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Friction Calculation and Simulation of Column Electric Power Steering System

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***Abstract*—**This study presents a procedure for friction calculation of column electric power steering (C-EPS) system which affects handling and comfort in driving. The friction losses estimation is obtained from experimental tests and mathematical calculation. Parts in C-EPS mainly involved in friction losses are bearings and worm gear. In the theoretical approach, the gear geometry and Hertz law were employed to measure the normal load and the sliding velocity and contact areas from the worm gears driving conditions. The viscous friction generated in the worm gear was obtained with a theoretical approach and the result was applied to model the friction in the steering system. Finally, by viscous friction coefficient and Coulomb friction coefficient, values of friction in worm gear were calculated. According to the Bearing Company and the characteristics of each bearing, the friction torques due to load and due to speed were calculated. A MATLAB Simulink model for calculating the friction in bearings and worm gear in C-EPS were done and the total friction value was estimated.

***Keywords*—**Friction, worm gear, column electric power steering system, Simulink, bearing, electric power steering, EPS.

I. Introduction

THE automotive steering system has two main functions: Controlling the vehicle in order to follow the desired path and feed-backing on the driving condition. Three types of modern power steering systems are commercialized: Hydraulic power steering (HPS), electrohydraulic power steering and EPS. Among these systems, the EPS system is the latest type of power steering system in which an electric motor is connected to the steering system by a reduction gear to provide steering assistance directly [1]. In order to have more comfortable driving conditions when parking at low speed and to have high sensibility and stability at elevated speed, EPS system is used [2]. The torque measuring system is placed within the steering column. EPS systems eliminate the hydraulic system and it is developed for fuel saving. Most EPS systems have variable assist, providing more assistance for decreasing vehicle speed. This functionality requires an accurate balance of power and control that has only been available in recent years. EPS provides fuel savings as high as 0.2 to 0.3 liters per 100 kilometers compared to HPS systems [3]. The system consists of a steering-assist motor on the column with a steering rack at the wheels and a torque-sensing device in the steering column. The column is split into two components: The part attached to the steering wheel connects to the lower portion with the outer shaft through a torsion bar equipped with a torque-sensing device. The upper and lower parts are instrumented with magnetic sensors and, as the upper half of the steering column starts rotating, the magnetic sensors send a signal to the power steering control computer (ECU). This system eliminates the lag and the cavitation, which can occur in hydraulic systems. When the steering shaft is rotated, torque is transferred from steer and causes the input shaft to rotate. The worm shaft in the motor performs the transmission of the motor torque to the column. Power steering systems for vehicles, which enable drivers to steer with modest effort, have been developed following market demand [3].

One of the most important factors affecting EPS behavior and quality of driving is friction. C-EPS components, like bearings and worm gear, are the most important for friction evaluation in this system. In this paper, a lumped parameter model is presented to estimate static friction in the C-EPS. The model will be developed basing on a commercial system, but it can be extended to other systems.

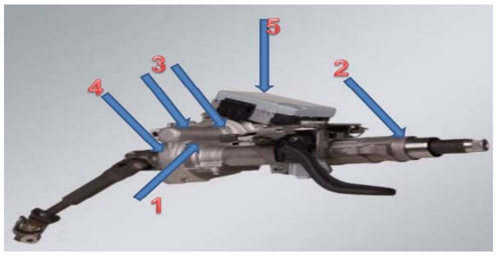

Fig. 1 C_EPS and friction contributions: 1) between worm and gear wheel, 2) between spindle shaft and outer column tube in bearing, 3) in bearings located on the worm shaft, 4) in bearing located on the output shaft, 5) in bearing inside the motor

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II. Analysis of the Worm Gear Forces

The material of the gear surface contacting with steel worm shaft is Nylon®. On both extremities of the worm shaft there are two bearings and two dampers; the bearings are deep groove single raw ball bearings. A worm gear with a plastic worm wheel is used in this application as a torque transmission system. Worm gearing is used in limited applications [4]. A worm gear system is used when a large speed reduction ratio is required between crossed axis shafts that do not intersect. A basic helical gear can be used but the transmitted power is low. The size of the worm gear set is based on the center distance between the worm and the worm wheel [5].

Lead angle and the number of worm threads, which are in contact with gear, can be used in order to calculate the efficiency. It is mentioned that the lead angle can affect frictional loses, gear speed and torque [6]. Lead angle γ, pressure angle and normal forces on the teeth in worm gear are important parameters. Lead angle γ:

$$\gamma = tan^{-1}\left(\frac{np}{\pi d_1}\right) \quad (1)$$

where d_1: Pitch diameter of worm; n: Number of threads; p: Pitch.

There are two ways to calculate the forces in worm gear, one is by knowing F_n and the other one is by knowing torque.

A. By Knowing F_n (Normal Force between Two Surfaces)

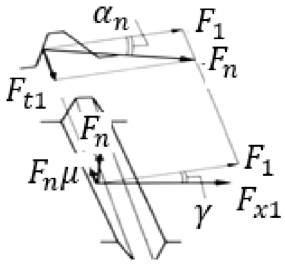


Fig. 2 Forces acting on the surface of the worm

As it is shown in Fig. 2, the forces between worm and gear are:

$$F_1 = F_n cos\alpha_n, F_{r1} = F_n sin\alpha_n \quad (2)$$

$$F_{t1} = F_1 sin\gamma + F_n\mu cos\gamma,\ F_{x1} = F_1 cos\gamma - F_n\mu sin\gamma \quad (3)$$

where α_n: Normal pressure angle; γ: Worm lead angle; μ: Friction coefficient. By using (2) and (3):

$$F_{t1} = F_n\ (cos\alpha_n sin\gamma + \mu cos\gamma),$$
$$F_{x1} = F_n\ (cos\alpha_n cos\gamma - \mu sin\gamma) \quad (4)$$

In a worm gear pair mesh with a shaft angle 90°, the axial force acting on driving gear F_{x1} equals the tangential force acting on driven gear F_{t2}. Similarly, the tangential force acting on driving gear F_{t1} equals the axial force acting on driven gear F_{x1}. The radial force F_{r1} equals to F_{r2}. The equations concerning worm-gear forces contain the friction coefficient μ. This coefficient has a great effect on the efficiency η_R of the transmission of a worm gear pair. Equation below, presents the efficiency when the worm is the driving element [7].

$$\eta_R = \frac{F_{t2}}{F_{t1}} tan\gamma = \frac{cos\alpha_n cos\gamma - \mu sin\gamma}{cos\alpha_n sin\gamma + \mu cos\gamma} tan\gamma \quad (5)$$

Changing the geometric parameters like pressure angle of teeth profiles can affect gearing friction. Highly optimized profiles are built with 40° pressure angle tools. Gear module is an important parameter too. Contact pressure, teeth bending and root resistance are designed to assure the mechanical resistance of the worm gear. Lowering the gear module causes teeth to become smaller and more numerous. Power losses in the worm and gear are associated primarily with tooth friction and lubrication-churning losses, which is relate to the peripheral speed of the gear passing through the fluid. The gear design, reduction ratio, pressure angle, gear size and coefficient of friction are the parameters, which effect on the frictional losses.

B. By Knowing Torque

Tangential force on worm, which is equal to axial force on gear, can be calculated as:

$$F_{wt} = F_{ga} = \frac{2M_1}{d_1} \quad (6)$$

where: M_1: Torque applied on the worm. Axial force on worm, which is equal to tangential force on gear:

$$F_{wa} = F_{gt} = F_{wt}\left(\frac{cos(\alpha_n)\,cos(\gamma) - \mu\, sin(\gamma)}{cos(\alpha_n)\,sin(\gamma) + \mu\, cos(\gamma)}\right) \quad (7)$$

The resulting torque for gear is:

$$M_2 = F_{gt}\left(\frac{d_2}{2}\right) \quad (8)$$

where d_2: Pitch diameter of gear.

The relation between tangential force on worm and gear is:

$$F_{wt} = F_{gt}\left(\frac{cos(\alpha_n)\,sin(\gamma) + \mu\, cos(\gamma)}{cos(\alpha_n)\,cos(\gamma) - \mu\, cos(\gamma)}\right) \quad (9)$$

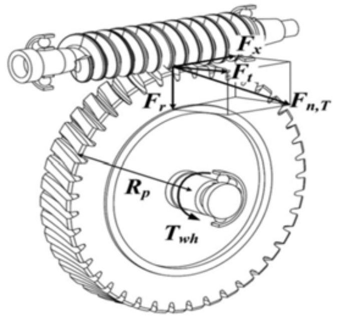


Fig. 3 Forces on the worm-gear pair

When steering, two components of the load act on the worm-gear tooth surface: The normal load due to the steering wheel torque and the other one due to the preload, which acts on the worm shaft. In order to calculate the normal load due to the torque as it is shown in Fig. 3, the equation is as [8]:

$$F_{n,T} = \frac{T_{wh}}{R_p}\sqrt{1 + tan\gamma^2 + tan\alpha_n{}^2} \quad (10)$$

where: T_{wh}: Torque on the gear; R_p: Radius of the gear; α_n: Pressure angle; γ: Lead angle.

III. Friction Study in Worm Gear

When the surface of one-component slips on the surface of another component and some forces are pressing the surfaces together, then a drag force, called friction force, develops between the two surfaces [9]. The mechanical friction in worm gear can be described considering two different contributions: viscous friction and coulomb (or dry) friction.

$$F_{viscous} = \mu_v V_s \quad (11)$$

where: V_s: sliding speed (m/s); μ_v: Viscous friction coefficient (Ns/m).

$$F_{coulomb} = \mu_c F_n \quad (12)$$

where: $F_{coulomb}$: Coulomb friction force; μ_c: Coulomb friction coefficient.

IV. Calculating the Contact Area and Normal Contact Pressure for Plastic Worm Gear

The roughness of the teeth is very important in case of friction. Heat generated in gearbox comes from friction in gear, bearing and seals. The contact point of a worm gear is on the pitch circles of the worm shaft and the gear. In addition, the two contact surfaces are shaped by four principal curves generated by the worm and the worm wheel at the contact point. One surface is formed from the two principal curves generated by the worm tooth surface. The other surface is formed from the two principal curves generated by the worm-wheel tooth surface [10].

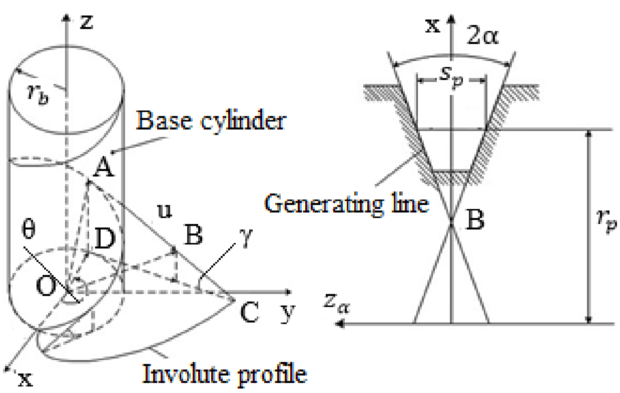


Fig. 4 Principal curves of the worm: screw involute surface of the ZI worm and geometry of the straight-lined blade

The surface equation of an involute worm that is called ZI worm is:

$$\overline{OB} = \overline{OD} + \overline{DA} + \overline{AB}, \overline{OD} = r_b(cos\theta\, i + sin\theta j),$$
$$\overline{DA} = r_b \theta tan\lambda_b,$$
$$\overline{AB} = ucos\lambda_b(sin\theta i - cos\theta j) - usin\lambda_b k \quad (13)$$

The surface of an involute worm is:

$$x = r_b cos\theta + ucos\lambda_b sin\theta, y = r_b sin\theta - ucos\lambda_b cos\theta,$$
$$z = -usin\lambda_b + r_b\theta tan\lambda_b \quad (14)$$

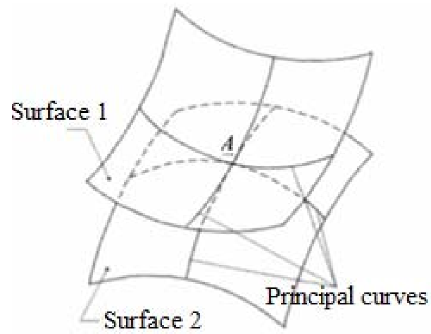


Fig. 5 Two tooth surfaces and four principal curves at contact point

In order to obtain the radius of the curvature, four principal curves take in to account and rely on Hertz's law to calculate the contact area [11].

$$x_\theta = -r_b sin\theta + ucos\gamma cos\theta, y_\theta = r_b cos\theta + ucos\gamma sin\theta$$
$$z_\theta = r_b tan\gamma \quad (15)$$

$$r_{\theta\theta} = -r_b cos\theta - ucos\gamma sin\theta, y_{\theta\theta} = -r_b sin\theta + ucos\gamma cos\theta$$
$$z_{\theta\theta} = 0 \quad (16)$$

where: r_b: Radius of the base cylinder; γ: Lead angle; u, θ: Surface parameters.

$$u = \frac{-r_b \cos(sin^{-1}\frac{r_b}{r_p})}{cos\gamma} \quad (17)$$

The surfaces of the worm wheel are defined as [11]:

$$x = r_h cos\theta + ucos\gamma sin\theta, y = r_h sin\theta - ucos\gamma cos\theta$$
$$z = -usin\gamma + r_h\theta tan\gamma, x_\theta = -r_h sin\theta + ucos\gamma cos\theta$$
$$y_\theta = r_h cos\theta + ucos\gamma sin\theta, z_\theta = r_h tan\gamma$$
$$x_{\theta\theta} = -r_h cos\theta - ucos\gamma sin\theta, y_{\theta\theta} = -r_h sin\theta + ucos\gamma cos\theta$$
$$z_{\theta\theta} = 0 \quad (18)$$

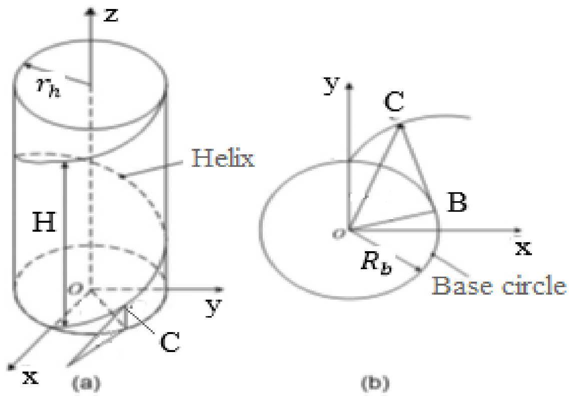


Fig. 6 Principal curves of the gear, (a) geometry of the gear surface, (b) involute curve of the gear

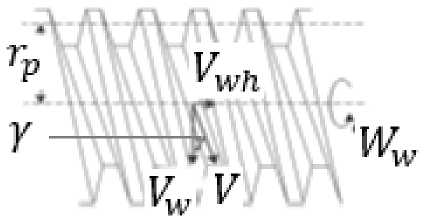


Fig. 7 Sliding velocity on the tooth surface

In case of a worm gear, applied in C-EPS systems, the gear is machined by an oversized hob to reduce the sensitivity of the worm gear to alignment errors. This gear is theoretically in point contact with the worm thread, whereas a gear processed by the hob whose generator surface is identical with the worm surface in line contact. The equation of the pitch line where the point contact occurs is:

$$u = \frac{-(r_h+r_p-r_b)\cos(sin^{-1}\frac{r_h}{(r_h+r_p-r_b)})}{cos\gamma} \quad (19)$$

When a worm-gear system is working, contact points are not exactly on the pitch circle due to vibration of the worm shaft and the tooth surface deformation of the gear, but it is good assumption to assume that they are on the pitch circle because their average position there. By applying Hertz's law, the contact area at the contact point is:

$$A_1 = \pi a^2 = \pi \left\{\frac{3}{4} R_{eq1}\left(\frac{1-\vartheta_1^2}{E_1} + \frac{1-\vartheta_2^2}{E_2}\right)\right\}^{\frac{2}{3}} . p^{\frac{2}{3}} \quad (20)$$

$$A_1 = \pi b^2 = \pi \left\{\frac{3}{4} R_{eq2}\left(\frac{1-\vartheta_1^2}{E_1} + \frac{1-\vartheta_2^2}{E_2}\right)\right\}^{\frac{2}{3}} . p^{\frac{2}{3}} \quad (21)$$

Consequently, the normal contact pressure is [10]:

$$P_n = \frac{\left[\frac{T\sqrt{1+tan\gamma^2+tan\alpha_n^2}}{R_p}\right]^{1/3}}{\pi\left\{\frac{1}{4}\left[\frac{(1-\vartheta_1^2)}{E_1}+\frac{(1-\vartheta_2^2)}{E_2}\right]\right\}^{2/3}\left\{\left[\frac{R_1R_3}{R_3-R_1}\frac{R_2R_4}{R_4+R_2}\right]\right\}^{1/3}} \quad (22)$$

V. Calculating the Sliding Velocity

In order to predict the friction coefficient of viscous friction, the sliding velocity on the tooth surface must be determined knowing the steering rate. Fig. 7 shows the sliding velocity on the tooth surface and it can be calculated by the following conversion equation [10].

$$V = \frac{w_w r_p}{cos\gamma} = \frac{V_w}{cos\gamma} \quad (23)$$

where: r_p: Pitch radius of the worm shaft.

Viscous friction varies with the steering conditions such as the steering torque and the steering rate. The normal pressure and the sliding velocities can be calculated from the obtained theoretical functions. The value of the friction coefficient between two surfaces in worm gear can be obtained by using a tribometer, as in [10] for Nylon-steel contact.

According to industrial aspects of producing worm gear, the value of pressure angle can be considered as 20° [12]. All the calculations related to worm gear are using data from Table I.

TABLE I
Calculated Values for Worm Gear

Item	Worm	Gear
Material	Alloy steel	Nylon
Normal Module [mm]	1.4	1.4
Number of Teeth	3	66
Pressure angle	20°	20°
Normal pressure angle	18.3°	18.3°
Lead angle	25°	25°
Hand	Right	Right
Pitch Diameter [mm]	9.3	92.2
Outer Diameter [mm]	15.5	95
Base circle diameter [mm]	8.74	86.6
Addendum [mm]	3.1	1.4

VI. Double Flank Worm Gear (Patent: US 6776064 B2)

The contact point in this type of worm-gear system is different from other types of worm-gear couplings. In this type, instead of one contact point, there are two contact points between teeth of worm and gear in both sides. This specific type is an undisclosed patent. The advantages of this type are:

- very low noise
- higher efficiency than the general types of worm and gear

As shown in Fig. 8, in this type of contact, there are two contact points respectively on the left and right side of each tooth. The corresponding force distribution and the shape of contact point varies with respect to the value of the load and pressure [13].

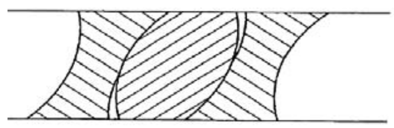
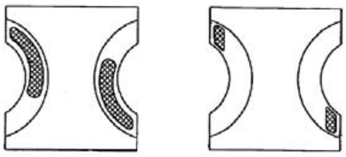


Fig. 8 Scheme of contact between teeth

The contact area can be high and low. The applied load can be divided in two categories: "Low or no load" and "High load".

Fig. 9 illustrates contact points and areas in case of the two different categories of load.



(a) (b)

Fig. 9 Contact area in case of (a) high load (b) low load or no load

In the present study, to estimate the pressure value between teeth, different experimental measurements were performed.

Pressure indicator sensor Fujifilm films were used. These films are sensible to different ranges of pressure and they are indicated by different colors [14]. In this case because of small area of teeth, the exact value for pressure cannot be given but it can give some information and estimation [14]. The selected film has a range between 10 to 50 MPa. In this experiment, the value of the pressure between two teeth can be evaluated by using the selected film. The result is that the pressure in the contact area is near the maximum value. Assuming pressure value equals to 50 MPa and a security correction factor, suggested by the film producer, equals to 1.5, final pressure value is 75 MPa.

As it is illustrated in Fig. 10, the viscous friction coefficient is assumed from [10], which the viscous friction coefficient is related to the sliding velocity.

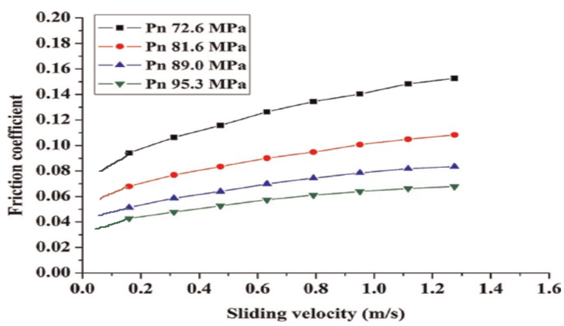


Fig. 10 Friction coefficient versus sliding velocity [9]

According to the value of sliding velocity, the empirical relation is as:

$$\mu = 0.04V_s^{-.025} \tag{24}$$

If sliding velocity is equal to 0.35 m/s, and the friction coefficient is 0.06 according to the Nylon plastic, viscous friction coefficient is equals to 0.06.

By considering this value of friction coefficient, value of worm gear efficiency can be calculated as:

$$\eta = \frac{cos18.26-0.06tan25}{cos18.26+0.06cot25}100\% = 85.46\% \tag{25}$$

According to Nylon® datasheet, the value for Coulomb friction without lubrication is between 0.38 and 0.42; in lubricated condition it is in the range 0.2 to 0.25 [15].

VII. Friction in Bearings

The equation for calculating the frictional torque, according to [16] is:

$$M_R = M_0 + M_1 \tag{26}$$

where: M_0: Friction torque as a function of speed; M_1: Friction torque as a function of load.

The equation for M_0 when vn≥2000 is:

$$M_0 = f_0(vn)^{\frac{2}{3}}.d_M^3.10^{-7} \tag{27}$$

where: f_0: bearing factor for frictional torque as a function of speed; n: operating speed (min^{-1}); v: kinematic viscosity (mm^2s^{-1}).

The equation for M_0 when $v{\cdot}n$<2000 is:

$$M_0 = f_0.160.d_M^3.10^{-7} \tag{28}$$

In order to calculate M_1 for ball bearing, the equation is:

$$M_1 = f_1.p_1.d_M \tag{29}$$

where: f_1: Bearing factor for frictional torque as a function of load; p_1: Load for frictional torque (N); d_M: mean diameter of bearing (mm).

Values for f_0 and f_1 for commercial bearings can be found in catalogues. With respect to bearings specifications, the value for f_0 for bearings in column EPS is 1.3 [17]. For f_1:

$$9*10^{-4}\left(\frac{P_0}{C_0}\right)^{0.5} \tag{30}$$

where: C_0: Static capacity (N); P_0: Static load (N).

Lubricating selection for bearings depends on combination of factors, the type of housing, operating temperature and operating speed. For the present automotive application grease lubrication is suited and for 40 °C the grease kinematic viscosity is assumed at $120\frac{mm^2}{s}$.

VIII. Calculation of Acting Forces on the Bearings

A. Force Calculation for Bearings on Worm Shaft

Even if there are two contact points between teeth of worm and gear but by consideration of one contact point; the values of the acting forces like tangential force on worm, normal force between worm and gear, radial and axial force in contact point in worm can be calculated. The bearings, which are used in worm shaft in present system, are deep groove ball bearing [18] and according to commercial catalogues, the equation for static bearing load in this type of bearing is as [16]:

$$P_0 = 0.6F_r + 0.5F_a \tag{31}$$

If $P_0 < F_r$ so $P_0 = F_r$ where F_r: Radial load, F_a: Axial load.

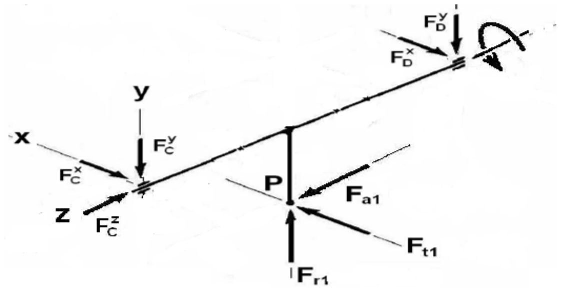


Fig. 11 Force analyses for bearings

B. Force Calculation for Bearings on the Gear Shaft

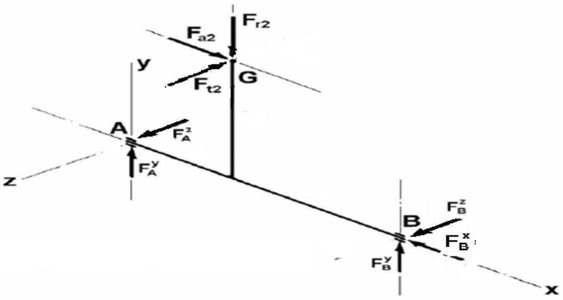


Fig. 12 Force analyses for bearings

The bearing A used after gear on the shaft is deep groove ball bearing, [18] thus the equation for calculating the static bearing load is [16]:

$$P_0 = 0.6F_r + 0.5F_a \quad (32)$$

According to the measured dimensions for bearing B, this is angular ball bearing, and according to specifications, the equation for static load calculation is as [15]:

$$P_0 = 0.5F_r + 0.26F_a \quad (33)$$

In order to calculate the friction torque due to speed, for bearings, equilibrium equations (27) and (28) can be used. According to specifications, the value for speed on worm gear in free turning condition is 30 rpm, so (27) should be used. For calculating the value of friction torque due to load, (34) is used:

$$M_1 = f_1 . p_1 . d_M \quad (34)$$

The formula for calculating f_1 is:

$$f_1 = 0.0009 * (\frac{P_0}{C_0})^{0.5} \quad (35)$$

Equation for calculating p_1 for deep groove ball bearings which are D, C, A is as:

$$3.3F_a + 0.1F_r \quad (36)$$

If $p_1 < F_r$ so $p_1 = F_r$. For bearing B which is an angular ball bearing; the equation for calculating is as:

$$p_1 = F_a - 0.1F_r \quad (37)$$

IX. Simulink Model Description

In this study, the friction behavior of a column power electric steering system is modeled by means of a lumped parameter model implemented by Simulink. The model simulates stationary working conditions. The model is composed of five parts:

1- Calculation of the forces on the shaft and bearings (solving (6)-(9))
2- Calculation of the friction parameters p_0, p_1, f_1 (solving (31)-(33), (35)-(37))
3- Calculation of the friction torque for each bearing (solving (28), (29))
4- Calculation of friction in worm gear (Coulomb and viscous friction contributions) (according to part VI)
5- Calculation of total friction torque

The input values for this Simulink model are: Torque applied on worm and speeds of worm and gear. The Simulink model output values are friction torque in bearings and worm gear.

In Fig. 13, the flowchart of the model for estimation the total friction torque is reported.

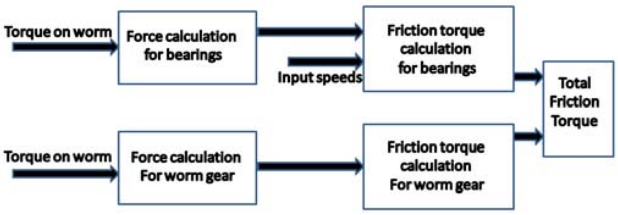


Fig. 13 Scheme of model for Simulink

X. Results and Comparison

A set of simulation was run for different values of input torque; from 1 Nm to 2.4 Nm for each bearing. The value of the input torque for steering in free turning torque is lower than 1.9 Nm. These simulations were run for the model component related to bearings A, B, C and D. it is mentioned that bearings nomination are as part VIII.

As it is shown in Fig. 14 in the bearings, by increasing input torque, the friction torque due to load will increase.

Another set of simulations was run for different values of the input speed for steering speed, which are from 600 rpm to 780 rpm for bearings D and C and from 30 rpm to 120 rpm for bearings A and B. These simulations were run for the model component related to bearings A, B, C and D. As it is shown in Fig. 15 the total friction torque increases by increasing the speeds.

Regarding the worm gear friction calculation, total friction torque results from both viscous friction and Coulomb friction contributions. A set of simulations was run for different values of Coulomb friction coefficient at constant input torque. The range of value for comparison is from 0.2 to 0.27.

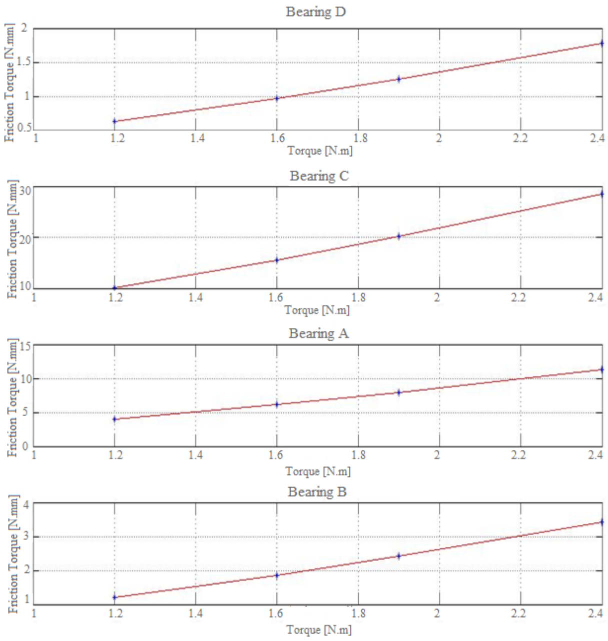


Fig. 14 Friction torque as a function of input torque [N.m]

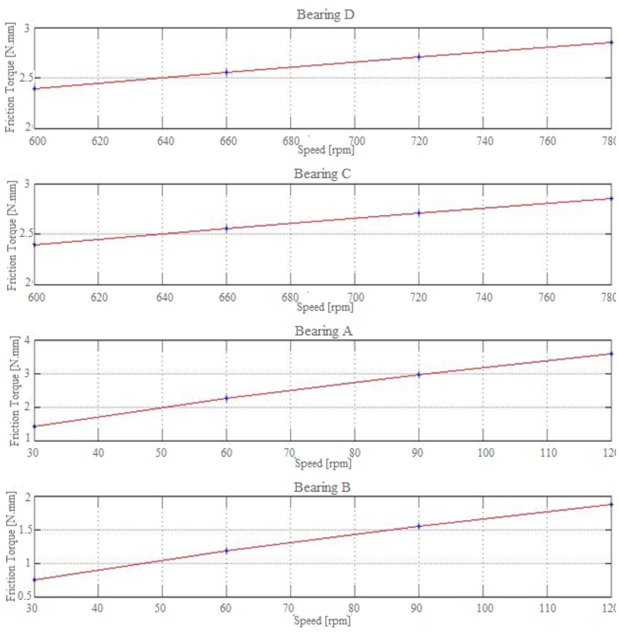


Fig. 15 Simulated friction torques for different input speeds [rpm]

As it is shown in Fig. 16, by increasing Coulomb friction coefficient, the value of total friction torque for worm gear will increase.

The last set of simulations was run for different values of input torque at constant value of Coulomb friction coefficient. As it is shown in Fig. 17, by increasing the input torque, the total friction torque will increase.

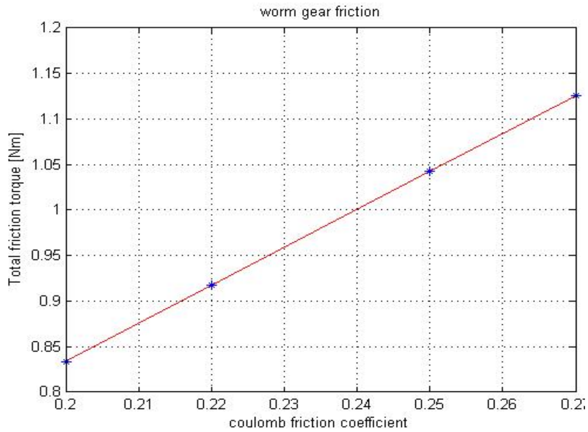


Fig. 16 Comparison of friction torque in worm and gear

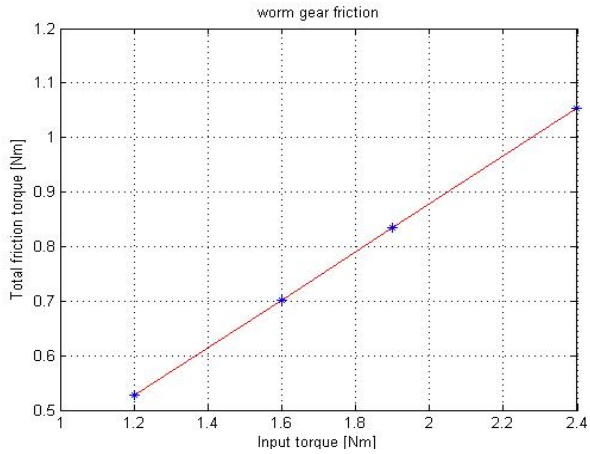


Fig. 17 Comparison of friction torque in worm and gear

XI. Conclusion

A calculation procedure to model the friction torque of bearings and worm gear in column steering power system was presented in this study. The viscous friction was estimated by some consideration.

The model estimates the friction behavior in stationary conditions.

Measuring the friction value for the automotive manufacturers is crucial as frictions phenomena affect the handling and comfort in the car. In this study, the prediction method and process are applied to the worm gear (with polymeric worm wheel) and to bearings. However, it is possible to predict the friction value of many different types of gears with polymeric materials and also different types of bearings with this model.

The viscous friction was applied to model the steering column by MATLAB/Simulink. In order to calculate the friction torque due to bearings, which depends on the input torque and the velocity, equations and model are applied. The estimated value of friction which results of the contribution of worm gear and bearings in the column EPS can thus be estimated. According to results, worm gear is the most important part in EPS from friction point of view.

At the end by comparing the friction torques due to different input data, it results that friction torque is related to input torque and input velocity.

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