Symmetrical In-Plane Resonant Gyroscope with Decoupled Modes

Shady Sayed, Samer Wagdy, Ahmed Badawy, Moutaz M. Hegaze

***Abstract*—**A symmetrical single mass resonant gyroscope is discussed in this paper. The symmetrical design allows matched resonant frequencies for driving and sensing vibration modes, which leads to amplifying the sensitivity of the gyroscope by the mechanical quality factor of the sense mode. It also achieves decoupled vibration modes for getting a low zero-rate output shift and more stable operation environment. A new suspension beams design is developed to get a symmetrical gyroscope with matched and decoupled modes at the same time. Finite element simulations are performed using ANSYS software package to verify the theoretical calculations. The gyroscope is fabricated from aluminum alloy 2024 substrate, the measured drive and sense resonant frequencies of the fabricated model are matched and equal 81.4 Hz with 5.7% error from the simulation results.

***Keywords*—**Decoupled mode shapes, resonant sensor, symmetrical gyroscope, finite element simulation.

I. Introduction

RESONANT vibratory gyroscopes depend on the Coriolis induced transfer of energy between the two vibration modes. Resonant gyroscopes are widely used in many low cost applications lately, especially the micro machined resonant gyroscopes, such as Automotive industry (Anti-lock braking, air bags, rollover detection) and Electronic devices (video camera stabilization, Robotics applications) [1], [2]. However, they cannot be used for high performance and long-term applications because the performance of micro gyroscopes is less than the conventional rotating wheel, as well as fiber-optic and ring laser gyroscopes [3]. Many researches were carried out for increasing the sensitivity and performance of the micro machined gyroscopes [4]–[8]. Mode matched resonant frequencies of the drive and sense modes are an important parameter for increasing the sensitivity of the resonant gyroscopes which are amplified by the mechanical quality factor of the sense vibration mode. Some approaches tried to achieve mode matched gyroscopes by symmetrical suspension beams design [6], [7] or by balance the drive and sense modes electrostatically to be close to each other [4], [5]. However, the difference between the drive and sense resonant frequencies is decreased, the mechanical coupling increases, which leads to an increase the zero-rate output shift and the device operation becomes more unstable. Decoupled resonant gyroscopes were discussed in [4], [6], where decoupled operation can be achieved by using independent suspension beams design. It is difficult to achieve all of these parameters with a symmetrical design; however, the unsymmetrical designs have more temperature induced frequency drifts and also it is difficult to get matched resonant frequencies.

In Section II, the mechanical design of the gyroscope model is discussed showing the calculated theoretical resonant frequencies. Section III presents the finite element simulations that are used to verify the theoretical results and show the amount of mechanical coupling between the drive and sense modes. In Section IV, the fabrication details and the experimental work are presented comparing the experimental results with the simulated results. Section V concludes the results and discusses the future work.

II. Mechanical Design of the Model

The vibrating gyroscope can be simplified to a mass-spring system. The vibrating gyroscope model consists of a proof mass and two sets of four flexible beams, as shown in Fig. 1. The model is symmetric and the driving and sensing modes have the same resonant frequencies and mode shapes. For each mode of the driving and sensing modes the spring system can be expressed as two beams which are fixed-guided beams and another two beams which are fixed-fixed beams.

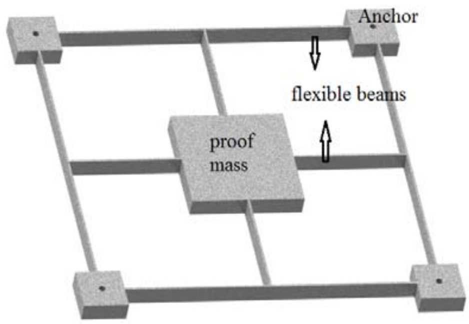

Fig. 1 Symmetrical single mass resonant gyroscope design

To determine the spring constant of a single fixed-guided beam, the governing differential equation for the elastic curve which characterizing the shape of the deformed beam is then expressed as:

$$\frac{d^2 y}{dx^2} = \frac{M(x)}{EI}. \quad (1)$$

where, M(x) is the bending moment, E is the modulus of elasticity, I is the moment of inertia of the cross section about

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its neutral axis, $\frac{d^2y}{dx^2}$....is the second derivative of the deflection of the beam y(x).

Consider a fixed-guided beam of length L under a lateral force F at the guided end, as shown Fig. 2.

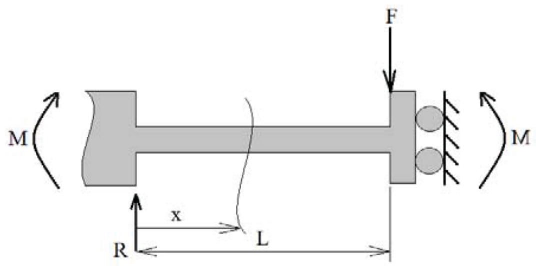


Fig. 2 Fixed guided beam

Then the equivalent stiffness of the fixed-guided beam can be determined as:

$$k = \frac{12EI}{L^3} = \frac{Etb^3}{L^3} \quad (2)$$

where b and t are the width and thickness of the beam, respectively.

By the same way we can determine the spring constant of a single fixed-fixed beam under a lateral force at its middle as:

$$k = \frac{192EI}{L^3} = \frac{16Etb^3}{L^3} \quad (3)$$

After determination of the spring constants of the fixed-guided beam and the fixed-fixed beam, we can determine the equivalent spring constant of the system.

The driving mode or sensing mode is illustrated by a vibration of single degree of freedom mass-spring system with four springs (two fixed-guided beams and two fixed-fixed beams) when neglecting the effect of the other beams in the axial direction, this assumption is acceptable because the axial stiffness of a beam (k_{axial}=Etb/L) is very large compared to the lateral stiffness [8]. The assumption is then validated by the finite element analysis and simulation of the gyroscope model using ANSYS software, the validation of this assumption is achieved by the design of the suspension beams which leads to the extreme decrease of the mechanical coupling between the drive and sense modes.

Then, the equivalent spring constant of the system is:

$$\mathrm{k_{eq}} = 2\mathrm{k_1} + 2\mathrm{k_2}.$$

where k_1, k_2 are the stiffness of the fixed-guided beam and fixed-fixed beam, respectively.

Using the following parameters of the fabricated gyroscope shown in Table I to determine the resonant frequency, the gyroscope model was fabricated from an aluminum alloy 2024 substrate.

TABLE I
GYROSCOPE MODEL PARAMETERS

Symbol	Quantity	Value [mm]
L_1	length of the fixed-guided beam	49.8
L_2	length of the fixed-fixed beam	129.8
b	width of the beams	1
t	thickness of the substrate	8.2
a	length of one side of a square proof mass	50.2

Then, the calculated resonant frequency of the fabricated gyroscope is 87.15 Hz. This value matches the driving frequency and sensing frequency because of the symmetrically gyroscope model. The matched resonant frequencies for the drive and sense vibration modes have a great effect on the gyroscope sensitivity because the sensitivity is amplified by the mechanical quality factor of the sense mode [1], [7]. The design of the suspension beams allows for decoupled vibration modes which is a very important point in the design of the gyroscope because it prevents undesired interaction between the drive and sense modes.

III. FINITE ELEMENT SIMULATION

Finite element simulations were performed using the ANSYS software package. The simulated results are used to verify the theoretical results and show the amount of mechanical coupling between the drive and sense modes. When neglecting the effect of the elasticity of beams directed in the direction of motion as discussed in Section II, the resonant frequencies of the drive and sense modes are the same and equal 87.2 Hz, as shown in Fig. 3. These results satisfy the theoretical calculations.

When taking the axial stiffness effect into consideration, the simulated model resonant frequencies are equal to 86.4 Hz, within 0.9% error compared with the theoretical results, the simulated model vibrations of the drive mode and sense mode are showed in Fig. 4. An important problem in the design of gyroscopes with symmetric shapes is the mechanical coupling between the two modes which causes the unstable operation of the gyroscope, the results of the simulation show that the mechanical coupling is non-considerable. Fig. 5 shows the finite element simulation of the relative displacement for the two vibration modes when only one mode is vibrating, it shows that during the vibration of one mode, the second mode is lightly affected from the first mode vibration. This unwanted effect can be eliminated using differential readout scheme, where the unwanted displacement in the sense mode due to coupling are measured in opposite directions, resulting in a zero output after the differential readout circuit [5], [9].

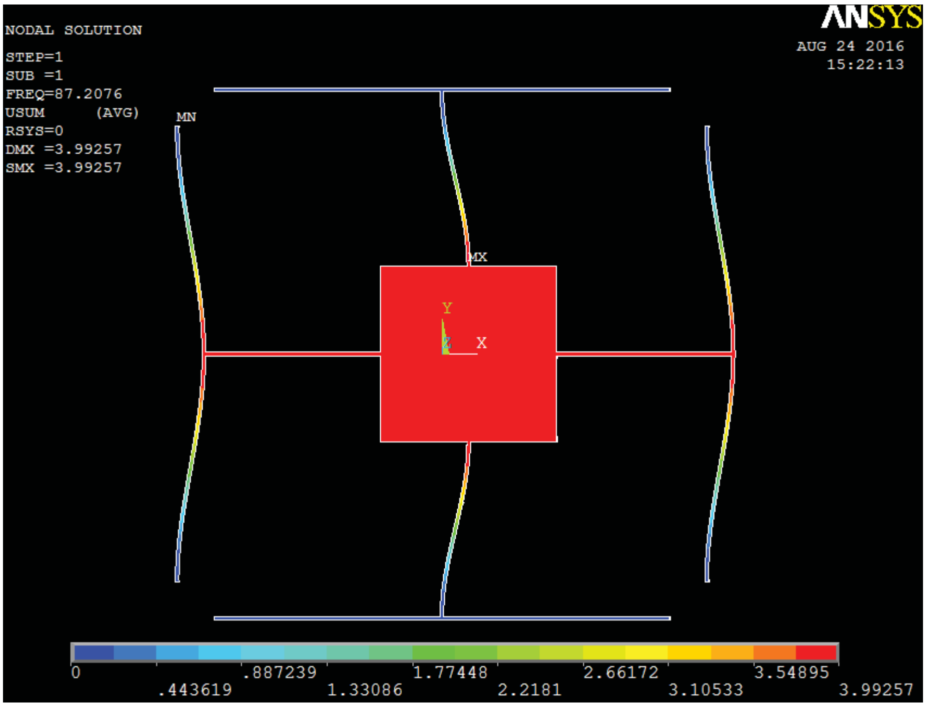
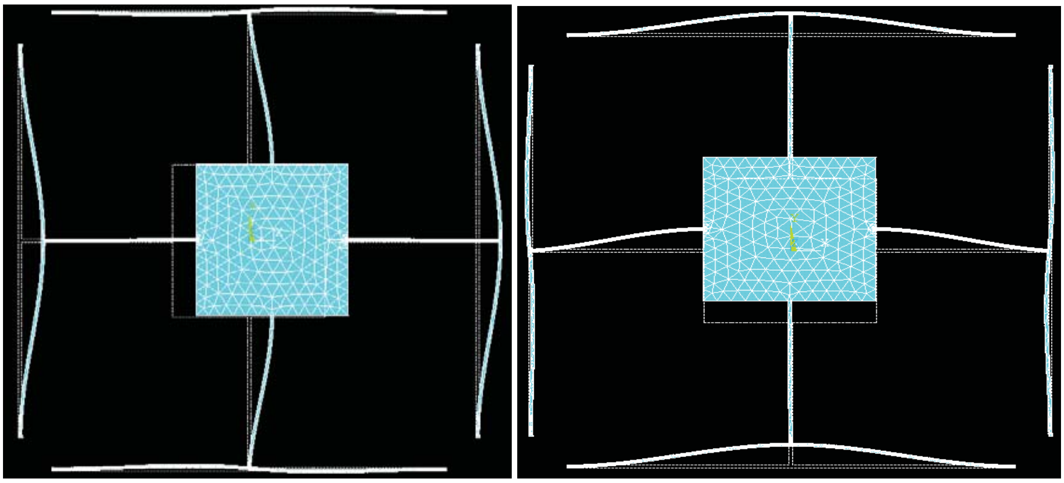


Fig. 3 ANSYS simulation of the vibration of one mode when neglecting the effect of the elasticity of beams directed in the direction of motion, resonant frequency is 87.2 Hz



(a) (b)

Fig. 4 ANSYS model simulation for (a) the drive mode and (b) the sense mode. Resonant frequencies are equal to 86.4 Hz for the two vibrating modes due to the design symmetry

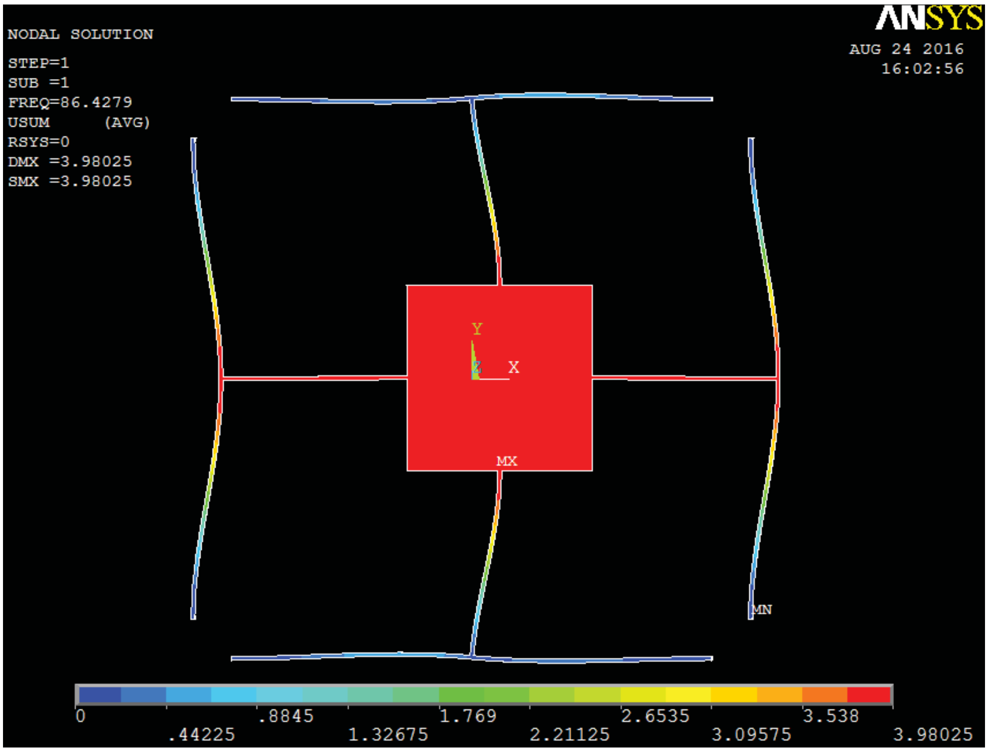
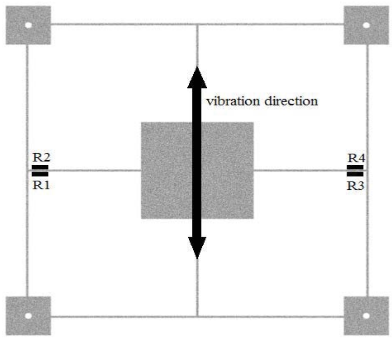


Fig. 5 ANSYS finite element simulation of the relative displacement for the two vibration modes when only one mode is vibrating, it is clear that the vibration of one mode does not have a significant effect on the other mode

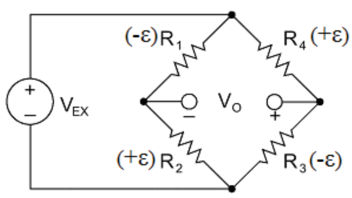
IV. Experimental Work

The gyroscope model is fabricated by a high speed wire cutting machine using aluminum alloy 2024 substrate. This process was performed with a high accuracy and small tolerances; the imperfections of the produced gyroscope model do not exceed 0.2% error of the designed model. Aluminum alloy 2024 is selected as a gyroscope material after many calculations and trials to get the required flexibility which is needed to decrease the mechanical damping and increase the driving amplitude, these parameters have a great effect on the quality factors of the drive and sense modes which affect the performance and resolution of the gyroscope. However, as it is required to decrease the mechanical damping, there was another important parameter which should be taken into consideration; it is the mechanical coupling that should be also decreased. It is clear that it is difficult to achieve all of these parameters with a symmetrical design. The special design of the suspension beams and the material used lead to achieve most of the required parameters. A number of tests were performed to verify the operation of the produced gyroscope by determining the resonant frequencies and performance, the test results will be shown later.

For the gyroscope tests, the resonance frequencies and quality factors for drive and sense modes are required. Comparing the designed values with the actual model results shows the process variations and the expected values of the quality factors. The resonance tests are performed using NI compactDAQ and LabVIEW sound and vibration suite, and a hammer test is used to determine the resonance frequencies and strain gauges are used as a sensing method. The strain gauges are located at the points of maximum strain and connected in a full Wheatstone bridge, as shown in Fig. 6.



(a)



(b)

Fig. 6 Strain gauge connection (a) location of strain gauges (b) full Wheatstone bridge scheme

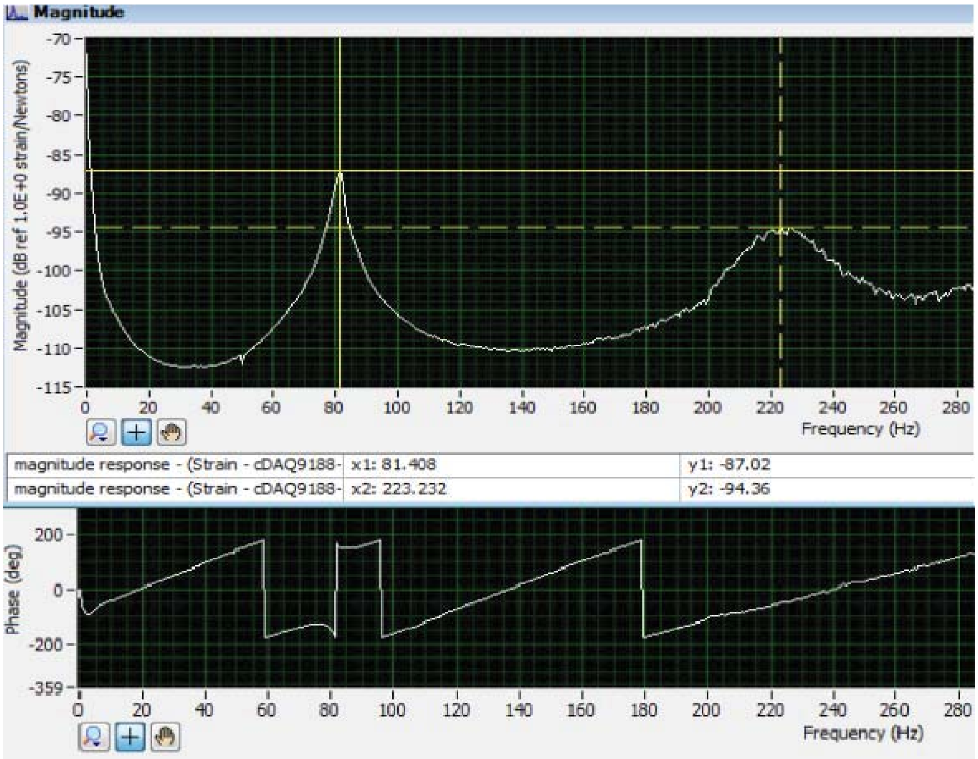
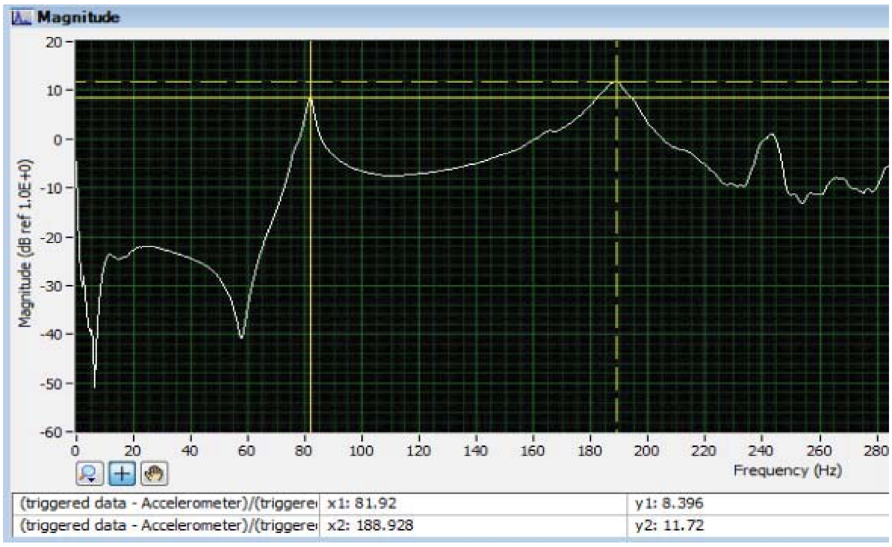


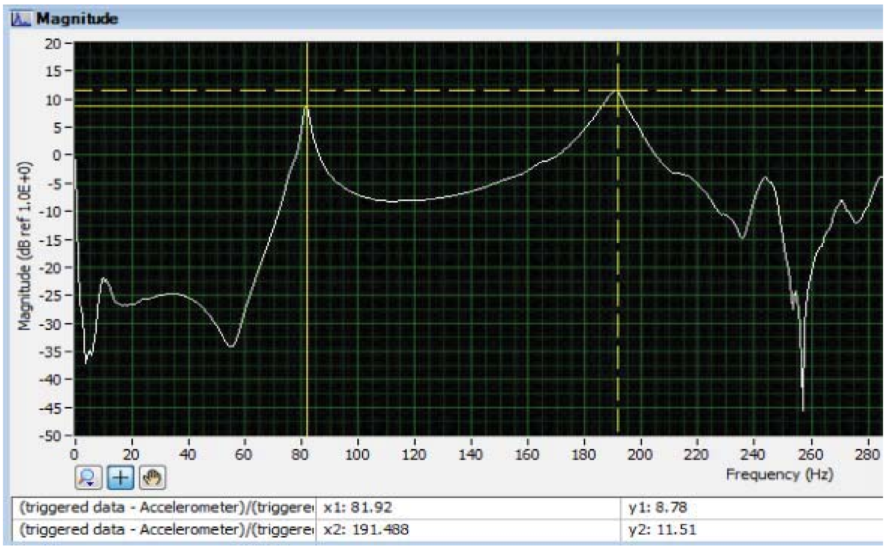
Fig. 7 Drive mode resonance characteristics using the strain gauges

Fig. 7 shows the drive mode resonance characteristics using the strain gauges, the first resonant frequency is equal 81.4 Hz with 5.7% error from the simulation results.

The experimental setup was repeated using an accelerometer as a sensing device, instead of the strain gauges, to verify the resonant frequencies. The first resonant frequencies of the drive and sense modes are equal, which satisfies the shape symmetry. Also, these values are approximately equal to the results determined by using strain gauges. The quality factors of the drive and sense modes are measured and found to be around 75. Fig. 8 shows the frequency response function FRF for the drive and sense modes.



(a) Drive mode



(b) Sense mode

Fig. 8 Frequency response function (FRF)

The error between the measured and simulated resonant frequencies is expected because of the extra mass of the sensing circuit and the accelerometer; it can be eliminated by using more light sensing circuits or compensating the extra mass in the simulated model.

V. Conclusion and Future Work

A symmetrical in plane single mass resonant gyroscope that allows not only mode matched of the drive and sense modes but also decoupled vibration modes to increase the sensitivity, resolution, operation stability, and to decrease the zero-rate output drift. The suspension beams are designed based on theoretical calculations. The gyroscope model is then simulated using ANSYS software package to verify the theoretical results, the resonant frequencies obtained from the modal analysis are matched and equal 86.4 Hz with 0.9% error from the theoretically calculated resonant frequencies due to the assumption of neglecting the elasticity of the beams directed along the vibration direction. The gyroscope is fabricated from aluminum alloy 2024 substrate using high speed wire cutting machine, the measured resonant frequencies are matched and equal 81.4 Hz with 5.7% error from the simulation results, the quality factors of the drive and sense modes are measured and found to be around 75. The sensitivity and resolution of the fabricated model are not measured and will be performed in the future; also the gyroscope model and the readout circuit will be modified to improve the performance of the resonant gyroscope.

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