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Posture Stabilization of Kinematic Model of Differential Drive Robots via Lyapunov-Based Control Design

Li Jie, Zhang Wei

***Abstract*—**In this paper, the problem of posture stabilization for a kinematic model of differential drive robots is studied. A more complex model of the kinematics of differential drive robots is used for the design of stabilizing control. This model is formulated in terms of the physical parameters of the system such as the radius of the wheels, and velocity of the wheels are the control inputs of it. In this paper, the framework of Lyapunov-based control design has been used to solve posture stabilization problem for the comprehensive model of differential drive robots. The results of the simulations show that the devised controller successfully solves the posture regulation problem. Finally, robustness and performance of the controller have been studied under system parameter uncertainty.

***Keywords*—**Differential drive robots, nonlinear control, Lyapunov-based control design, posture regulation.

I. Introduction

DIFFERENTIAL drive robots are an important and widely used class of mobile robots. Simplicity and low cost of production, as well as their simple mechanism of work, make them a popular class of robots for mobile robotics applications. The mechanical structure of these robots is comprised of two active wheels on the sides of the robot and a passive caster wheel which is responsible for balancing the weight of the robot through the ground reaction forces. The name “differential drive” comes from the fact that the difference in the angular velocity of the wheels on the two sides of the robot determines the direction of motion of this robot. Differential drive robots have been widely used in many applications such as service robotics [1], rescue robotics [2], [3], and mapping and surveillance applications [4]-[6]. More importantly, many other systems such as wheelchairs can be modeled as differential drive robots [7]-[9]. Although these robots are structurally simple, they have been widely studied. Researchers have studied various aspects of differential drive robots including their modeling and control. Modeling of wheeled robots is a challenging task mainly due to the complexities associated with the slipping phenomenon and static friction force [1], [9]-[11], [28]. On the other hand, control of these devices is a big challenge too. Many of the studies in the literature study the kinematic model of these robots. This is because the kinematics of this robot is responsible for the challenges associated with their control. The non-holonomic nature of the kinematic constraints and the under-actuated nature of these systems cause complications in the design of controls. For example, linearization of the differential drive robot kinematics about the point of operation yield a linear system that is not controllable and thus cannot be used for the design of a local linear controller [1], [2].

Control problems for mobile robots can be roughly categorized into three groups which includes stabilization [12]-[18], [23], [25], path following [9], [13] and their tracking control [20]-[23], [26], [27]. In the posture stabilization, the goal is to take robot from an initial position and heading for a final a specified position and heading, whereas tracking control is the problem of following a predefined trajectory in the task space. The path following problem is similar to tracking control with the difference that only the path associated with the trajectory needs to be followed in the task space, not the timing. In fact, stabilization of differential drive robots is more challenging than the path following and tracking problems [1], [13], [23]. This complexity stems from the non-holonomic and under-actuated nature of the differential drive robots which is deeply studied in Brockett’s prominent work on controllability of non-holonomic system [24]. Based on Brockett’s theorem, stabilization of non-holonomic systems requires a number of inputs equal to the number of states of the systems. As a result of this conditions, it is known that stabilization of differential drive robots is not possible via smooth time-invariant feedback of state variables.

Stabilization of differential drive robots has been studied in many research works. In general, it is not possible to stabilize differential drive robots by smooth time invariant control. Therefore, stabilizing controls are either discontinuous [12]-[14] or time-dependent [15]-[19]. On the other hand, several studies have used standard nonlinear control techniques such as back-stepping [26], sliding mode control, adaptive control [27] and feedback linearization [20], [22], [23], [25] to design stabilizing controls. Quite many of the research works in the literature use the simple unicycle model of the differential drive systems for control design. This research work uses a more comprehensive model of the differential drive robots which includes the physical aspects of the mechanical structure. Then the problem of posture regulation is studied, and a stabilizing controller is designed using Lyapunov control design method. The advantage of using a model containing system’s physical parameters is the ability of the designer to study the effect of parametric uncertainties on the

Li Jie and Zhang Wei are with the Department of Electrical Engineering, Beijing Institute of Technology, China (e-mail: jielilii2010@gmail.com, zhangweieebit@gmail.com).

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control performance. Finally, this paper studies the robustness and performance of the controller in stabilizing the posture of differential drive robots under parametric uncertainty.

II. Modeling Differential Drive Robots

Many research studies in the literature have focused on the so-called unicycle model [1], [13], [14], [20]-[23]. Unicycle model is the simplest model for differential drive robots which expresses the rolling without slip kinematic constraint of a rolling disc. The unicycle model that is commonly used is:

$$\begin{pmatrix}\dot{x}\\ \dot{y}\\ \dot{\beta}\end{pmatrix} = \begin{pmatrix}cos\beta & 0\\ sin\beta & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}v\\ \omega\end{pmatrix} \quad (1)$$

where x and y represent the coordinates of the robot and β denotes the heading of the robot. The control inputs of this model are v and ω which denote the forward velocity of the robot and its rotation rate respectively. However, unicycle is a very abstract model which is only focused on describing the rolling without slip phenomena. In this paper, we use a more comprehensive model for differential drive robots which incorporates the physical dimensions of the system into the model [13], [28]. Consider the schematics of a general differential drive robot in Fig. 1 where (i,j) denotes the frame of reference, (e_1,e_2) is a rotating frame located on the axle line connecting the two wheels, and β is the heading of the robot. By applying zero relative velocity between the point of contact of the wheel and the ground the kinematic model is found to be:

$$\begin{pmatrix}\dot{X}\\ \dot{Y}\\ \dot{\beta}\end{pmatrix} = B(\beta)\begin{pmatrix}\dot{\varphi}_L\\ \dot{\varphi}_R\end{pmatrix} \quad (2)$$

$$B(\beta) = \begin{pmatrix}\frac{rLcos(\beta+\phi)}{d} & rcos(\beta) - \frac{rLcos(\beta+\phi)}{d}\\ rsin(\beta) - \frac{rLsin(\beta-\phi)}{d} & \frac{rLsin(\beta-\phi)}{d}\\ -\frac{r}{d} & \frac{r}{d}\end{pmatrix}$$

where (X, Y) denote the position of the robot's center of mass G, β is the robot heading, r is the radius of the wheels and $(\dot{\varphi}_L, \dot{\varphi}_R)$ are the angular velocity of left and right wheels respectively. Other parameters in matrix B are the physical dimensions of the robot specified in Fig. 1.

Rolling without slipping constraint in the unicycle model limits the velocities the robot can achieve only to the longitudinal direction. In other words, under the assumption of rolling without slipping, lateral velocity of the robot is always zero. In order for our model to be compatible with unicycle model in this aspect, we modify the comprehensive model proposed in [13], [28]. For this purpose, we consider the point between the wheels of the robot as the point around which the motion of the system is modeled. There are two possible approaches for deriving the model. Either one could follow similar steps by imposing zero velocity for the contact point of wheels or set ϕ to zero in (2).

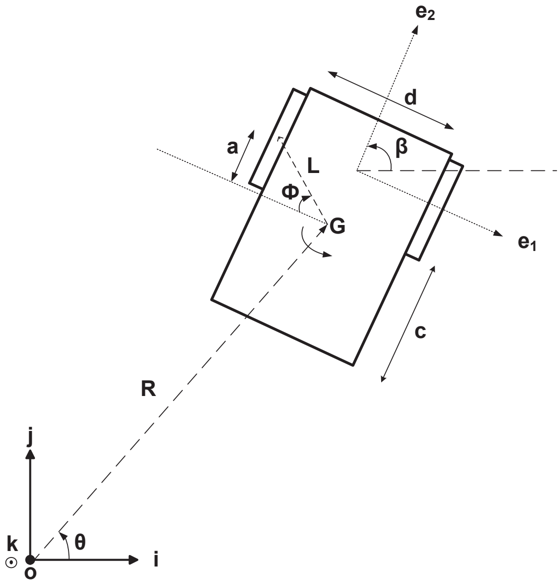

Fig. 1 Schematics of the differential drive robot [13], [28]

Either method yields the following model for the rolling without slipping kinematics of differential drive robots:

$$\begin{bmatrix}1 & 0 & Lcos\beta\\ 0 & 1 & -Lsin\beta\\ 0 & 0 & d\end{bmatrix}\begin{bmatrix}\dot{X}\\ \dot{Y}\\ \dot{\beta}\end{bmatrix} = \begin{bmatrix}0 & rcos\beta\\ rsin\beta & 0\\ -r & r\end{bmatrix}\begin{bmatrix}\dot{\varphi}_L\\ \dot{\varphi}_R\end{bmatrix} \quad (3)$$

Similar to the unicycle model, this model is under-actuated since the number of inputs is 2 which is less than the number of system states to be controlled in the stabilization process. Also, (2) describes a set of non-holonomic constraints as expected since the Lie bracket of the vectors in the driftless form representation of the system is not included in their spanned set.

III. Nonlinear Control Background

As shown in Section II, the kinematic model of differential drive robots is a non-holonomic underactuated nonlinear system. Lyapunov theory is a useful tool for stability analysis and design of control for nonlinear systems. In this section, we briefly review this theorem and the Barbalat's lemma which is commonly used when the result of Lyapunov analysis is inconclusive, and the derivative of Lyapunov function is negative semidefinite. First, the definition of stability in the sense of Lyapunov is introduced, and then the formal theory is included.

Definition 1: Consider the dynamical system:

$$\dot{x} = f(x,t), \quad x(0) = x_0 \quad (4)$$

where $f\ (x,t)$ is Lipschitz continuous with respect to x, and piecewise continuous in t. The equilibrium point $x^* = 0$ of (4) is stable (in the sense of Lyapunov) at $t = t_0$, if for any $\varepsilon > 0$ there exists a $\delta(t_0,\varepsilon)$ such that:

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$$\|x(t_0)\| < \delta \quad \rightarrow \quad \|x(t)\| < \varepsilon, \quad \forall t > t_0 \tag{5}$$

Definition 2: Consider the dynamical system in (4). An equilibrium point $x^* = 0$ is asymptotically stable at $t = t_0$ if:

1. $x^* = 0$ is stable, and
2. $x^* = 0$ is locally attractive; i.e., there exists $\delta(t_0)$ such that:

$$\|x(t_0)\| < \delta \quad \rightarrow \quad \lim_{t\to\infty} x(t) = 0 \tag{6}$$

Lyapunov's Theory [29]: Let $V(x, t)$ be a non-negative function with derivative $\dot{V}(x,t)$ along the trajectories of the system:

1. If $V(x, t)$ is locally positive definite and $\dot{V}(x,t) \le 0$ locally in x and for all t, then the origin of the system is locally stable (in the sense of Lyapunov).
2. If $V(x, t)$ is locally positive definite and decreasing, and $\dot{V}(x,t) \le 0$ locally in x and for all t, then the origin of the system is uniformly locally stable (in the sense of Lyapunov).
3. If $V(x, t)$ is locally positive definite and decreasing, and $-\dot{V}(x,t)$ is locally positive definite, then the origin of the system is uniformly locally asymptotically stable.
4. If $V(x, t)$ is positive definite and decreasing, and $\dot{V}(x,t)$ is positive definite, then the origin of the system is globally uniformly asymptotically stable.

Barbalat's Lemma [29]: Suppose $f(t) \in C^1(a,\infty)$ and $\lim_{t\to\infty} f(t) = \alpha < \infty$. If f' is uniformly continuous, then: $\lim_{t\to\infty} f'(t) = 0$

IV. Lyapunov-Based Control Design

Using Lyapunov analysis for the design of controllers is a common and effective approach for nonlinear systems. The idea is using a positive definite function and devising the control signals such that the time derivative of the function along the system trajectories is negative definite. To apply this idea to the problem of posture stabilization, we need to express the model of the system in Polar coordinates. Let's consider the following change of variables:

$$\begin{cases} R = \sqrt{X^2 + Y^2} \\ \alpha = \theta - \beta + \pi \\ \delta = \theta + \pi \end{cases} \tag{7}$$

where θ is defined using the four quadrant inverse tangent as:

$$\theta = ATAN2(Y, X)$$

Using this change of variables, the new set of state equations can be found. To this end, the time derivative of R is calculated as:

$$\dot{R} = {(X\dot{X} + Y\dot{Y})}\Big/{\sqrt{X^2 + Y^2}} \tag{8}$$

From the kinematic model of the robot in (3), the time derivative of the position of robot is found as:

$$\begin{cases} \dot{X} = \left({r}/{2}\right)(\dot{\varphi}_L + \dot{\varphi}_R)\, cos\phi \\ \dot{Y} = \left({r}/{2}\right)(\dot{\varphi}_L + \dot{\varphi}_R)\, sin\phi \\ \dot{\beta} = -\left(\frac{r}{d}\right)(\dot{\varphi}_L - \dot{\varphi}_R) \end{cases}$$

Substituting these equations into (8):

$$\dot{R} = \left({r}/{2}\right)(\dot{\varphi}_L + \dot{\varphi}_R)\left(\frac{(Xcos\phi + Ysin\phi)}{\sqrt{X^2 + Y^2}}\right) \tag{9}$$

From Fig. 1, the following geometric relationship can be easily deduced:

$$\begin{cases} \frac{X}{\sqrt{X^2 + Y^2}} = cos\theta = -cos\delta \\ \frac{Y}{\sqrt{X^2 + Y^2}} = sin\theta = -sin\delta \end{cases} \tag{10}$$

Thus, (9) can be simplified to:

$$\dot{R} = -\left({r}/{2}\right)(\dot{\varphi}_L + \dot{\varphi}_R)\, cos(\delta - \phi) \tag{11}$$

Similarly, the time derivative of δ can be calculated as:

$$\dot{\delta} = \dot{\theta} = \left({r}/{2}\right)(\dot{\varphi}_L + \dot{\varphi}_R)\,\frac{sin(\delta - \phi)}{R} \tag{12}$$

Therefore, model of the robot after change of variables is found to be:

$$\begin{cases} \dot{R} = -\left({r}/{2}\right)(\dot{\varphi}_L + \dot{\varphi}_R)\, cos(\delta - \phi) \\ \dot{\delta} = \left({r}/{2}\right)(\dot{\varphi}_L + \dot{\varphi}_R)\,\frac{\sin(\delta - \phi)}{R} \\ \dot{\phi} = -\left({r}/{d}\right)(\dot{\varphi}_L + \dot{\varphi}_R) \end{cases} \tag{13}$$

Defining $\alpha = \delta - \phi$, (13) can be written as:

$$\begin{cases} \dot{R} = -\left(\frac{r}{2}\right)(\dot{\varphi}_L + \dot{\varphi}_R)\, cos\alpha \\ \dot{\alpha} = \left(\frac{r}{2}\right)(\dot{\varphi}_L + \dot{\varphi}_R)\,\frac{sin\,\alpha}{R} + \left(\frac{r}{d}\right)(\dot{\varphi}_L - \dot{\varphi}_R) \\ \dot{\phi} = -\left(\frac{r}{d}\right)(\dot{\varphi}_L - \dot{\varphi}_R) \end{cases} \tag{14}$$

Using the transformed system in (14), Lyapunov-based control design is used for synthesizing a posture stabilizing control. This procedure is outlined in Theorem I.

Theorem I: Consider the transformed rolling without slipping kinematic of the differential drive robots in (14). Under the control laws:

$$\begin{cases} \dot{\varphi}_L = \frac{\lambda}{r} Rcos\alpha - \frac{d}{2r}\left(k\alpha + 0.5\frac{sin2\alpha}{\alpha}(\alpha + h\delta)\right) \\ \dot{\varphi}_R = \frac{\lambda}{r} Rcos\alpha + \frac{d}{2r}\left(k\alpha + 0.5\frac{sin2\alpha}{\alpha}(\alpha + h\delta)\right) \end{cases} \tag{15}$$

where k, λ, and h are positive scalars, the differential drive robot is asymptotically stabilized (i.e. states of system reach the origin asymptotically).

Proof: The proof is based on the Lyapunov analysis. Consider the following positive definite function:

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$$V = \frac{1}{2}\lambda R^2 + \frac{1}{2}(\alpha^2 + h\delta^2) \quad (16)$$

The first term is the norm of the distance error and the second term is the norm of the alignment error. The time derivative of this positive definite function along the system trajectories is found to be:

$$\dot{V} = \lambda R\dot{R} + \alpha\dot{\alpha} + h\delta\dot{\delta} \quad (17)$$

Substituting (14) into (17), the time derivative of V along system trajectories is found as:

$$\dot{V} = -\lambda R\left(\frac{r}{2}\right)(\dot{\varphi}_L + \dot{\varphi}_R)\cos(\delta-\phi) + \alpha\left(\left(\frac{r}{2}\right)(\dot{\varphi}_L + \dot{\varphi}_R)\frac{\sin(\delta-\phi)}{R} + \left(\frac{r}{d}\right)(\dot{\varphi}_L - \dot{\varphi}_R)\right) + h\delta\left(\left(\frac{r}{2}\right)(\dot{\varphi}_L + \dot{\varphi}_R)\frac{\sin(\delta-\phi)}{R}\right) \quad (18)$$

Using the control signals of (15) in (18) and after long algebraic calculations, it can be shown that:

$$\dot{V} = -\lambda^2 R^2\cos^2\alpha - \lambda^2(\cos^2\alpha)e^2 - k\alpha \le 0 \quad (19)$$

Although the first term of (19) is a negative definite term, all the function together is negative semidefinite. Since the Lyapunov function V is positive definite, it is lower bounded by zero. On the other hand, (19) shows that the value of V is non-increasing. Therefore, the value of V is converging asymptotically toward a finite non-negative value. V is a radially unbounded function. Therefore, state trajectories for any initially bounded condition will remain bounded and resultantly $\dot{V}$ is uniformly continuous in time. Consequently, based on Barbalat's Lemma, $\dot{V}$ converges to zero which means that the state of system will converge to $[0, 0, \hat{\theta}]$ for some $\hat{\theta}$. The next step is showing that $\hat{\theta} = 0$ is the only possible convergence point. To this end, the closed-loop equations of the system is studied by substituting (15) into (14). The closed-loop system is:

$$\begin{cases} \dot{R} = -\lambda R\cos^2\alpha \\ \dot{\alpha} = -k\alpha - 0.5\,\gamma h\,\dfrac{\sin 2\alpha}{\alpha} \\ \dot{\theta} = \gamma\cos\alpha\sin\alpha \end{cases} \quad (20)$$

We showed that the first and third equations both converge to zero and from the second equation it is concluded that $\dot{\alpha}$ is converging to a finite limit. Since state trajectories are bounded, therefore $\dot{\alpha}$ is a uniformly continuous function in time. Applying Barbalat's lemma it follows that $\dot{\alpha}$ converges to zero. Convergence of $\dot{\alpha}$ to zero is equivalent to convergence of θ to zero because of (20). Therefore $\hat{\theta} = 0$ and the system is asymptotically stabilized to origin.

V. Simulation Results

To verify the performance of the stabilizing controller proposed in this paper, simulation results are provided. In this section the Lyapunov-based control is applied to the robot model. The physical parameters of the differential drive robot used for simulation are $r = 0.1\ ^m$ and $d = 2L = 0.3\ ^m$. The posture stabilization scenario considered here is a stabilization scenario where the initial and final configurations of the robot are defined as:

$$q_0 = (-3, -2, {}^{-\pi}/_4)',\ q_f = (0,0,0)'$$

As the next step, we use the Lyapunov-based controller devised in this paper (15) for stabilization of the differential drive robot. The gains of the controller are chosen as $k = 3$, $\gamma = 1$ and $h = 2$. Fig. 2 shows the path of the robot during the posture stabilization where black arrows show a snapshot of the motion of robot axle and Fig. 3 shows the state trajectories for the stabilization process. Fig. 4 shows the control signal used by this control to solve the stabilization problem.

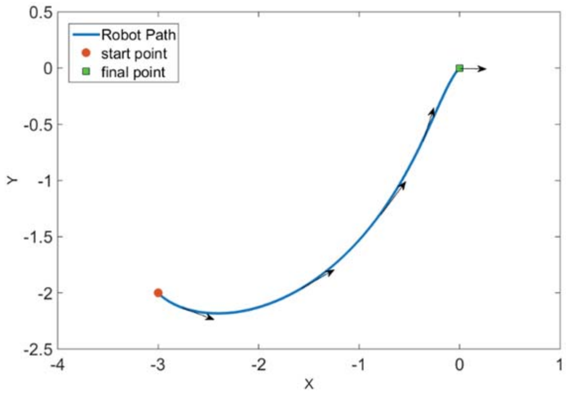

Fig. 2 Posture Stabilization Path-Lyapunov based control

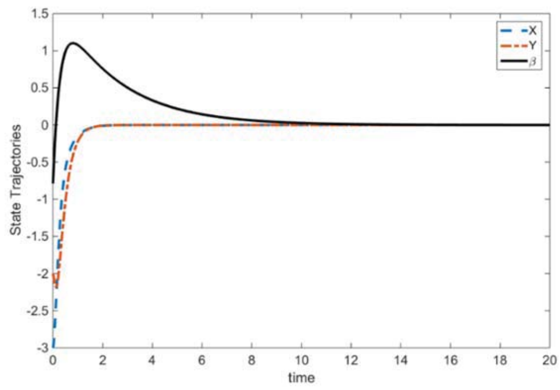

Fig. 3 State Trajectories

As Figs. 2-4 show, the controller successfully stabilizes the system. It should be taken into consideration that tuning the gains of the Lyapunov-based control is challenging. Unlike other controllers that have some level of linearity in their structure, Lyapunov-based control is a pure nonlinear controller, and the gains are not linearly affecting the transient performance of the system.

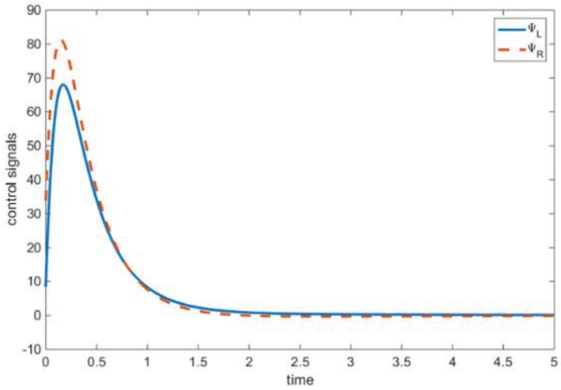

Fig. 4 Stabilizing Control Signals

To evaluate the efficiency of the proposed control, we have used a well-known control, dynamic compensator, from literature to compare their performance [23]:

$$\begin{cases} \dot{\xi} = u_1 cos\beta + u_2 sin\beta \\ v = \xi \\ \omega = {(u_2 cos\beta - u_1 sin\beta)}/{\xi}, \ \xi_0 = 0.4 \end{cases}$$

The results of the simulation are given next. Fig. 5 shows the path of the robot during the stabilization and Fig. 6 shows the time evolution of the state variables. Finally, Fig. 7 shows the control signals used by the designed controller. As the Figs. 5-7 show, the performance of our proposed control is as good as the dynamic compensator. The transient performance of Lyapunov-based control is as good as the dynamic compensator. Also, the magnitude of the control signals is within the same order of magnitude.

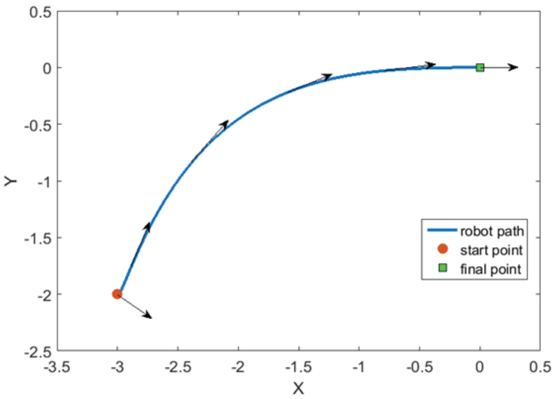

Fig. 5 Stabilization Path

To study the effect of the parameter uncertainty in the performance of the stabilizing controller, the same posture stabilization scenario is simulated where a 20% parametric uncertainty has been added to the dimensions of the robot. In other words, the estimates of the parameters of the system used by the controller are: $\hat{r} = 0.12^m, \hat{d} = 0.24^m$. For the case of dynamic compensator, the uncertainty results in residual additive terms to the linearized double integrator model. In other words, the dynamic compensator is not able to completely cancel out the nonlinear terms and some additive terms remain in the system model. For the Lyapunov based control this uncertainty acts as a parametric uncertainty in the system parameters. The 20% uncertainty is successfully compensated by both controllers.

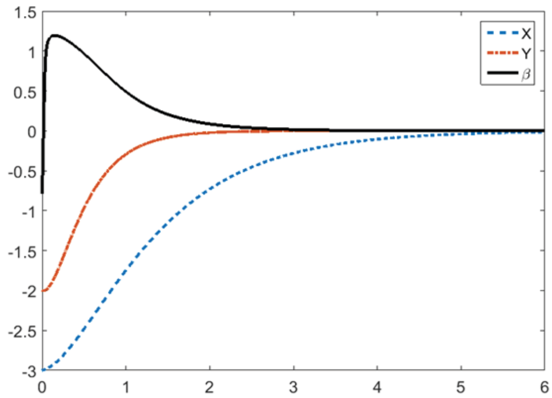

Fig. 6 State Trajectories

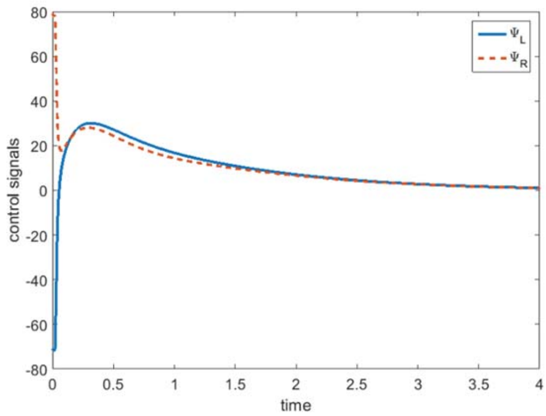

Fig. 7 Stabilizing Control Signals

In addition to parametric uncertainty, the effect of additive uncertainty is also studied on these controllers. Additive uncertainty can be realized by a violation of the rolling without slipping assumption used for modeling the kinematics of the robot. This physical phenomenon can happen at high speeds or when the traction force between the wheels and the ground are small. To simulate nonparametric additive uncertainty, a random noise is added to the kinematic model of the robot. Both controllers fail to compensate the additive uncertainty, and the unactuated degree of freedom is not stabilized as soon as the uncertainty is introduced into the system. The design of a controller that is robust to additive uncertainties is a future goal of our research.

As our future research goals optimizing the proposed posture stabilization control with respect to the energy consumption or the stabilization time can be an interesting issue. Using intelligent optimization methods such as Ant Colony optimization [30], Genetic Algorithms [31], [32] and Particle Swarm optimization [33] to search the space of stabilizing controls can be a possibility for solving the mentioned optimization problem.

VI. Conclusion

Differential drive robots are a class of mobile robots that are widely studied by many researchers. Within this paper, a new model of the kinematics of the differential drive robots under the assumption of rolling without slipping was used. It was also shown that the developed model is an under-actuated nonholonomic system just as the unicycle model. The paper also develops a posture stabilization control for the comprehensive model of the differential drive robots using the standard frameworks of Lyapunov-based control. Incorporation of the physical parameters of the system into the robot model enables studying the effect of parameter uncertainty in the behavior of the control. Robustness of the developed controls is studied via simulation. The results show that Lyapunov-based control is robust to parametric uncertainty but not additive uncertainty.

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