International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:10, No:8, 2016

A Quadratic Programming for Truck-to-Door Assignment Problem

Y. Fathi, B. Karimi, S. M. J. Mirzapour Al-e-Hashem

Abstract**—**Cross-docking includes receiving products supplied by a set of suppliers, unloading them from inbound trucks (ITs) at strip doors, consolidating and handling these products to stack doors based on their destinations, loading them into outbound trucks (OTs); then, delivering these products to customers. An effective assignment of the trucks to the doors would enhance the advantages of the cross-docking (e.g. reduction of the handling costs). This paper addresses the truck-to-door assignment problem in a cross-dock in which assignment of the ITs to the strip doors as well as assignment of the OTs to the stacks doors is determined so that total material handling cost in the cross-dock is minimized. Capacity constraints are applied for the ITs, OTs, strip doors, and stack doors. We develop a Quadratic Programming (QP) to formulate the problem. To solve it, the model is coded in LINGO software to specify the best assignment of the trucks to the doors.

Keywords**—**Cross-docking, truck-to-door assignment, supply chain, quadratic programming.

I. Introduction

CROSS-DOCKING is a logistics procedure where products from a supplier or manufacturing plant are distributed to customers or retailers through cross-dock facility with a little or no storage in-between. Cross-docking is just one strategy that can be implemented to help to achieve a competitive advantage. Implemented appropriately and in the right condition, cross-docking can provide significant improvements in efficiency and handling times. This strategy includes receiving products supplied by a set of suppliers, unloading them from ITs at strip doors, consolidating and handling these products to stack doors based on their destinations, loading them into OTs; then, delivering these products to a set of customers. The movement of shipments from the strip doors to the stack doors is done by forklift trucks. These movements have to be minimized. The core idea is to transfer the products from supply side to the OTs and deliver them to the customers with no sorting operations in-between. Shipments typically stay in the cross-dock up to 24 hours. Keeping little or no storage in the cross-dock, an effective cross-docking application explores eliminating or at least reducing the storage costs.

Cross-docking has attracted significant attention of the researchers in the past few years. These studies can be grouped according to the problem types [1]: The first group is *strategic level* which includes problems such as determining the optimal location of the cross-docks and the best layout design for the cross-docks. The second group is *tactical level* which includes problems such as design of the cross-docking networks. The last one is *operational level* which includes various problems such as dock-door assignment, truck scheduling, vehicle routing, and the location of the products in the temporary storage.

In this paper, we focus on truck-to-door assignment problem in which the ITs pick up the products supplied by one or more than one supplier; then arrive at the cross-dock. Each product can be picked up by only one IT and each origin could be assigned to no more than one strip door. Next, the assignment of the ITs to the stack doors is done. When assigned, products are unloaded and consolidation process is performed by employees. These products are transferred to the stack doors by some equipment like forklifts. Then, consolidated products are loaded into the OTs. Similar to the pickup side, each OT is allowed to serve more than one destination. Determining the assignment of the ITs to the strip doors and the OTs to the stack doors, this problem tries to find the minimum amount of the material handling cost (time or distance) incurred in the cross-dock.

To the best of our knowledge, dock-door assignment was discussed by Peck [2] for the first time. The author presented a simulation model to assign the trucks to the inbound and outbound dock doors. The model's aim is to minimize the time which is spent to transfer the goods from the ITs to the OTs. They formulated the problem as a bilinear integer program. To solve it, a greedy heuristic is proposed. Considering the inbound doors on one side and the outbound doors on the other side, Tsui and Chang [3] introduced a bilinear program to determine the assignment of the dock doors that would minimize the total material handling cost. They assumed that each inbound door is assigned to only one origin and vice versa. Also, the same assumption is considered for the outbound doors and the destinations. Then, a near optimal algorithm is used to solve the problem. Afterward, Tsui and Chang [4] developed a branch-and-bound algorithm to solve their previous problem. Bermudez [5] developed a genetic algorithm to assign the doors in order to minimize the weighted travel distances in a cross-dock.

Oh et al. [6] took a different dock-door assignment into account in a mail distribution center. In this problem, there is only one receiving door where all ITs arrive at. Hence, there is no need to assign the ITs to the receiving doors. Each shipping

Y. Fathi is with the Industrial Engineering Department, Amirkabir University of Technology (Tehran Polytechnic), 424 Hafez Avenue, 1591634311, Tehran, Iran (phone: +989188567694; e-mail: younesfathi@aut.ac.ir).

B. Karimi and S.M.J. Mirzapour Al-e-Hashem are with the Industrial Engineering Department, Amirkabir University of Technology (Tehran Polytechnic), 424 Hafez Avenue, 1591634311, Tehran, Iran (e-mail: b.karimi@aut.ac.ir, b.mirzapour@gmail.com).

door can be assigned to no more than one destination. The destinations are clustered into groups which are assigned to the shipping doors. A nonlinear mathematical model is developed with the objective of minimizing the travel distance of the pallets in the mail distribution center. To solve the problem, they proposed two heuristics. The first one is a decomposition heuristic which consists of three phases. The second solution method is based on genetic algorithm. The computational results show that both heuristics can improve the current situation by reducing the travel distance. Bozer and Carlo [7] studied trailer-to-door assignment in the cross-docks in which there are as many inbound (outbound) trailers as inbound (outbound) doors. Each trailer is assigned to the exactly one door and vice versa. No limitation is imposed by the model itself on the number of loads per inbound trailer. Their objective is to minimize the sum of the flows × rectilinear distances. To solve the problem, a simulated annealing algorithm is presented. Zhu et al. [8] introduced a new model for the Cross-Dock Door Assignment Problem (CDAP). Their model is an extension of the model proposed in [3]. In this new model, it is possible to assign multiple origins (destinations) to a single inbound (outbound) door, as long as its capacity could accommodate them. Additionally, there is a certain capacity for each in(out)bound door that must not be exceeded. They tested their method with only one instance including 8 origins, 8 destinations, 4 strip doors, and 4 stack doors, and different door capacities. Considering weight restriction for each trailer, Cohen and Keren [9] suggested a new optimal nonlinear mixed integer formulation and a new heuristic approach to assign the cross-dock doors to the trailers. On delivery side, more than one trailer could serve the same destination but, on the pickup side each incoming trailer is assigned to one and only one receiving door. They assign capacity to the load doors while unloading doors have no capacity. They state that their heuristic compared with other heuristic assignments shows average savings of about 10%. Guignard et al. [10] proposed two heuristics for the problem considered in [8]. The first one is a multi-start local search and the second one employs a Convex Hull Relaxation (CHR) to linearize the quadratic CDAP function. Then, they compared the result obtained by the heuristics with the optimal solution. For small instances the branch and bound search is as efficient or possibly more efficient than any of the heuristics but, as the size of the problem gets larger, the heuristics outperform the branch and bound search. Nassief et al. [11] studied CDAP with the objective of minimizing the total material handling cost. They provided a linear mixed integer programming formulation and solve it by proposing a Lagrangian Relaxation (LR) method. The MIP formulation obtained solutions of five benchmark instances for which the optimal solution was not known before. The LR method shows high quality of performance on the instances up to 50 origins (destinations) and 30 strip (stack) doors.

II. Problem Description

Following assumptions are introduced to develop the model.

1) Cross-dock has two sides (I-shape). Strip doors on one side and stack doors one the other side.
2) The problem is multi-product and each origin supplies one type of product.
3) ITs are allowed to visit more than one origin and OTs are allowed to serve more than one destination.
4) Each origin (destination) can be assigned to no more than one strip (stack) door.
5) Strip and stack doors have capacity and so do ITs and OTs.
6) The number of ITs (OTs) are not forced to be equal to the number of strip (stack) doors.

Mathematical model

Sets:

- M: Set of origins
- N: Set of destinations
- I: Set of strip doors
- J: Set of stack doors
- P: Set of ITs
- Q: Set of OTs

Parameters

- s_m: The amount of product supplied by origin m
- w_{mn}: The amount of product m destined for n
- a_i: Capacity of strip door i
- b_j: Capacity of strip door j
- CI_p: Capacity of IT p
- CO_q: Capacity of OT q
- d_{ij}: Length of path between strip door i and stack door j
- r_n: Demand of destination n

Decision Variables

- x_{mpi}: Binary variable for assignment of product m to strip door i which is carried by IT p
- y_{nqj}: Binary variable for assignment of destination n to stack door j which is carried by OT q

The following nonlinear mathematical model is developed:

$$MIN \sum_m \sum_p \sum_i \sum_n \sum_q \sum_j x_{mpi} * w_{mn} * d_{ij} * y_{nqj} \quad (1)$$

Subject to

$$\sum_m \sum_p s_m * x_{mpi} \le a_i \quad \forall i \in I \quad (2)$$

$$\sum_n \sum_q r_n * y_{nqj} \le b_j \quad \forall j \in J \quad (3)$$

$$\sum_p \sum_i x_{mpi} = 1 \quad \forall m \in M \quad (4)$$

$$\sum_q \sum_j y_{nqj} = 1 \quad \forall n \in N \quad (5)$$

$$\sum_m \sum_i s_m * x_{mpi} \le CI_p \quad \forall p \in P \quad (6)$$

$$\sum_n \sum_j r_n * y_{nqj} \le CO_q \quad \forall q \in Q \quad (7)$$

$$x_{mpi} = 0 \; or \; 1 \quad \forall m \in M \quad \forall p \in P \quad \forall i \in I \quad (8)$$

$$y_{nqj} = 0 \; or \; 1 \quad \forall n \in N \quad \forall q \in Q \quad \forall j \in J \tag{9}$$

The objective function (1) minimizes the total weighted distance in the cross-dock. Constraints (2) and (3) ensure that the capacity of the strip and stack doors, respectively, is not exceeded. Constraint (4) makes sure that each origin is assigned to only one strip door and one IT. Constraint (5) guarantees that each destination is assigned to only one stack door and one OT. Constraints (6) and (7) ensure that the capacity of the strip and stack doors, respectively, is not exceeded. Constraints (8) and (9) restrict the decision variables to zero or one.

III. Contribution and Main Advantages of The Paper

This paper presents a model for truck-to-door assignment problem in which a fleet of vehicles (trucks, trailers and so on) pick up and deliver the products. Almost all the papers that deal with CDAP ignore the vehicles. They treat origins and destinations like vehicles and assume that each vehicle carrying product supplied by only one origin arrives at the cross-dock. On the other side, each vehicle that leaves the cross-dock carries demand of only one destination [1], [2], [7], [8], [10], and [11]. These assumptions force the number of ITs (OTs) to be equal to the number of origins (destinations) which is not very common in the real world. In the example presented in [8], origin1 supplies 65,572 goods while origin8 supplies 2,229 goods. Assume there are trucks with capacity of 50,000 goods. These trucks are small for origin1 and too big for origin8. In our model, there are two sets of trucks each on one side of the cross-dock. Each IT is allowed to pick up the products from several origins and each OT can deliver the demand of several destinations. In the CDAP as stated in [1], time aspects are not taken into account. Hence, we follow this rule in our proposed model.

IV. Computational Experiment

In this section, we appraise the proposed programming model to see how good the solution quality is and how long the running time of the problems lasts. We randomly generate some numerical instances which the proposed model is tested with. Using LINGO software version 16, instances are solved on a Laptop with a AMD A6 and 2.70 GHz CPU processor and 4.00 GB memory. Input data which are required to the proposed programming model is presented in Table I. In this table, c is a constant and is added to the capacity of the trucks and doors to make the problem feasible if it is infeasible. As the cross-dock is I-shaped, the distance between strip door i and stack door j is $3+|i-j|$.

The optimal solution and CPU time for each instance is reported in Table II. In this table, LINGO software is capable of solving the problem. It can be seen that as the size of the instances increases, the running time to obtain the optimal solution increases considerably. As a result, obtaining the optimal solution would be tough for large size of the problem. Hence, a heuristic method could help to find the near-optimal solutions.

TABLE I
INPUT DATA FOR THE MODEL

Inputs	Value
s	U[100,300]
r	U[50,200]
w	U[50,150]
CI	$\left\lceil \frac{\sum_m s_m}{P} \right\rceil + c$
CO	$\left\lceil \frac{\sum_n r_n}{Q} \right\rceil + c$
a	$\left\lceil \frac{\sum_m s_m}{I} \right\rceil + c$
b	$\left\lceil \frac{\sum_n r_n}{J} \right\rceil + c$
d	$3+|i-j|$

P= Number of Its, Q=Number of OTs, I=Number of strip doors, J=number of stack doors

TABLE II
EXPERIMENTAL RESULTS FOR A SET OF 20 INSTANCES

	Number of						LINGO Software	
Instance	Origins	ITs	Strip doors	Destinations	OTs	Stack doors	Optimal Solution	Time (s)
1	1	2	2	2	2	2	600	0.01
2	2	2	2	2	2	2	1130	0.14
3	3	2	2	3	2	2	1740	0.28
4	4	2	2	4	2	2	2370	0.26
5	4	2	3	4	2	3	2780	12.5
6	4	3	3	4	3	3	2780	116
7	5	2	2	5	2	2	3565	4.61
8	5	3	2	5	3	2	3565	78.63
9	5	3	3	5	3	3	3815	778
10	6	2	3	5	2	3	4120	173.15
11	6	3	2	5	3	2	3815	95.81
12	6	3	3	5	3	3	4120	795
13	6	2	2	6	2	2	3650	6.09
14	6	3	2	6	3	2	3650	100
15	7	2	2	7	2	2	4790	26.42
16	7	3	2	7	3	2	4790	676
17	7	2	3	7	2	3	4885	358
18	7	2	4	7	2	4	5040	4226
19	8	2	2	8	2	2	5330	93.7
20	8	2	3	8	2	3	5470	1544

V. Conclusion and Future Work

This paper considers a Truck-to-Door Assignment Problem in a cross-dock center in which assignment of the trucks to the doors is determined to minimize the total material handling cost. In this problem, each origin supplies one product and each destination can have a known demand for each product (possibly for all the products). A fleet of trucks is available to pick up the products supplied by a set of suppliers and a fleet of trucks is available to deliver the products requested by a set of customers. To formulate the problem, we develop a QP in which there are two sets of the variables. The first set is related to the supply side and the second one is related to the demand side. To show that the developed model is applicable, we test it with a set of 20 instances which randomly generated and are small sizes of the problem. For future work, some algorithms can be proposed to solve the large scale of the problem effectively.

REFERENCES

[1] Van Belle, J., Valckenaers, P., & Cattrysse, D. (2012). Cross-docking: State of the art. *Omega*, *40*(6), 827-846.

[2] Peck, K. E. (1983). *Operational analysis of freight terminals handling less than container load shipments* (No. 83-09998 UMI).

[3] Tsui, L. Y., & Chang, C. H. (1990). A microcomputer based decision support tool for assigning dock doors in freight yards. *Computers & Industrial Engineering*, *19*(1), 309-312.

[4] Tsui, L. Y., & Chang, C. H. (1992). An optimal solution to a dock door assignment problem. *Computers & Industrial Engineering*, *23*(1-4), 283-286.

[5] Bermudez, R., & Cole, M. H. (2000). A genetic algorithm approach to LTL breakbulk terminal door assignment. In *Proceedings of the 2000 industrial engineering research conference*.

[6] Oh, Y., Hwang, H., Cha, C. N., & Lee, S. (2006). A dock-door assignment problem for the Korean mail distribution center. *Computers & Industrial Engineering*, *51*(2), 288-296.

[7] Bozer, Y. A., & Carlo, H. J. (2008). Optimizing inbound and outbound door assignments in less-than-truckload cross docks. *IIE Transactions*, *40*(11), 1007-1018.

[8] Zhu, Y. R., Hahn, P. M., Liu, Y., & Guignard, M. (2009, September). New approach for the cross-dock door assignment problem. In *in Proceedings of the XLI Brazilian Symposium on Operations Research*.

[9] Cohen, Y., & Keren, B. (2009). Trailer to door assignment in a synchronous cross-dock operation. *International Journal of Logistics Systems and Management*, *5*(5), 574-590.

[10] Guignard, M., Hahn, P. M., Pessoa, A. A., & da Silva, D. C. (2012). Algorithms for the cross-dock door assignment problem. In *Proceedings of the Fourth International Workshop on Model-Based Metaheuristics*.

[11] Nassief, W., Contreras, I., & As' ad, R. (2016). A mixed-integer programming formulation and Lagrangean relaxation for the cross-dock door assignment problem. *International Journal of Production Research*, *54*(2), 494-508.