numerical results obtained by modeling the seismic isolators using the BWM (i.e., PFM-BWM), whereas the second one shows the results obtained by simulating the dynamic behavior of LRBs adopting the proposed NEM (i.e., PFM-NEM).

The solution algorithm and the seismic isolator models have been implemented on the same computer (Intel® Core™ i7-4700MQ processor, CPU at 2.40 GHz with 16 GB of RAM) by using the computer program Matlab. In the PFM, the adopted convergence tolerance value is equal to 10^{-8} and in the BWM the first order ordinary nonlinear differential equation given by (10) has been solved by using the unconditionally stable semi-implicit Runge-Kutta method [11] with a number of steps equal to 50.

The comparison of the maximum and minimum values of the MC_b displacements and MC_4 accelerations in X and Y directions, obtained using the PFM-BWM and the PFM-NEM, reveals that the NLTHA performed by modeling the seismic isolators with the proposed model provides results that are close enough to those obtained adopting the BWM.

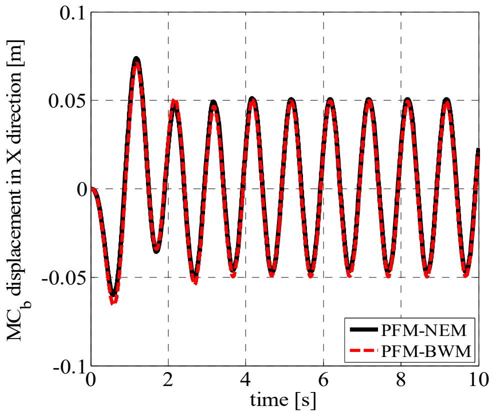
As far as the computational efficiency is concerned, the total computational time, *tct*, required by the PFM-NEM is significantly reduced in comparison to the PFM-BWM. It must be observed that the comparisons using the *tct* are meaningful only qualitatively because it depends on the CPU speed, memory capability, and background processes of the computer used to obtain the previous results. Thus, in order to normalize the computational time results, Table II also shows the percentage of the PFM-NEM *tct* evaluated with respect to the PFM-BWM *tct* as follows:

$$\text{PFM-NEM}\, tctp\,[\%] = \frac{\text{PFM-NEM}\, tct}{\text{PFM-BWM}\, tct} \cdot 100 \qquad (25)$$

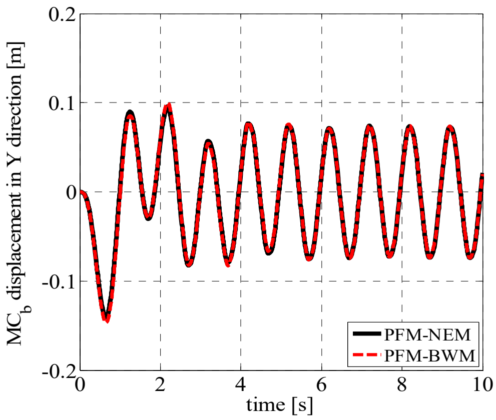
The PFM-NEM *tctp*, evaluated with (25), is equal to 0.88%.

Figs. 5 and 6 illustrate, respectively, the displacement time history of the MC_b and the acceleration time history of the

MC_4 for a time duration of the harmonic earthquake excitation t_d = 10 s. It is evident the good agreement between responses obtained using the BWM and the proposed NEM.

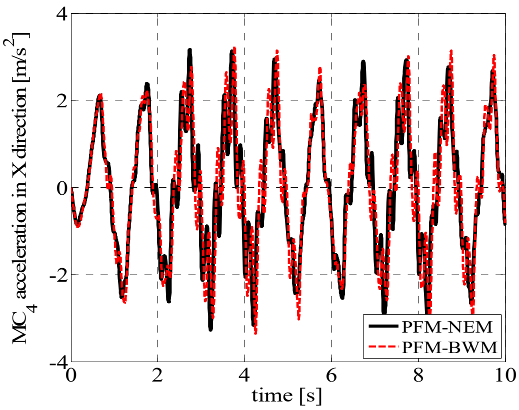


(a)

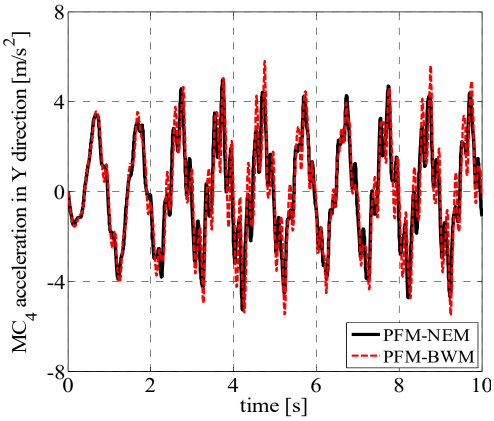


(b)

Fig. 5 Displacement time history of the base isolation system mass center in (a) *X* and (b) *Y* directions



(a)



(b)

Fig. 6 Acceleration time history of the superstructure fourth story mass center in (a) *X* and (b) *Y* directions

VI. Conclusions

The use of the BWM to simulate the dynamic behavior of seismic devices can increase significantly the computational effort of a nonlinear time history analysis of a base-isolated structure due to the numerical solution of the first order ordinary nonlinear differential equation required by the model for each time step of the analysis.

In this paper, the NEM developed by [8] has been described and then adopted in order to speed up the nonlinear dynamic analysis of base-isolated structures. Indeed, the proposed model is able to simulate the dynamic behavior of seismic isolators by avoiding the solution of the nonlinear differential equation required in the BWM.

In order to demonstrate the accuracy of the proposed analytical model and its capability to reduce the computational effort of the analysis, the nonlinear dynamic response of a 3D base-isolated structure with a lead rubber bearing system subjected to harmonic earthquake excitation has been simulated by modeling each seismic device with the NEM. Comparing the numerical results and computational time obtained by simulating the dynamic behavior of each LRB adopting the proposed NEM with those obtained by using the BWM, the following conclusions can be drawn:

(1) The nonlinear time history analysis performed by modeling the LRBs with the proposed analytical model provides results that are close enough to those obtained modeling the seismic isolators adopting the BWM;

(2) Simulating the dynamic behavior of the seismic isolation devices with the proposed NEM allows decreasing very significantly the computational effort of the analysis: indeed, the nonlinear time history analysis performed by modeling the LRBs with the proposed model requires a total computational time percentage, evaluated with respect the total computational time obtained by adopting the BWM, equal to 0.88%.

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