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An Optimization Model for the Arrangement of Assembly Areas Considering Time Dynamic Area Requirements

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***Abstract*—**Large-scale products are often assembled according to the job-site principle, meaning that during the assembly the product is located at a fixed position, while the area requirements are constantly changing. On one hand, the product itself is growing with each assembly step, whereas varying areas for storage, machines or working areas are temporarily required. This is an important factor when arranging products to be assembled within the factory. Currently, it is common to reserve a fixed area for each product to avoid overlaps or collisions with the other assemblies. Intending to be large enough to include the product and all adjacent areas, this reserved area corresponds to the superposition of the maximum extents of all required areas of the product. In this procedure, the reserved area is usually poorly utilized over the course of the entire assembly process; instead a large part of it remains unused. If the available area is a limited resource, a systematic arrangement of the products, which complies with the dynamic area requirements, will lead to an increased area utilization and productivity. This paper presents the results of a study on the arrangement of assembly objects assuming dynamic, competing area requirements. First, the problem situation is extensively explained, and existing research on associated topics is described and evaluated on the possibility of an adaptation. Then, a newly developed mathematical optimization model is introduced. This model allows an optimal arrangement of dynamic areas, considering logical and practical constraints. Finally, in order to quantify the potential of the developed method, some test series results are presented, showing the possible increase in area utilization.

***Keywords*—**Dynamic area requirements, facility layout problem, optimization model, product assembly.

I. Introduction

LARGE-SCALE products are characterized by its relatively large size and heavy weight. According to Behrens et al. [1], a large-scale product is distinguished from a regular product by the effect that the production costs of a large-scale product disproportionately increase in relation to a further enlargement of a characteristic product feature (e.g. dimension). Examples for large-scale products can be found in machine and plant engineering such as packaging machines, printing machines, or electrical transformers. The assembly of large-scale products is characterized by a high level of technological and structural complexity [2], [3]. Furthermore, due to the high handling expenses, the assembly is usually organized in the job-site principle. This means that the assembly is carried out at a fixed position within the factory and the necessary resources are moved to the product [4]. When conducting multiple simultaneous product assemblies, the available area may become the bottleneck resource because of the high area requirements for this type of assembly organization. In such case, the different products compete for the available area, and the amount of possible assemblies within a time horizon gets extremely limited. Consequently, the challenge here is to find the best possible arrangement of products and handling areas within the factory in order to maximize the number of parallel assemblies and thus the possible output.

Typically, the size of large-scale products is increasing throughout their assembly process. In addition, specific assembly steps may require auxiliary areas for handling processes during certain periods. These may become abundant in following assembly steps, the latest when starting functionality tests or delivery preparations. The same applies to the material buffer zones, which are only required for certain assembly steps. In short, the assembly of large-scale products is characterized by significant fluctuations in area requirements over time, while facing limited available area on the shop floor. Thus, the intelligent arrangement of multiple assembly areas is a very important factor; especially when a subsequent relocation is technically not possible or if it would demand a great expense in cost and time [5]. This is the reason why, in current practice, a fixed area within the factory is assigned for each product assembly. The geometry of this reserved area results from the superposition of all product, handling and buffer areas [6] meaning that it corresponds to the maximum extend of all area requirements for each assembly step. When using this procedure, the first step is to define the area requirements for each product. This area is reserved over the entire assembly for only one product. Then, the position of each product within the factory is planned precisely, not allowing for any subsequent changes as they are not common due to the high associated cost. Consequently, this allows the use of operations-research (OR) approaches (e.g. Facility Layout Problem, see II.A). Nevertheless, this procedure ignores the described fact that the required areas for the products may vary over time (Fig. 1). A huge part of the reserved area is temporarily unused and could be used for the other purposes.

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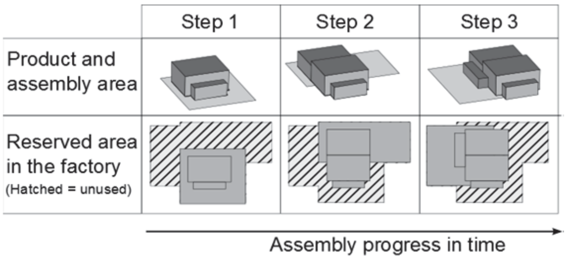


Fig. 1 exemplary assembly steps and area reservation

Applying a systematic arrangement approach that is based on the real area requirements may lead to better planning results: unused areas could then be used for the assembly of other products, which would result in a higher area utilization and higher productivity of the factory, as well as reduction of logistical costs. The arrangement of product assembly areas based on dynamic area requirements is highly complex because overlaps of currently occupied areas have to be avoided. Thus, even for small problem instances, a manual search for the optimal solution is nearly impossible, in particular when the areas are composed out of irregular geometries. An appropriate solution for solving the described problem is to transfer it into a specific mathematical optimization model.

II. Literature Review

In context of the stated problem definition, the arrangement of areas within a factory can be understood as a special case of a classical facility layout planning problem (FLP). Numerous research papers have coped with the solution of various types of FLP models using methods of mathematical optimization. In the following, current approaches that are applicable to the problem are described and evaluated, and afterwards, the most appropriate approach is selected.

A. Current Approaches

The first to model the problem of optimally locating facilities were Koopmanns and Beckmann in 1957 [9]. In their research, they arranged several facilities to predetermined positions, such that the costs for the material flow between the locations would be minimized. In following research activities this model has been concretized and applied to the arrangement of different areas within factories, resulting in the static facility layout problem (SFLP) [10]. The quality of the planning of a facilities layout is an important factor for its effective utilization [11], [12] and reducing operating costs [13]. Thus, the FLP has been sharpened in many other articles. Here, special requirements such as considering undefined [14], irregular shapes [15], or input/output points [16] were integrated in the original problem formulation. Extensive state of the art surveys for FLP approaches are, among others, published by Matai et al. [7] and Drira et al. [8].

For the problem described in this paper, the research of Rosenblatt [17] is significant, as it describes the SFLP by considering dynamic material flow intensities. Specifically, it is assumed that the material flow between the different factory areas (which form the basis for optimization in the SFLP) may vary over time, and thus, the layout has to be adjusted to the new situation. For this purpose, the problem is described as a sequence of SLPs that are connected via relocation costs. By integrating the relocation costs an adjustment is not realized until significant variations justify the additional expenses. Such time dynamic versions of the SFLP are described as dynamic facility layout problems (DFLP). While Rosenblatt [17] describes the assignment of individual elements in a predetermined framework, numerous following research works have addressed this specific problem and extended several constraints. For example, Lacksonen [18] extended the DFLP to consider freely allocable areas by defining the area requirements as an input variable while the corresponding (rectangular) geometries are being defined by a linearization. The possible applications of the DFLP are not limited to the general planning of a factory layout. Yang and Peters for example use a DFLP based approach for finding the optimal arrangement of equipment in a specific area [19]. Here, the machines can be arranged in different orientations and the material flow intensities are taken into account under uncertainty, which means that they are based on stochastic scenarios. An approach, which includes time dynamic space requirements, is described by Montreuil and Venkatadri [20]. They primarily address the expansion phase of a factory and therefore assume that the side lengths of the different areas depend on time. This approach involves both, expanding and shrinking areas, as well as avoiding overlaps in time, though, ignoring any possibility of dislocations and limited need of the areas. Another model is from Dunker et al., who developed a mixed integer optimization problem (MIP) for layout planning [21]. Machine groups are being clustered and arranged in an initial layout, which is gradually improved by using genetic algorithms. Here, spatial constraints such as restricted areas, access for material supply and removal, neighborhood constraints, as well as varying area sizes and orientations are considered. A differentiated consideration of working and buffer zones and specifications of the site assembly are not taken into account. Also specific details, such as transportation paths, which should be considered when installing large scale products in the course of the material handling, are also not considered. However, the modeling of existing operational frameworks combined with the possible use of genetic algorithms makes this model a possible source for adaption. Another suitable approach is described by Bock and Hoberg [22], who formulated the SFLP as a quadratic assignment problem (QAP) for the allocation of machines with irregular shapes considering discrete transportation paths. Furthermore, specific areas of the layout can be blocked or contain special requirements. The optimization is driven by the minimization of transportation paths and relocation costs.

B. Evaluation

All current models have in common that they focus on a given number of areas/machines, which are being arranged simultaneously. Thus, the time dynamics in the DFLP-models are limited to particular changes in the arrangement due to changes in the material flow or other external events. Whereas, changes in the geometries and size of the arranged areas as well as their project like character is not yet sufficiently considered. To evaluate existing approaches in respect to an adaptation for the issue of this article, various requirements were formulated and potentially suitable approaches were evaluated by their degree of fulfillment. The result of the evaluation is shown in Table I.

TABLE I
EVALUATION OF EXISTING APPROACHES

	Lacksonen & Enscore [23]	Solimanpur et al. [24]	Chiang [25]	Lacksonen [26]	Kim & Kim [16]	Dunker & Weskämper [21]	Garces et al. [27]	Montreuil & Venkatadri [20]	Bock & Hoberg [22]
Free positioning of the areas	-	-	o	+	+	+	+	+	+
Different area sizes	-	+	+	+	+	+	+	+	+
Irregular geometries	-	-	-	-	-	-	-	-	+
Various orientations	-	-	+	+	+	+	-	+	+
Time dynamic arrangement	-	-	-	-	-	o	-	+	-
Restricted areas	o	o	o	o	o	o	-	-	+
Transportation paths	-	-	-	-	-	-	-	-	+

- not considered; o Partially considered; + considered

According to the evaluation, the work of Dunker, Radon and Westkämper [21], as well as Bock and Homberg [22] are most suitable. Similar to the SFLP the DFLP can be modelled in different ways [10]. For choosing which approach fits best for the current problem, the type of modelling needs to be evaluated, too. One option for modelling a DFLP is formulating it as a mixed integer programming problem. Here, the variables and therefore the positions of areas in the layout are integers. The advantage of this method is the low number of variables and the possibility of an effective and fast solving through heuristics. However, the geometries of the areas, which have to be arranged, are limited to rectangular shapes. The other option is to formulate the model as a QAP, which describes the objects or areas, which have to be arranged in a raster of geometrically homogeneous rectangular elements. Thus, the position of areas is discrete and the results of the model depend strongly on the resolution of the area elements. Numerous opportunities such as the consideration of irregular shapes and a free surface orientation provide an appropriate basis for solving the presented problem. Due to the high number of decision variables, solving larger problem instances is more difficult. Ultimately, the QAP model by Bock and Hoberg [22] has been chosen because of the large number of possible variations as the groundwork for the here described research. Although this approach does not yet consider it, time dynamics can be easily implemented by adding an appropriate index and reformulating the models constraints and objective function.

III. THE MODEL

A. General Assumptions

In order to realize an optimal arrangement of assembly objects in the layout, a QAP-based operations research approach is used. The following assumptions are set:

1) All areas (layout & product) are known and defined
2) There are no interferences during the assembly processes
3) There are no other bottleneck resources, except the area
4) The production plan is known and is not changed
5) Transportation paths are predefined

These assumptions allow an efficient modelling and evaluation of the method, while in return confining the possibility of a practical application.

B. Indices, Parameters and Variables

TABLE II
INDICES, PARAMETER AND VARIABLES OF THE MODEL

Index	Range	Meaning
p	1,…,P	product
a	1,…,A	area type
s	1,…,S	assembly step
o	1,…,O	orientation
t	1,…,T	time-period
x	1,…,X	position in the layout raster
y	1,…,Y	
xp	1,…,XP	position in the product raster
yp	1,…,YP	
Parameter	**Type**	**Meaning**
$PE_{p,a,s,o,xp,yp}$	Binary	1 if element is part of the assembly area
$BE_{x,y}$	Binary	1 if element is banned for assembly
$TE_{x,y}$	Binary	1 if element is a transportation path
$PF_{p,a}$	Binary	1 if area cannot be dislocated
$PC_{p,a,t}$	Integer	1 if product must be completed in t
Variable	**Type**	**Meaning**
$PP_{p,a,s,o,t,x,y}$	Binary	1 if Product is placed on element
$AE_{p,a,s,o,t,x,y}$	Binary	Combined term for area assignment

The developed model includes a number of indices and parameters. Indices set the dimensions of the model, while parameters describe the specific conditions, such as the product and area geometries or transportation routes. The user must define both, the indices dimensions and the parameter values in order to apply the model to a specific problem situation. When solving the model, the decision variables are set by an optimization algorithm, which optimizes the objective function while fulfilling all constraints. Table II shows all indices, parameters, and variables which are used in this model.

All indices are integers and are in a logical sequence in accordance with their arrangement. For example, the coordinates in the layout are arranged in accordance with the indices x and y in ascending order, t=2 means the second period, which is followed by t=3 and the assembly begins with step s=1 and ends with step s=S.

C. Modelling Procedure

The starting point of modelling a specific problem situation is the transfer of the layout of an existing factory onto a raster (subsequently referred to as layout raster). Each element in this raster is uniquely located and described by the combination of the indices *x* and *y*. The dimensions of the raster elements can be freely defined (e.g. 2m x 2m), wherein it must be weighed between a high resolution (better solution quality) and the required computing power, which increases with the indices dimensions. Fig. 2 shows the conversion of an exemplarily facility layout into a layout raster.

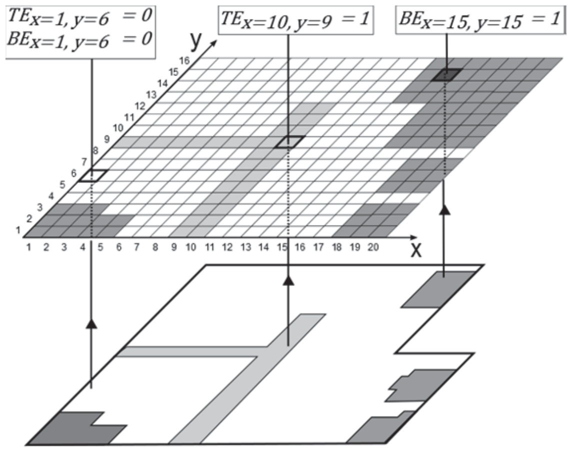


Fig. 2 Transforming a given layout into a raster

Elements that are not usable for the allocation of an assembly area are characterized by the binary parameter $BE_{x,y}$ *"banned element"*. This parameter is defined to the value 1, when a layout position, which is equivalent to raster element (x, y), is not appropriate for placing assembly areas. Examples are solid objects or areas located outside the factory. Special types of blocked elements are transportation paths. Transportation paths are defined by the binary parameter $TE_{x,y}$ (*"transportation element"),* which is set to the value 1, when the layout raster element *(x, y)* is part of a transportation path in the underlying facility layout. The dynamics are depicted by index *t*, which implies the time period. The length of the time periods can be defined freely (e.g. 1 day, 1 week or 1 month), but has to be constant for the whole time horizon. The modeling of the product areas is done analogous to the modelling of the factory in a raster, hereinafter referred to as product raster, which elements are referred by the combination of the indices *xp* and *yp*. The dimensions of the elements of the product raster correspond to those of the layout raster.

In total, *P* different products must be arranged within the layout. Each product *p* may consist of several areas (e.g. product, material handling, buffer), described by index *a*. During the assembly process the geometries of these areas are changing. To accommodate this effect, the assembly process is divided into several assembly steps, described ascending by index *s*. The length of each assembly step *s* corresponds to the length of the time periods *t*. The dimension of the index s corresponds to the longest duration of assembly over all products. When starting a product assembly, all areas are started simultaneously, and the stages are contiguous. If the required time for the assembly of a product is shorter than S, the unused assembly steps remain vacant by setting all elements of the product raster to value 0. Analogously for areas that are needed in later steps, the product raster for the unused earlier steps remains also vacant.

The geometries for each assembly step of each product area are described by the binary parameter $PE_{p,a,s,o,px,py}$. This parameter is only set to value 1, if the product raster element *(xp, yp)* is required for area *a* of product p in assembly step s (see Fig. 3). The size and geometry of the product area of each assembly step is derived of the superposition of the real area requirements within its duration. Thus, a higher resolution (shorter time periods) may lead to better planning results. To depict different orientation directions, this parameter furthermore depends on index *o*. Bock and Hoberg integrate this automatically in their model by arithmetic operations, while here for each orientation *o* direction a new product raster is formed via simple algorithms assuming that 90° rotation increments its dimension corresponds to 4. For orientation *o=1*, the positive elements of the product raster have to be set manually, the parameter values for the other product raster (index *o>1)* can be derived out of these. Thus, other dimensions of *O* are possible and furthermore this index may be used for the consideration of alternative shapes instead of orientations.

The arrangement of the product areas within the layout is modelled via the binary variable for $PP_{p,a,s,o,t,x,y}$ (*"product position").* If this variable is set to the value 1, assembly step *s* of area type *a* of product *p* is assigned to the raster element *(x, y)* in period t and orientation o. This variable does not represent a direct assignment of a product assembly area to the correspondent layout element (*x, y*), but rather forms an anchor point, for a projection of the product raster to the layout raster. The area assignment is done via the term

$$AE_{p,a,s,o,t,x,y} = \sum_{xp=1}^{XP} \sum_{yp=1}^{YP} PP_{p,a,s,o,t,x-xp+1,y-yp+1} * PE_{p,a,s,o,px,py} \quad (1)$$

This term equals value 1 exactly when a product area is placed on the element (*x, y*) in the period *t*. The simplified logic behind this is outlined in Fig. 3.

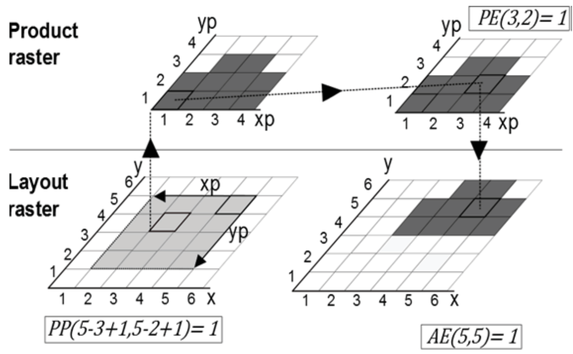


Fig. 3 Transfer of product areas into the layout raster

Indices *p,a,s,o* and *t* are limited to 1, so they are not mentioned in the figure. To check if a layout raster element (in the example for element x=5 and y=5) is covered by a product area, it is first checked via the sum in the formula, whether the element can theoretically belong to a product surface. This is exactly the case when one of the layout raster elements (*x – xp + 1, y - yp + 1*) , marked in light gray in the figure, is set to value 1 for one expression of variable $PP_{p,a,s,o,t,x,y}$ (*"position point"*). In the illustrated example in Fig. 3, that is the case for the layout element (*x=3, y=4*) and corresponding *xp=3* and *yp=2*. Then, the origin of the product raster (*xp=0,yp=0*) is literally projected to this position. If the product raster element (*xp=3, yp=2*) is defined as an existing product area the corresponding parameter $PE_{p,a,s,o,px,py}$ has value 1. With the product in the formula being value 1 and a limit of only one position point (see constraints described below), the corresponding value for $AE_{p,a,s,o,t,x,y}$ is 1. This means that the examined element in the layout raster is marked as occupied. This process is done for each element in the layout raster.

D.Model Constraints

To avoid collisions of areas and to take account of logical relationships and restrictions, the model is based on several constraints, shown in Table III.

First, constraint (2) demands that, for each step of each product area, at least one position point is placed in the layout raster. If an area is not needed in this step, the corresponding product raster is empty. Setting the number of allowed position points to exactly 1 is impermissible, as multiple position points are allowed to model planned interruptions (see (7) and (8)).

Constraint (3) ensures that no product area is placed on transportation paths or restricted areas by limiting the maximum number of different area types placed on a layout raster element to 1. Thus, a simultaneous arrangement of product areas, transportation paths and blockings on the raster element is prohibited.

To ensure that all of the modelled product areas are placed in the layout, constraint (4) is necessary. This constraint ensures that the sum of the product elements placed in the layout raster corresponds to the number of positive elements in the product raster. Corresponding to this, exactly one product position is set for each product in such a way that no product element is set outside of the boundaries of the layout raster.

Since each product area can be placed in different orientations, constraint (5) ensures that for each product area exactly one orientation is used. Otherwise, the projection on the layout raster could consist of parts of multiple product raster.

During assembly, a product may need various types of areas (if A > 1), such as storage or auxiliary areas. For each assembly step these areas have to be implemented simultaneously, but may be positioned freely in the layout. This is enforced by constraint (6). In addition, the logical time sequence of each product must be adhered. When the position point for the first assembly step ($s=1$) is set the following steps must be placed in the subsequent periods in ascending order. Possible interruptions are modeled by a multiple placement of the corresponding position point over several periods. This is achieved due to constraint (7) and (8). Specifically, in the subsequent period, the position point of the following stage must be placed, or otherwise the assembly operation would be interrupted, and the same stage would be placed again. This is to respect the predetermined sequence of the products assembly, where planned interruptions of the assembly process are permitted by the possibility of multiple placements of the same level.

Even though a subsequent change of the products location *(x,y)* in the layout during assembly is theoretically possible, it is often not appropriate because of the high expenses of relocation. In the model, both options are considered by constraint (9). For this purpose, the binary parameter $PF_{p,a}$ is used. This Parameter is set to 1 if the associated product area is not allowed to be relocated. In this case, each product position must be placed on the same layout raster element as its predecessor. In addition, further logical or organizational constraints may be relevant. A possible constraint could be the requirement that all product assemblies have to be started or completed in a predetermined order. Constraint (10) ensures this by enforcing the product positions in a sequence, depending on their index *p*. Based on the products completion, the product raster must be defined as such that the completion is performed at the last assembly step. Furthermore, the completion of products can be linked to completion dates. For this, constraint (11) defines that the assembly completion time may not lie behind the parameter $PC_{p,a,t}$.

Constraint (12) is excluded in the model and only used for replacing constraint (2) when using objective function (15).

Transport routes are usually specified due to building regulations and the building's geometry. Constraints which force a direct link of the surfaces to the transport paths were discarded. The reason for this is that several test series have shown that connections to transport routes are usually given due to the large product size.

TABLE III
CONSTRAINTS OF THE MODEL

$\sum_{o=1}^{O}\sum_{t=1}^{T}\sum_{x=1}^{X}\sum_{y=1}^{Y} PP_{p,a,s,o,t,x,y} \geq 1$	$\forall p \in \{1,\dots,P\}$ $\forall a \in \{1,\dots,A\}$ $\forall s \in \{1,\dots,S\}$	(2)
$\sum_{p=1}^{P}\sum_{a=1}^{A}\sum_{s=1}^{S}\sum_{0=1}^{O} AE_{p,a,s,o,t,x,y} + BE_{x,y} + TE_{x,y} \leq 1$	$\forall t \in \{1,\dots,T\}$ $\forall x \in \{1,\dots,X\}$ $\forall y \in \{1,\dots,Y\}$	(3)
$\sum_{o=1}^{O}\sum_{t=1}^{T}\sum_{x=1}^{X}\sum_{y=1}^{Y} AE_{p,a,s,o,t,x,y} \geq$ $\sum_{o=1}^{O}\sum_{xp=1}^{XP}\sum_{yp=1}^{YP}\left(PE_{p,a,s,o,xp,yp} * \sum_{t=1}^{T}\sum_{x=1}^{X}\sum_{y=1}^{Y} PP_{p,a,s,o,t,x,y}\right)$	$\forall p \in \{1,\dots,P\}$ $\forall a \in \{1,\dots,A\}$ $\forall s \in \{1,\dots,S\}$	(4)
$\sum_{0=1}^{O}\sum_{x=1}^{X}\sum_{y=1}^{Y} AE_{p,a,s,o,t,x,y} = \sum_{0=1}^{O}\sum_{x=1}^{X}\sum_{y=1}^{Y} PP_{p,a,s,o,t,x,y} * \sum_{xp=1}^{XP}\sum_{yp=1}^{YP} PE_{p,a,s,o,px,py}$	$\forall p \in \{1,\dots,P\}$ $\forall a \in \{1,\dots,A\}$ $\forall s \in \{1,\dots,S\}$ $\forall t \in \{1,\dots,T\}$	(5)
$\sum_{o=1}^{O}\sum_{x=1}^{X}\sum_{y=1}^{Y} PP_{p,a,s,o,t,x,y} = \sum_{o=1}^{O}\sum_{x=1}^{X}\sum_{y=1}^{Y} PP_{p,\tilde{a},s,o,t,x,y}$	$\forall p \in \{1,\dots,P\}$ $\forall a,\tilde{a} \in \{1,\dots,A\}$ $\forall s \in \{1,\dots,S\}$ $\forall t \in \{1,\dots,T\}$	(6)
$\sum_{o=1}^{O}\sum_{x=1}^{X}\sum_{y=1}^{Y} PP_{p,a,s,o,t,x,y} \leq$ $\sum_{o=1}^{O}\sum_{x=1}^{X}\sum_{y=1}^{Y} PP_{p,a,s,o,t+1,x,y} + \sum_{0=1}^{O}\sum_{x=1}^{X}\sum_{y=1}^{Y} PP_{p,a,s+1,o,t+1,x,y}$	$\forall p \in \{1,\dots,P\}$ $\forall a \in \{1,\dots,A\}$ $\forall s \in \{1,\dots,S-1\}$ $\forall t \in \{1,\dots,T\}$	(7)
$\sum_{o=1}^{O}\sum_{x=1}^{X}\sum_{y=1}^{Y} PP_{p,a,s,o,t-1,x,y} + \sum_{0=1}^{O}\sum_{x=1}^{X}\sum_{y=1}^{Y} PP_{p,a,s-1,o,t-1,x,y} \geq$ $\sum_{o=1}^{O}\sum_{x=1}^{X}\sum_{y=1}^{Y} PP_{p,a,s,o,t,x,y}$	$\forall p \in \{1,\dots,P\}$ $\forall a \in \{1,\dots,A\}$ $\forall s \in \{2,\dots,S\}$ $\forall t \in \{1,\dots,T\}$	(8)
$PF_{p,a} * PP_{p,a,s,o,t,x,y} \leq PP_{p,a,s,o,t+1,x,y} + PP_{p,a,s+1,o,t+1,x,y}$	$\forall p \in \{1,\dots,P\}$ $\forall a \in \{1,\dots,A\}$ $\forall s \in \{1,\dots,S-1\}$ $\forall o \in \{1,\dots,O\}$ $\forall t \in \{1,\dots,T-1\}$ $\forall x \in \{1,\dots,X\}$ $\forall y \in \{1,\dots,Y\}$	(9)
$\sum_{\tilde{t}=1}^{t}\sum_{o=1}^{O}\sum_{x=1}^{X}\sum_{x=1}^{Y} PP_{p,a,s,o,\tilde{t},x,y} \leq \sum_{o=1}^{O}\sum_{x=1}^{X}\sum_{y=1}^{Y} PP_{p+1,a,s,o,t,x,y}$	$\forall p \in \{1,\dots,P-1\}$ $\forall a \in \{1,\dots,A\}$ $\forall s \in \{1,\dots,S-1\}$ $\forall t \in \{1,\dots,T\}$	(10)
$\sum_{\tilde{t}=1}^{t}\sum_{o=1}^{O}\sum_{x=1}^{X}\sum_{x=1}^{Y} PP_{p,a,s,o,\tilde{t},x,y} \geq PC_{p,a,t}$	$\forall p \in \{1,\dots,P\}$ $\forall a \in \{1,\dots,A\}$ $\forall s \in \{1,\dots,S\}$ $\forall t \in \{1,\dots,T\}$	(11)
$\sum_{\tilde{o}=1}^{O}\sum_{t=1}^{T}\sum_{x=1}^{X}\sum_{y=1}^{Y} AE_{p,a,s,\tilde{o},t,x,y} \geq$ $\sum_{dx=1}^{DX}\sum_{dy=1}^{DY} PE_{p,a,s,o,px,py} * \sum_{\tilde{o}=1}^{O}\sum_{t=1}^{T}\sum_{x=1}^{X}\sum_{y=1}^{Y} PP_{p,a,s,\tilde{o},t,x,y}$	$\forall p \in \{1,\dots,P\}$ $\forall a \in \{1,\dots,A\}$ $\forall s \in \{1,\dots,S\}$ $\forall o \in \{1,\dots,O\}$ $\forall t \in \{1,\dots,T\}$	(12)

E. Objective Functions

In terms of optimization, there are different possibilities for the design of the objective function. First, the following function which generates a simple and sufficient compression of all assembly operations can be used.

$$min \sum_{p=1}^{P}\sum_{a=1}^{A}\sum_{s=1}^{S}\sum_{o=1}^{O}\sum_{t=1}^{T}\sum_{x=1}^{X}\sum_{y=1}^{Y}\left(PP_{p,a,s,o,t,x,y} * t\right) \quad (13)$$

This objective function minimizes the production horizon and thus maximizes simultaneously the area utilization, enabling it to be used for tests to identify the potential consideration of dynamic area requirements. When using this function, constraints (10) and (11) should be excluded.

An alternative approach is to minimize the schedule deviations, which is achieved by the following objective function.

$$\min \sum_{p=1}^{P}\sum_{a=1}^{A}\sum_{s=1}^{S}\sum_{o=1}^{O}\sum_{t=1}^{T}\sum_{x=1}^{X}\sum_{y=1}^{Y} \left(PP_{p,a,s,o,t,x,y} * \sum_{\tilde{t}=1}^{t}(t-\tilde{t}) * PC_{p,a,\tilde{t}}\right) \quad (14)$$

This objective function is useful for practical application by focusing adherence to schedules rather than appropriateness for the evaluation of the method. When using this function, constraint (11) should be excluded.

Next to the first objective function, which minimizes the amount of time necessary for a given production plan, the output quantity in a defined time period can also be maximized.

$$\max \sum_{p=1}^{P}\sum_{a=1}^{A}\sum_{s=1}^{S}\sum_{o=1}^{O}\sum_{t=1}^{T}\sum_{x=1}^{X}\sum_{y=1}^{Y} PP_{p,a,s,o,t,x,y} \quad (15)$$

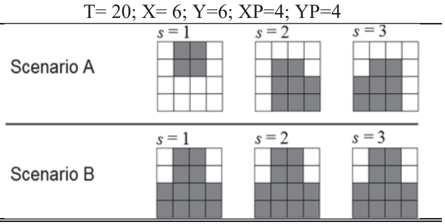
In this case, there must be enough products available to fully occupy the time frame. To avoid a prioritization due to favorable geometries, only one type of product should be considered, meaning the geometries of all products are identical. For this, the dimension of index p must be limited to 1. By excluding constraint (2) and reformulating constraint (4) to (12), this product can be arranged more than once.

IV. Implementation and Validation

To solve the described mathematical model, this model has been implemented in the software GAMS and then is solved by using the CPLEX solver. The definition of the parameters was supported by using an EXCEL-based GUI, specially designed for this problem. This was necessary because the binary character of the parameter combined with several indices would require great efforts when employing a manual data input. This tool is also used to display the results from the GAMS solution in a GUI, where the arrangement of the areas in the layout is visualized to the user.

To show the potential of considering time dynamic area requirements in layout planning, a test scenario for a fictitious product was solved. Here, a total of 16 products had to be assigned in a square layout raster. All products have the same geometries and each assembly requires three time periods. Table IV shows the dimensions of indices and layout, as well as the product raster.

TABLE IV
Evaluation Settings

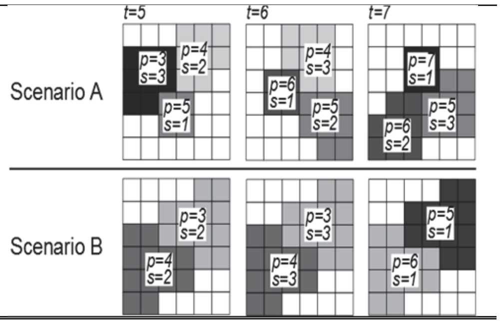
Index dimensions	P=10; A=1; S= 3; O=4 T= 20; X= 6; Y=6; XP=4; YP=4
Product raster	

In the layout, there are no transportation paths or blocked elements. Thus, constraint (6) was ignored, as well as constraint (10) and (11). Dislocations are prohibited by setting $PF_{p,a}$=1.

The evaluation was performed by solving the model for two different scenarios using objective function (13). The first scenario (Scenario A) is considering time dynamics by using dynamic area geometries. To compare this with the current method, a second test scenario (Scenario B) was implemented. Under identical conditions, the product raster was set to be static. According to the current practical procedure, the product shapes consist of the superposition of all assembly steps. The results of this comparison are shown in Table V.

In total, the competition of all 16 product assemblies in Scenario A took 3 periods less than in scenario B, and three products can be assembled simultaneously instead of two. Thus, area utilization increases from 37% up to over 54.3%, which proves that considering time dynamic area requirements when arranging assembly areas can enhance the productivity of a limited area. The dimensions of the layout are chosen in a way that exactly two products (Scenario B) fit within the parameters. If the dimensions would be greater (e.g. $x=7$, $y=7$) the improvement could be even higher. But for an operational use, especially when there is a need to quickly react on unplanned interruptions or problems, the solution must be determined promptly.

TABLE V
Results of Evaluation

	Scenario A (real area)	Scenario B (reserved area)
elapsed time for solving	200.000s	1800s
objective value	456	600
periods to complete all assemblies	18	24
total area utilization	54.3%	37%
Solution (extract)		

In order to achieve this, the research now focuses on the conception and implementation of an efficient heuristic, which enables a quick solution for larger problem instances.

V. Conclusion

The presented mathematical model is able to realize an optimal arrangement of temporary assembly areas complying with dynamic area requirements within the factory layout. The validation shows that an increase in the area utilization can be achieved by using this method. Thus, it is shown that, in the described initial situation, a use of the method is economically advisable. Since higher area utilization is associated with a lower interference resistance, the method also needs to be examined in terms of the impact of possible interferences such as delays in delivery of materials. The formulation of the model as QAP allows an easy modification of the constraints to specific conditions.

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