

# Control of Underactuated Biped Robots Using Event Based Fuzzy Partial Feedback Linearization

Omid Heydarnia, Akbar Allahverdizadeh, Behnam Dadashzadeh, M. R. Sayyed Noorani

**Abstract**—Underactuated biped robots control is one of the interesting topics in robotics. The main difficulties are its highly nonlinear dynamics, open-loop instability, and discrete event at the end of the gait. One of the methods to control underactuated systems is the partial feedback linearization, but it is not robust against uncertainties and disturbances that restrict its performance to control biped walking and running. In this paper, fuzzy partial feedback linearization is presented to overcome its drawback. Numerical simulations verify the effectiveness of the proposed method to generate stable and robust biped walking and running gaits.

**Keywords**—Underactuated system, biped robot, fuzzy control, partial feedback linearization.

## I. INTRODUCTION

RESEARCHERS are still interested in biped robots because of their friendly and human-like appearance and their capability to move in uneven environments. Through three decades of research on biped robots, effective control methods are sought to improve stability and robustness of biped walking like human or animals. Underactuated biped robots walking and running have been noticed by many researchers [1], [2].

Systems that have fewer number of actuators than the degree of freedom (DOF) are defined as underactuated systems. Control of these systems is more challenging than fully actuated systems and it is an open research problem. Second order sliding mode controller [3], sliding mode tracking control [4], and optimal sliding mode [5] were designed for the underactuated systems. The hierarchical sliding mode is designed to overcome uncertainty and disturbances of a class of the underactuated systems [8]. Partial feedback linearization was also proposed by Spong [6], [7] as a base to control the underactuated systems. A collocated form of partial feedback linearization was applied to a flexible link, and asymptotic stability of its zero dynamic was proved [9].

Biped robots can be divided into three classes: 1- passive bipeds, 2- underactuated bipeds and 3- fully actuated bipeds. There are no actuators in passive robots, and they use gravity force to continue walking. Engineers usually use springs to improve performance of these robots [10], [11]. A robot with

point feet provides an example of a common underactuated bipeds, since there is no actuation on ankle. While flat feet biped robots are often controlled by zero moment point (ZMP) stability criterion showing unnatural and slow walking and running gaits [12], underactuated robots demonstrate more natural dynamics and faster gaits [2].

Tzafestas et al. [13] showed that for a 5-link biped robot sliding mode controller is more robust than feedback linearization. A robust tracking control algorithm was presented for underactuated biped robots, making them self-balance in presence of disturbances. Then, stability criteria were derived based on linearization of the one nonlinear equation [14]. An adaptive controller was developed for underactuated biped locomotion in which recursive least square error was used for parameter estimation [15]. Stability analysis of compass gait walking has been investigated with the partial feedback linearization in [1], [16], and the convergence of its state vector to a reference limit cycle for both feedback linearization and partial feedback linearization was investigated. Poincaré map is one of the best tools for analysing the stability of periodic orbits of dynamic systems. It converts the hybrid dynamic model of biped walking or running to a discrete map. The fixed point of the map corresponds to periodic walking or running gait of the robot. The method of Poincaré has been used to study the stability of underactuated motion in several studies [17], [18]. Poincaré map has been used to study the stability of both passive and underactuated bipeds [18], [19].

Initiating by Zadeh's pioneering work [25], fuzzy logic has been utilized in control engineering for four decades. As a consequence, an Adaptive Network Based Fuzzy Interface System (ANFIS) control strategy has been proposed based on a hierarchy of walking gait planning and joint control level which do not require detailed kinematics and dynamics biped models [20]. Fuzzy logic has been used to eliminate chattering phenomena in classic sliding mode and was applied to a biped robot [21].

In this paper, a fuzzy partial feedback linearization controller is presented for biped models to enhance controller performance and improve its disturbance rejection. This controller is applied to a compass gait biped and a 5-link biped robot. We use two different gait generation methods to investigate the effect of gait generation method on controller performance. The rest of this paper is organized as follows. Section II describes dynamic modelling of the considered biped robots, including dynamic model of stance phase and touch-down. Section III describes reference trajectories generation for walking. Section IV discusses controller design

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using fuzzy logic and its implementation on biped models. Section V provides simulation results of our control strategy for walking on flat ground, for both compass gait and 5-link biped robot. Finally, Section VI is the conclusion that discusses about effect of fuzzy logic on enhancement of partial feedback linearization in walking of biped robots.

## II. DYNAMIC MODELLING

At this paper, we study dynamic model of two biped robots in sagittal plane. Walking gait includes two phases, single support phase (SSP) and double support phase (DSP). Here, it is assumed that transition from SSP to DSP occurs instantaneously. So, dynamic model of this type of biped walking gait is divided in two parts as following:

- Stance phase
- Touch-down (collision of leg with ground)

Walking dynamic model of an  $n$ -DOF biped robot with point feet can be expressed as follows. Let  $q$  be a vector of generalized coordinates in  $n$ -dimensional configuration space  $Q$ , and  $u$  be a vector of forces and torques, which is  $(n-1)$ -dimensional.

$$\begin{aligned} M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) &= B(q)u \quad \text{for } q \notin \mathfrak{S} \\ \left. \begin{aligned} q^+ &= \Delta_q q^- \\ \dot{q}^+ &= \Delta_{\dot{q}}(q^-)\dot{q}^- \end{aligned} \right\} & \text{whenever } q^- \in \mathfrak{S} \end{aligned} \quad (1)$$

where  $M(q)$  is inertia matrix,  $C(q, \dot{q})$  is matrix of Coriolis and centrifugal terms,  $G(q)$  is the gradient of the potential energy field, and  $B(q)$  describes the effects of actuators on the generalized coordinates. The set of  $\mathfrak{S}$  represents switching surface which is chosen to be:

$$\mathfrak{S} = \{q \in Q, \dot{q} \in \dot{Q} \mid P_e^v = 0, P_e^H > 0\} \quad (2)$$

where  $P_e^v$  and  $P_e^H$  denote the vertical and horizontal position of the end of the swing leg, respectively.

Several assumptions are considered for touch-down phase [22]. To find angular velocities after collision, Lagrange Impact equation and two additional equations coming from no slip and rebound constraint has been used. So, we have:

$$\Delta_{q_e}(q_e^-) = \begin{bmatrix} I_{1 \times N} & -D_a^{-1} J_a^T \cdot (J_a \cdot D_a^{-1} \cdot J_a^T)^{-1} J_a \end{bmatrix} \quad (3)$$

where  $D_a$  is the inertia matrix of biped robot when both legs in the air and we need to add the Cartesian coordinates of the robot body.  $(x_p, y_p)$  is the robot position which can be located at the end of swing leg, center of mass, or hip. Also,

$$J_a = \frac{\partial p_e}{\partial q_e} \quad \text{and} \quad q_e = [q, x_p, y_p]^T$$

$$\dot{q}_e^+ = \Delta_{\dot{q}_e}(q_e^-)\dot{q}_e^- \quad (4)$$

The vector of  $\dot{q}^+$  in (1) is the first  $n$  element of  $\dot{q}_e^+$  in (4).

More details about dynamic modelling of biped robots can be found in Refs. [13], [23].

### A. Compass Gait Biped Robot

This robot has two DOF and there is only one actuator on the hip which makes this robot an underactuated system. Dynamic model of compass gait can be derived using Lagrange method and is available in literature [23], [24].

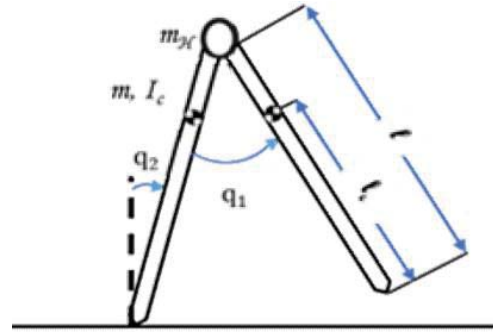


Fig. 1 Schematic of compass gait

We donate  $q = [q_1, q_2]^T$  as configuration vector of the robot so  $M(q) \in R^{2 \times 2}$ ,  $C(q, \dot{q}) \in R^{2 \times 2}$ ,  $G(q) \in R^{2 \times 1}$ ,  $B = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$ .

### B. 5-Link Biped Robot

One of the famous anthropomorphic biped models is 5-link biped robot. Various prototypes have been made for this robot like RABBIT and MABEL [23]. Fig. 2 depicts a schematic view of this robot.

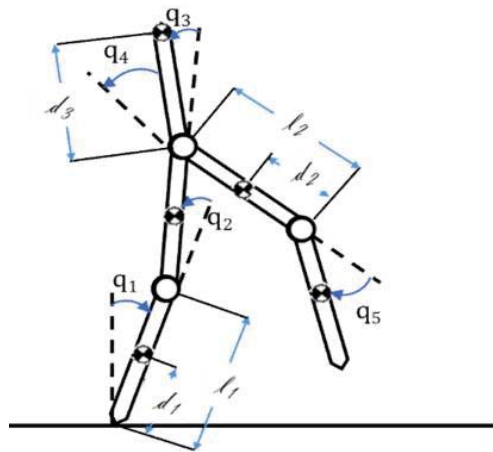


Fig. 2 Schematic of 5-link biped robot

Vector of generalized coordinate is considered as  $q = [q_1 \ q_2 \ q_3 \ q_4 \ q_5]^T$  and dimension of matrixes in (1) are as  $M(q) \in R^{5 \times 5}$ ,  $C(q, \dot{q}) \in R^{5 \times 5}$ ,  $G(q) \in R^{5 \times 1}$ ,  $B = \begin{bmatrix} 0_{1 \times 4} \\ I_{4 \times 4} \end{bmatrix}$ .

### III. REFERENCE TRAJECTORY OF WALKING

In order to find reference trajectory of walking, we used two convenient methods: 1- Poincaré Map [2], 2- intellectual trajectory [13]. Here, we consider both of these methods to understand effects of reference trajectories on controller abilities. We used Poincaré map to find the reference trajectory of compass gait and intellectual method to find the reference trajectory of 5-link biped robot.

#### A. Active Poincaré Map

Poincaré map is a powerful mathematical tool to transform problem of finding periodic orbits into finding fixed points of a discrete map. Fixed point can be considered as equilibrium point of a specific discrete nonlinear system. Active Poincaré map can be used to find the reference trajectories of walking that consists of finding an initial condition and control commands which enables the robot to walk periodically.

$$x(k+1) = P(x(k), u(k)) \quad (5)$$

We use active Poincaré map to find reference trajectories of compass gait biped robot. Poincaré section is selected as start of stance phase, and a nonlinear optimization method is used to find the root or minimum of (6).

$$Er = x(k+1) - x(k) \quad (6)$$

Optimization parameters includes initial condition of stance phase and five points which determine motor torques during this phase. We apply these initial condition and torque to robot and save the reference trajectory of robot. The desired reference trajectory of walking is demonstrated in Fig. 3.

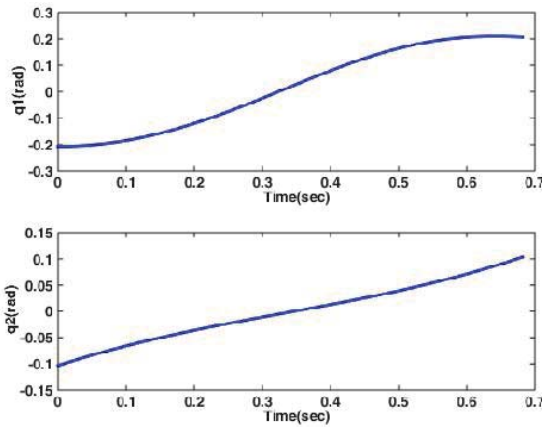


Fig. 3 Reference trajectories of compass gait

#### B. Intellectual Method

We name “intellectual trajectory” any desired reference trajectory of walking which is proposed by an expert person without using analytical tools like inverse kinematics, splines, and so on. We use this method similar to Tzafestas et al. [13] to find joints reference trajectories for the 5-link biped robot. These trajectories are designed so that torso remains vertical,

and the angular momentum of biped about support point increases by gravitational forces. Fig. 4 indicates the reference trajectories which are designed by intellectual method [13].

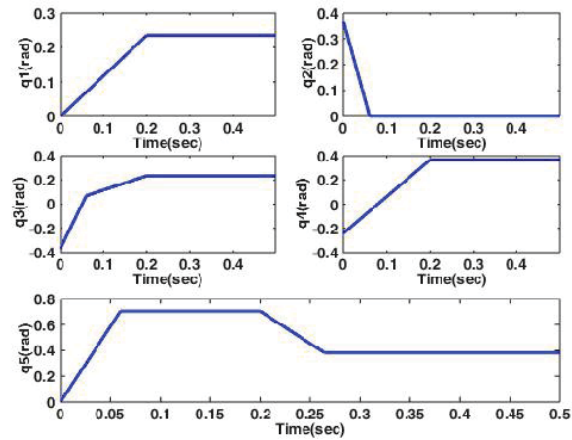


Fig. 4 Reference trajectories of 5 link biped robot

### IV. FUZZY PARTIAL FEEDBACK LINEARIZATION

Linearization of underactuated systems is impossible with exact feedback linearization. So, partial feedback linearization was proposed to linearize the controllable part of underactuated systems so that it makes the remaining zero dynamic stable. Partial feedback linearization is divided into three categories: 1- Collocated form, 2- Non-collocated form and 3- Task space form.

In the collocated form, control signal linearizes dynamics of actuated degrees of freedom. Non-collocated form refers to linearizing dynamics of passive degrees of freedom, and in task space form, a combination of some active and passive degrees of freedom are linearized and controlled. Here, we used fuzzy logic to enhance capabilities of collocated partial feedback linearization abilities.

General form of underactuated systems can be written as

$$\begin{cases} M_{aa}\ddot{q}_a + M_{a0}\ddot{q}_0 + H_a(q, \dot{q}) + G_a(q) = \tau \\ M_{a0}^T \ddot{q}_a + M_{00}\ddot{q}_0 + H_0(q, \dot{q}) + G_0(q) = 0 \end{cases} \quad (7)$$

where  $q_a$  and  $q_0$  represent actuated and unactuated variables, respectively. Note that because  $M$  is uniformly positive definite,  $M_{aa}$  and  $M_{00}$  are also positive definite and hence invertible.

$$\ddot{q}_0 = -M_{00}^{-1} (M_{a0}^T \ddot{q}_a + H_0 + G_0) \quad (8)$$

By substituting (8) into first term of (7), the dynamic equation of the system can be expressed as

$$N\ddot{q}_a = \tau - D \quad (9)$$

where

$$N = M_{aa} - M_{a0} M_{00}^{-1} M_{a0}^T$$

$$D = H_a(q, \dot{q}) + G_a(q) - M_{a0} M_{00}^{-1} (H_0 + G_0).$$

A feedback linearization control law can linearize the actuated subsystem. Therefore, it can be defined for (9) according to

$$\tau = N v + D \quad (10)$$

where  $v$  is an additional outer loop control input. Similar to Ref. [7],  $v$  can be defined as:

$$v = \ddot{q}_a^d - K_d(\dot{q}_a - \dot{q}_a^d) - K_p(q_a - q_a^d) \quad (11)$$

with  $K_d, K_p > 0$ .

The complete dynamic model of the system can be written as

$$\ddot{q}_a = v$$

$$M_{00} \ddot{q}_0 + H_0 + G_0 = -M_{a0}^T v \quad (12)$$

Similar to [6], we define new variables as

$$z_1 = q_a - q_a^d, \quad \dot{z}_1 = z_2 = \dot{q}_a - \dot{q}_a^d$$

$$\eta_1 = q_0, \quad \dot{\eta}_2 = \dot{\eta}_1 = \dot{q}_0 \quad (13)$$

So, we have

$$\dot{z} = A z$$

$$\dot{\eta} = \omega(\eta, z, t) \quad (14)$$

Matrix  $A$  is Hurwitz because we defined  $K_d, K_p > 0$ . So, there is a zero dynamics  $\dot{\eta} = \omega(\eta, 0, t)$  in our problem. Stability of this zero dynamics is discussed in Ref. [6].

Because of highly nonlinear dynamics and presence of a discrete event at the end of the gait, control of a point foot biped robot is more difficult than other underactuated systems. So choosing control parameters properly is very important. As depicted in Fig. 5, we use a supervisory fuzzy controller to adjust the partial feedback linearization controller parameters.

The supervisory controller finds  $\lambda$  which will be used in partial feedback linearization as:

$$k_d = 2\lambda$$

$$k_p = \lambda^2 \quad (15)$$

The fuzzy controller will be updated at the beginning of SSP which has less computational effort. The membership functions of input variables  $e, \dot{e}$  and the membership functions of output linguistic variable  $\lambda$  are shown in Fig. 6 (a) and Fig. 6 (b), respectively.

The input variables of fuzzy controller are defined as weighted combination of actuated and non-actuated states

$$e = \|q_a - q_a^d\|_2 + \alpha \|q_0 - q_0^d\|_2$$

$$\dot{e} = \|\dot{q}_a - \dot{q}_a^d\|_2 + \beta \|\dot{q}_0 - \dot{q}_0^d\|_2 \quad (16)$$

where  $\alpha, \beta$  are positive constants. The fuzzy rule base is designed as Table I, and a general form is used to describe the fuzzy rules as:

If  $e(t)$  is  $E^j$  and  $\dot{e}(t)$  is  $D^k$  then  $\lambda$  is  $Y^s$ ,

where  $E^j, D^k$  and  $Y^s$  are triangular membership functions that are depicted in Fig. 6.

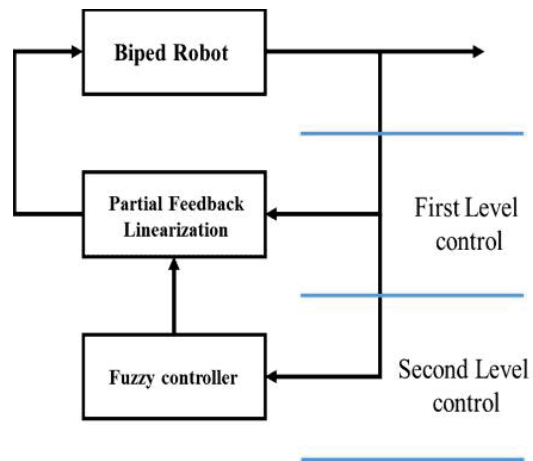


Fig. 5 Structure of fuzzy partial feedback linearization

TABLE I  
RULE TABLE OF FUZZY PFL

| $\lambda'$                   |          | $e$ (j index) |          |       |       |       |
|------------------------------|----------|---------------|----------|-------|-------|-------|
|                              |          | $E^{-2}$      | $E^{-1}$ | $E^0$ | $E^1$ | $E^2$ |
| $\frac{d}{dt}e$<br>(k index) | $D^{-2}$ | B             | B        | B     | M     | S     |
|                              | $D^{-1}$ | B             | B        | M     | M     | S     |
|                              | $D^0$    | B             | M        | S     | M     | B     |
|                              | $D^1$    | S             | M        | M     | B     | B     |
|                              | $D^2$    | S             | M        | B     | B     | B     |

## V. SIMULATION RESULTS

Validity of our designed controller is checked by simulations. We apply our controller on “compass gait biped robot” and “5-link biped robot”. In order to be sure about our controller performance, two different methods are used to find the reference trajectories of walking.

### A. Compass Gait Biped Robot

Dynamic model of compass gait is expressed in Refs. [23], [24] and biped characteristics are chosen similar to Ref. [24]. A non-constrained optimization approach (OPTIMSEARCH) is used to find the Poincaré map fixed point. Stick diagram of 10 steps of walking is showed in Fig. 7.

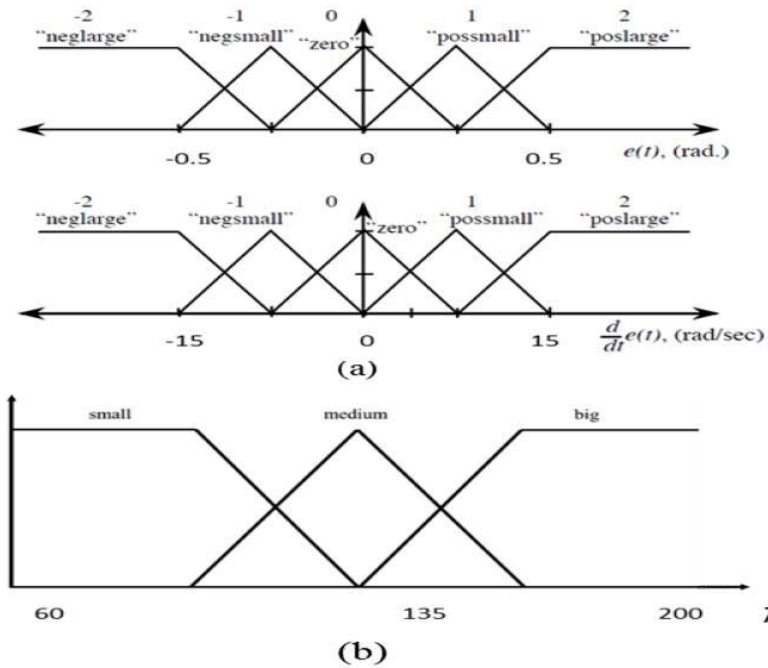


Fig. 6 (a) Membership function of input variables, (b) Membership function of output variable

Phase plane of one leg during 10 steps is shown in Fig. 8. At the beginning of stance phase, a 30% deviation of state vector from reference trajectory is considered. Our fuzzy partial feedback linearization controller is able to stabilize it, whereas a conventional partial feedback linearization controller can stabilize initial deviations up to 20%. As shown in Fig. 8, the system state converges to a stable limit cycle.

Fig. 9 depicts the control torque at the hip that has rational magnitudes in Newton-meters considering robot size and mass.

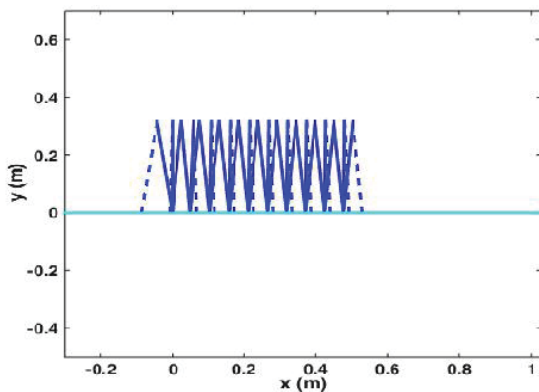


Fig. 7 Stick Diagram of 10 step walking for compass gait biped robot

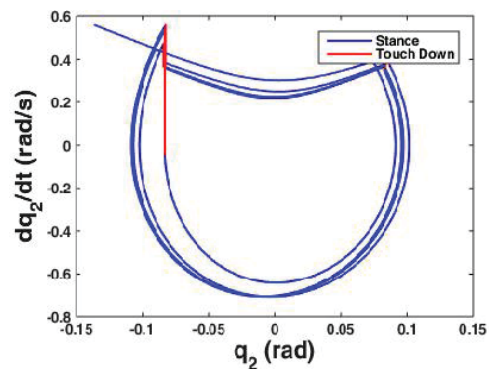


Fig. 8 Phase plane of one leg for 10 steps walking

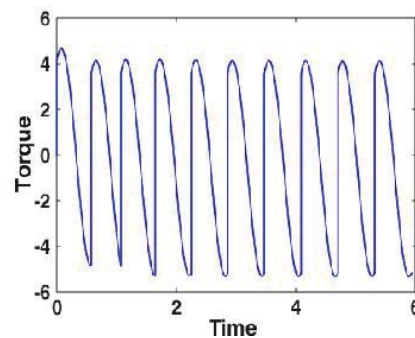


Fig. 9 Torque of hip actuator

#### B. 5-link Biped Robot

Dynamic model and characteristics of 5-link biped model are illustrated in Refs. [13], [23]. Again, we apply both convenient partial feedback linearization and fuzzy partial feedback linearization controllers to this robot and we obtain



similar results to compass gait biped robot. Stick Diagram of stable walking of this model using our designed controller is shown in Fig. 10.

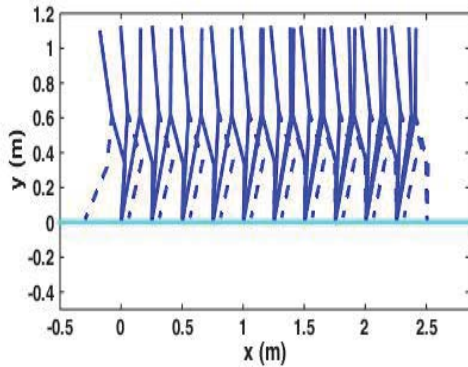


Fig. 10 Stick diagram of 10 steps walking for 5-link biped robot

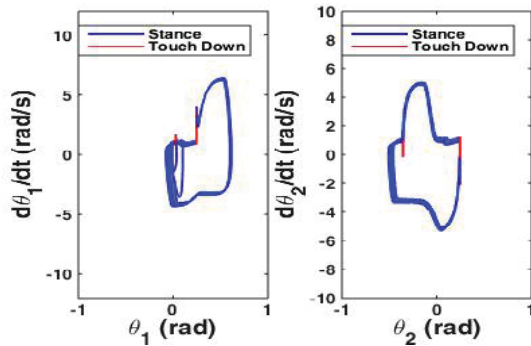


Fig. 11 Phase plane of one leg for 10 steps walking of 5-link biped robot

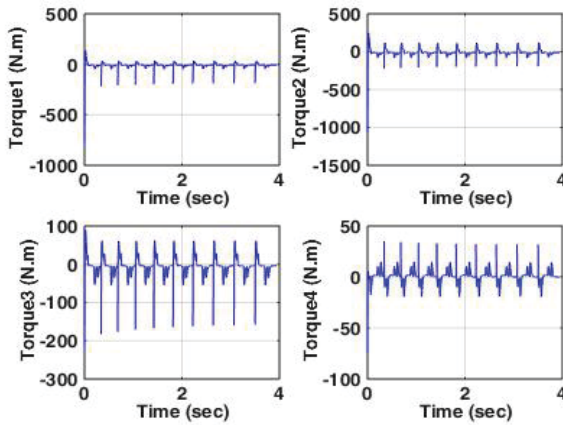


Fig. 12 Driving torques of 5-link biped robot

In the phase portraits shown in Fig. 11, absolute angles of the links belonging to the stand leg with respect to the vertical axis has been considered to investigate the stability of the limit cycles well. Clearly, they are taken as:

$$\begin{aligned}\theta_1 &= q_1 \\ \theta_2 &= q_1 - q_2\end{aligned}\quad (17)$$

Phase diagram convergence of these angles to a stable limit cycle is obvious in Fig. 11. Fig. 12 shows the required actuator torques at the hip and knee joints. These actuation torque profiles are comparable to the previous published numerical results using the other non-fuzzy controller results [13], and their magnitudes are reasonable for a biped robot with the mass of 50 kg and the length of 1.3 m.

## VI. CONCLUSION

Fuzzy partial feedback linearization controller was proposed and applied to underactuated biped robots walking. We used fuzzy logic as supervisory controller to enhance the capabilities of partial feedback linearization controller. According to the simulation results, the fuzzy partial feedback linearization showed better performance in comparison with a convenient partial feedback linearization controller. Two types of reference trajectories of walking including fixed point of active Poincaré map and intellectual method were considered. The designed controller was able to stabilize the desired walking gait in both models. The basin of attraction was larger with the fuzzy partial feedback linearization method.

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