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Characterizing Multivariate Thresholds in Industrial Engineering

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***Abstract*—**This paper highlights some of the normative issues that might result by setting independent thresholds in risk analyses and particularly with safety regions. A second objective is to explain how such regions can be specified appropriately in a meaningful way. We start with a review of the importance of setting deterministic trade-offs among target requirements. We then show how to determine safety regions for risk analysis appropriately using utility functions.

***Keywords*—**Decision analysis, thresholds, risk, reliability.

I. Introduction

DECISION analysis is a rigorous normative method for ranking, valuing, and selecting the best decision alternative cased on the expected utility criterion (for references on the theory of decision analysis see [1]-[3]; for a reference on multiattribute utility theory, see [4], and for applications of decision analysis, see [5]).

Decision analysis has been applied in numerous areas including industrial engineering design and industrial risk and reliability analysis. Often deviations of expected utility occur in organizations for simplicity or for tactical and operational decisions. Some of the most common deviations result from target-based incentives, design by threshold requirements, and setting threshold safety regions in risk analysis. The flow down of requirements in engineering design is a common approach to centralizing design decisions but, if conducted incorrectly, it might also pose issues with the overall design. See for example [6]-[8] for application of decision analysis to milling profit maximization that uses value instead of thresholds; and [9] for a references that includes common errors in thresholds; and [10]-[12] for normative methods for setting thresholds.

The purpose of this paper is to draw attention to the implications of setting arbitrary multivariate threshold requirements in industrial engineering applications and to provide some perspective on normative methods for setting thresholds. In particular, we shall discuss deterministic thresholds in a design decision, as well as thresholds set independently for safety regions in risk analysis.

II. Multivariate Deterministic Thresholds in Engineering Design

To start, consider the following example from [12] with two direct value attributes of a product, say X and Y, and where more of each attribute is preferred to less over the domain of the attributes. The level sets (isopreference contours) corresponding to this preference must have a non-positive slope as shown in Fig. 1.

If requirement thresholds are set independently for each attribute, such that each attribute must exceed a threshold value, then the acceptable designs are limited to those in the rectangular (shaded) region. Now consider two design configurations, A and B, superimposed on the contours of constant value. Design B lies within this region (and therefore meets the target requirements that have been set) but it has a lower value than design A, which is outside this region and yet lies on a higher value contour. Thus, the design that would be induced by this requirement has a lower value than one that would be rejected by the requirement. This difference in choice comprises a value gap a loss of value to the design enterprise. It is plausible that these requirements are physical requirements, but quite often requirements are set arbitrarily. To avoid this type of behavior, a simple axiom must be introduced.

Axiom 1: *There shall be no design outside the requirements region that is preferred to a design within the requirements region.*

Axiom 1 necessitates assigning trade-offs to design requirements and aligns the value function approach for engineering design with the requirements approach. The following proposition results from Axiom 1.

Proposition 1: *If* Axiom 1 *is satisfied by the requirements for all possible designs, then the requirements must define a space that is bounded by a contour of constant value as determined by the corporate value function.*

Proposition 1 illustrates the importance of constructing a value function and assessing representative trade-offs among the design requirements. The proposition also implies that threshold requirements cannot be set independently for each attribute, and must be associated with the trade-offs amongst them.

III. Multivariate Safety Thresholds in Risk Analysis

Now consider, the common risk-scenario plot that is widely used in risk analysis and that incorporates two axes: severity of a consequence and the probability of loss (or probability of success of an attack) as shown in Fig. 2. The figure shows four rectangular regions: the bottom left region is commonly colored green and referred to as the safe region. The top right region is commonly colored red and represents a danger zone, and the remaining two regions are usually colored orange and represent intermediate risk regions.

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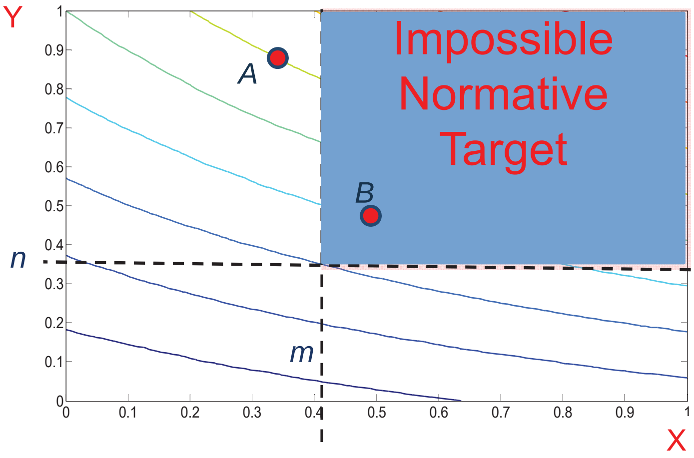

Fig. 1 Alternative *A* has a higher value than *B*, but the target rewards *B* and penalizes *A*

A. What is the Problem with This Formulation?

There are several issues with this representation. First of all, it is important to observe that an attack might have multiple possible consequences. For example, an attack might involve multiple levels of severity with different probabilities. Representing an attack scenario by only one point in this graph is therefore misleading. This implies that an attack should be a cloud of points on this space and not just one point representing only one possible loss scenario with a given probability.

Second, even if a successful attack represented only one loss point, the contours represented by this chart cannot be meaningful. The chart implies that we are indifferent between two scenarios having the same probability of success but one has a larger damage. To illustrate, Fig. 2 represents three possible attack scenarios, *A*, *B*, and *C*. The chart shows indifference between prospects *B* and *C*, as they both lie in the Red region. It is not possible to consider those scenarios equally because, although they have the same probability, one has a much higher severity for the consequence. Therefore, *C*, would be a much worse scenario than B but they lie in the same region and have the same risk code. Using rectangular regions is misleading, and rectangular color codes to represent risk is inappropriate and misleading. Furthermore, it is difficult to provide a trade-off between probability of success and severity of a consequence without calculations using tools such as utility values. To illustrate, we will now show how to calculate indifference contours and demonstrate that the indifference regions defined by rectangular contours in Fig. 2 can never occur with reasonable utility functions.

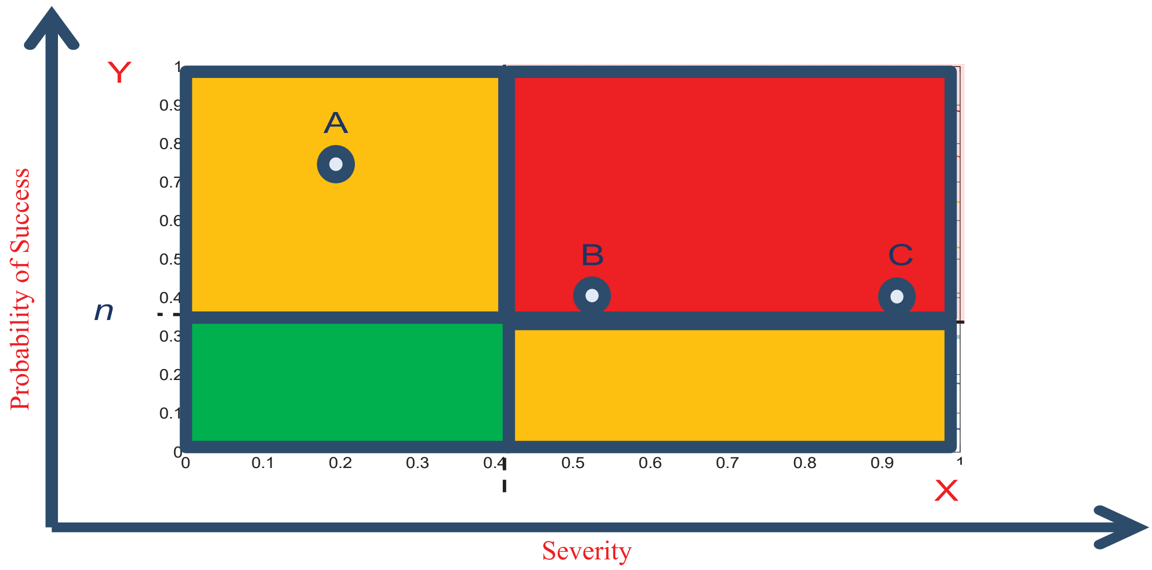

Fig. 2 Safety regions set by independent threshold levels of likelihood and severity

Using expected utility theory, we can plot the relation between probability of success and severity. For additional reading on utility functions for individuals and groups, see [13]-[15], and for probability encoding, see [16]-[19].

To start, consider a scenario where there is 90% chance of a loss (or damage) of severity worth $1 Million and a 10% chance that the scenario would not occur resulting in $0 loss. Now suppose we wish to identify the severity of the loss, l, with probability 80% that would make us indifferent to this $1 Million loss scenario. Using the principle of "gain some - lose some", if we are indifferent, and if the new loss has a lower probability of occurrence, then we should expect that the loss value l would be more than $1 Million. Fig. 2 shows this scenario. Indifference between the two scenarios implies that the expected utility of each deal must be equal. Therefore, we need the utility values of the prospects involved.

Suppose that a decision maker has an exponential utility function with risk aversion coefficient γ equal to 10^{-6}. For example, the utility function has the form $U(x) = 1 - e^{-10^{-6}x}$.

The expected utility of the deal on the left-hand side of Fig. 3 is:

$$0.9\left(1-e^{10^{-6}.10^{6}}\right)+0.1\left(1-e^{-10^{-6}.0}\right)=0.9\left(1-e\right).$$

The expected utility of the second lottery is

$$0.8\left(1-e^{-10^{-6}.l}\right)+0.2\left(1-e^{-10^{-6}.0}\right)=0.8\left(1-e^{-10^{-6}.l}\right).$$

Equating the expected utilities of both deals gives

$$0.9\left(1-e\right)=0.8\left(1-e^{-10^{-6}.l}\right)\Rightarrow$$
$$l=\frac{-1}{10^{-6}}\ln\left(1-\frac{9}{8}\left(1-e\right)\right)=-1.076 \text{ Million}$$

Indeed, the loss amount is more than $1 Million as expected.

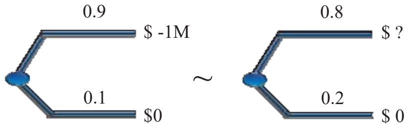

Fig. 3 Indifference between two deals having different probabilities of loss

We can repeat the same analysis for different probabilities of loss. Fig. 4 shows the deals for a loss probability of 0.7. Using similar analysis as above shows that the indifference loss amount for a probability of 0.7 is

$$l=\frac{-1}{10^{-6}}\ln\left(1-\frac{9}{7}\left(1-e\right)\right)=-1.166 \text{ Million}.$$

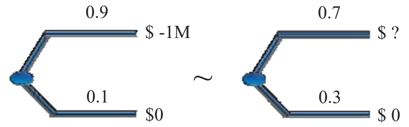

Fig. 4 Indifference between two deals having different probabilities of loss

By repeating the same steps for various probabilities of loss, we can get the indifference contour for all values of probability of loss vs. loss. Fig. 5 plots the indifference contour for losses and probabilities of loss that are equivalent to a 90% chance of losing $1 million. It is clear that the indifference region is not rectangular.

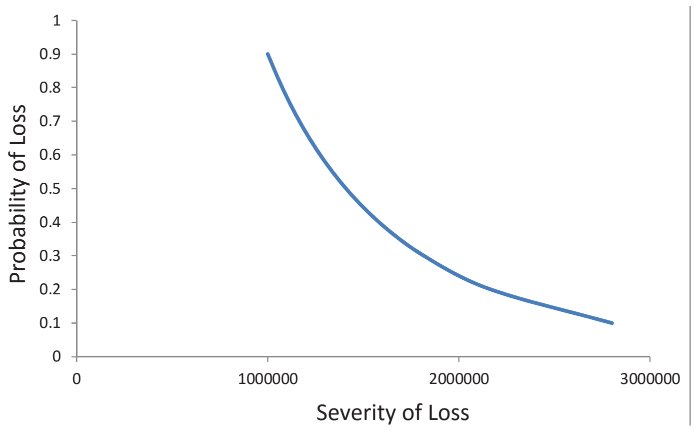

Fig. 5 Trade-offs between probability of loss and severity of loss

In principle, we can also repeat these steps starting with different probabilities of loss and determine the corresponding iso-preference contours. Fig. 6 shows the analysis conducted starting with probabilities of loss of $1,000,000 corresponding to 0.1, 0.3, 0.5, 0.7, and 0.9 for comparison. As we can see these trade-offs were calculated by using utility values and the shapes of these contours would vary based on the risk aversion coefficient that is used.

Another problem with the rectangular representation of Fig. 2 is why there is a cut-off threshold region in the first place. If we are given a loss probability and a consequence with severity, such as *B* or *C*, then a small reduction in loss probability will result in a change of the risk situation from Red to Orange. This sudden change in state of the risk results in a reduction in readiness and alert. It is best to think of readiness and preparations as a decision about allocation of resources using full probability distributions instead of cut-off threshold values.

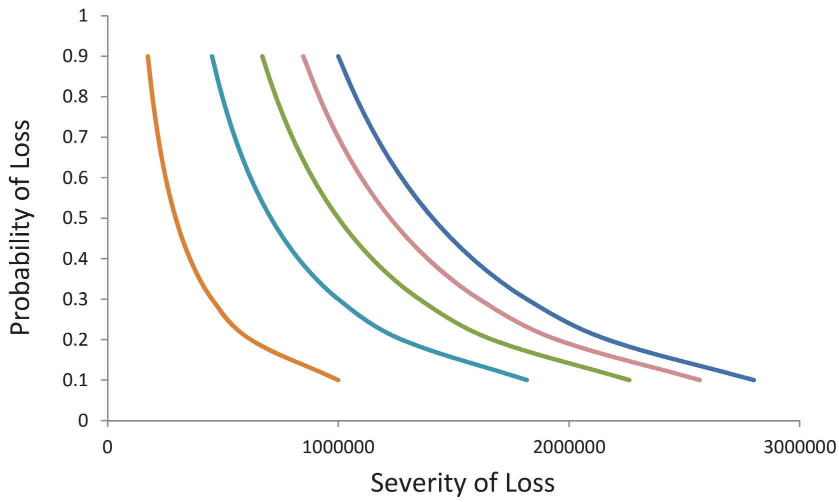

Fig. 6 Trade-off contours between probability of loss and severity of loss

IV. Conclusion

Multivariate thresholds are widely used in industrial engineering applications. Independent thresholds should not be set alone but specified with trade-offs among the threshold values using isopreference contours. In deterministic settings, this requires a deterministic value function. In probabilistic settings, this requires a utility function. Setting independent thresholds over each variable, or attribute of interest, can result in loss of value.

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