

# Axiomatic Systems as an Alternative to Teach Physics

Liliana M. Marinelli, Cristina T. Varanese

**Abstract**—In the last few years, students from higher education have difficulties in grasping mathematical concepts which support physical matters, especially those in the first years of this education. Classical Physics teaching turns to be complex when students are not able to make use of mathematical tools which lead to the conceptual structure of Physics. When derivation and integration rules are not used or developed in parallel with other disciplines, the physical meaning that we attempt to convey turns to be complicated. Due to this fact, it could be of great use to see the Classical Mechanics from an axiomatic approach, where the correspondence rules give physical meaning, if we expect students to understand concepts clearly and accurately. Using the Minkowski point of view adapted to a two-dimensional space and time where vectors, matrices, and straight lines (worked from an affine space) give mathematical and physical rigorosity even when it is more abstract. An interesting option would be to develop the disciplinary contents from an axiomatic version which embraces the Classical Mechanics as a particular case of Relativistic Mechanics. The observation about the increase in the difficulties stated by students in the first years of education allows this idea to grow as a possible option to improve performance and understanding of the concepts of this subject.

**Keywords**—Axiom, classical physics, physical concepts, relativity.

## I. INTRODUCTION

THE high school education does not include the General Relativity (GR) or Special Relativity as content to be developed. This means that Modern Physics, which includes the GR, Quantum Physics, and Nuclear Physics, has not been strongly taught in secondary school. The only topics embraced by the different divisions of secondary school are Kinematics, Dynamics, Work, and Energy. The GR seems to be a huge question for students, and a major challenge is to find the right way to introduce them to the subject.

The continuing development in science and technology forces to go beyond what is conventionally adopted as basic pillars in science teaching. In the last few years, it has been observed that the difficulty to understand speed-time equations for a given position of the particles results in a huge time investment on the teachers' behalf and in a greater energy investment on behalf of the students who reach the academic level without the relevant knowledge in subjects like Physics, Chemistry, and Mathematics.

Our purpose as regards this topic is to generate material and didactic proposals which can be carried out in the classrooms where the geometric conception of space time can be

reintroduced [1]. It is an attempt to solve them or at least reduce them. Although we believe that due the difficulties which have been previously mentioned, it is necessary to update the approach and address Physics from an axiomatic perspective, it is interesting to observe the Classical Physics teaching from an algebraic perspective.

It is possible to examine the Classical Physics from a geometric aspect if we can define an axiomatic system in which one of the main concepts is the affine space. Furthermore, if we incorporate the concept of "elementary event", we are entering into the Minkowski field [2]. This perspective is not a minor issue because this point of view is able to be seen from the relativistic phenomena as a consequence of non-Euclidean geometry. Proposals:

1. Train teachers in the new theories about physics/science teaching, involve not only Newtonian Physics but also compare it with the solutions offered by the General Theory of Relativity.
2. Include conceptions of Euclidian and non-Euclidian geometries in the syllabus of the entry course to the degree course. Accordingly, the Minkowski space-time approach can be easily introduced.
3. Show students how these tools are necessary to understand more general aspects of the macroscopic world.
4. Make the change in the new conceptions easier through the use of postulates and intuitive ideas (in some cases) in regard to speed, rest, and movement. Students bring very classical ideas of these concepts where the relativity of time and speed is not part of their training.
5. Define the Minkowski reference framework and compare it with the Newtonian framework. Students must clearly see the differences between them.
6. Introduce matrices basically as a changing mechanism of Cartesian Systems and its use in the transformation of coordinates and uniform motion diagrams.

These proposals should be considered as a good preparation in linear algebra where students are really convinced of the usefulness and importance in different fields of study and especially in physics.

If we make a comparison between Classical and Relativistic Kinematics, we are forced to introduce the idea of "simultaneity" and define it such that the idea of "simultaneity" is absolute in Classical Kinematics, while, in the Relativistic Kinematics, it is relative. These definitions together with the ones about the systems of references, time, and space should be addressed from an inclusive perspective and the disjointed ideas.

L. M. Marinelli and C. T. Varanese are with the Universidad Tecnológica Nacional – Facultad Regional Delta, Argentina (e-mail: lmarinelli@frd.utn.edu.ar, cvaranese@gmail.com).

We should consider that it is necessary to make a radical change in the students' way of thinking regarding the classical concepts due to their rigid structures derived from their education.

At first instance, we could start the development of the related kinematics by defining concepts, proving theorems, and proposing postulates that could serve as theoretical basis for the development of related Relativistic Kinematics.

The teaching suggestions in the teaching of science so far have failed to clearly reflect the needs [3].

If we conceive, for example, laboratory practices as the base to build learning; they limit markedly the idea proposed in this piece of work with respect to an axiomatic development of Classical Physics as a particular case of Relativistic Physics.

This proposal is intended to revive the symbolic language of geometry and arithmetic because they are important elements forgotten in secondary education assuming that it is an indisputable mediator in the teaching of science in general. It is further understood that the lack of symbolic language clearly hinders the transposition of the structure to the meaning that one wants to grant.

Understanding symbology, interpreting its meaning (from a physical perspective), and transporting it to different theoretical situations are extremely important to shape concepts; particularly, theoretical and abstract physical thinking. Taking into account this perspective, Physics shall not be taught by heart and it cannot be rote, but it is supposed to be rather rational and inferential. The mechanisms to be used need to be focused on achieving a theoretical thinking which can be applied and transferred to any future situation, as the basis to build scientific concepts.

Setting situations and matters that result in epistemological thoughts (necessary for the training of engineers) could be an example of a valuable option for this practice. Furthermore, we do not need to expose statements and axioms as blatant truths.

Using an axiomatic system could guarantee from certain deductions the consistency of the structure where it would be impossible to get any contradiction from these axioms. Therefore, as it is unlikely to obtain contradictions the importance of axiomatic systems to build knowledge would be evident.

The basis for any theory is strictly a new axiomatic system, born as a pure mathematical model which then, through adequate correspond rules, establishes contact with physical reality. Most axiomatic systems of Special Relativity which have been offered until now mix mathematical and physical notions, which minimize epistemological elegance [2].

For this, it would be necessary to build an axiomatic system which allows students to meet with certain objective and get the mechanisms established to introduce significant statements. So, if the statements that serve as axioms are chosen, rules defining the system are fully established, and new statements according to the established rules will found an axiomatic system. When all agree with these principles then, students can build a kinematics possible to be seen from

both views.

Students lack important algebraic tools that facilitate certain passages or changes of variables. For example, the matrix of passage could be useful for students to see the physical application:

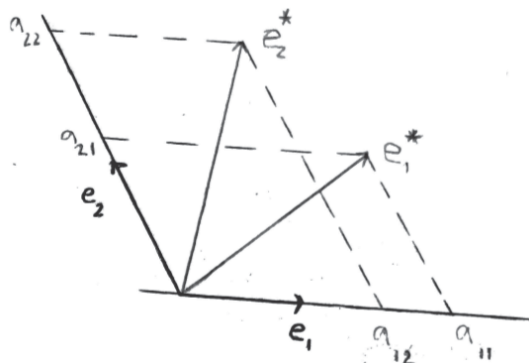


Fig. 1 Matrix of passage

In this case, the matrix of passage would be:

$$M_{(S:S')} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

where  $a_{12}$  and  $a_{22}$  are the coordinates of the tip of the vector  $e_2$ . In the same way,  $a_{11}$  and  $a_{21}$  are the coordinates of the tip of the vector  $e_1$ . Therefore, it can be seen that if the element  $a_{11}$  is nonzero and speed relative to the other system is equal to the ratio between  $a_{21}$  and  $a_{11}$ , then we can call that speed "v" and:

$$v_{(S:S')} = \frac{a_{21}}{a_{11}}$$

The fundamental theorem of general kinematics could be introduced from an axiomatic perspective, and from there to demonstrate the existence of a number k wherein the matrix of passage is as follows:

$$M_{(S:S')} = \begin{pmatrix} a & kva \\ va & a \end{pmatrix} = a \begin{pmatrix} 1 & kv \\ v & 1 \end{pmatrix}$$

where the value of k is determined uniquely and is the same for all ET systems space-time [2]. That number k represents the possibility of seeing Classical Mechanics as a special case of General Mechanics. If "k" is greater than 0, then it is Special Relativity, if "k" is zero, one thinks in terms of Classical Mechanics, and if "k" is less than zero, it is in a Non-Einsteinian Relativity [2].

Another interesting question that appears in the proposal is to involve theorems development and summary where they can relate issues that appear to have no link in the study of matrices and linear transformations [4].

The close relationship between vectors and matrices as a basic tool of linear algebra should be clearly and conclusively seen from the eyes of students.

To teach a new basic geometry (non-Euclidean) in the incoming course, it will be essential to have students who have already passed an average training level where they were able to develop a logical thinking which serves as a basis for the new geometry. If this condition is not met, then there should be a course of academic support which can meet those needs.

Many models can be viewed by students from a purely theoretical discipline and then be converted into requirements of physics. They facilitate the understanding of the phenomena and at the same time they are suspended as potential tools that can be used when the application is required.

How is the constant value of the speed of light in vacuum explained? From an axiomatic perspective under the faithful observance of the principles and proposed axioms and under the rules of proper correspondence, one can arrive at the concept of invariant speed and signs attributed to it.

We all agree that building a formal system is not an easy task. It is necessary to define the borders of deductive structure to ensure that a formal system faithfully expresses the defined structure which is required to match the statements of the formal system with those in the domain of scientific objects [5].

Generating questions that refer to the comparisons between what classical mechanics offers and the variables that incorporates or changes Relativistic Mechanics, does reflect not only the physical concepts and observable phenomena, but also they also reflect the deductive field supporting the different versions or perspectives of the events that occur in nature. On the other hand, knowledge of the existence of another possible framework (Minkowskian) will allow students to have a broader view of possibilities of geometric theory. For this reason, in a world where information is at our fingertips, it is necessary to rethink about the way how the science is taught.

The teaching of physics in secondary education and even higher education is subject to Aristotelian conceptions schemes, where the proposals impose always from truisms and from these obvious truths. What is proposed here is to consider a formal axiomatic model in which the process is just the reverse. First, develop strictly the mathematical derivations obtained from logical procedures, and then establish the truth through interpretation [6]. The aforementioned process would also facilitate to study the difference between the General Theory of Relativity and the Special Theory of Relativity. We mark special attention in that because while the latter responds to the comparison between measurements made in different inertial reference systems moving with constant velocity relative to each other, the first one refers to accelerated reference systems and gravity [7].

How a student could do some questions about what is being taught, if the information received is already full of meaning? For example, to ask the student a problem of type: two events occur at the same point  $x_0$  on  $t_1$  and  $t_2$  in the system  $s$  which is moving with speed  $v$  with respect to  $S$ : What is the spatial separation of these two events in the system  $S$ ? What is the temporal separation of these two events on the same system? [7]

To analyze the possible answers, considering the equations known as time dilation and Lorentz transformations, could start from the axiomatic structure, could give meaning and significance from the matrix of passage, and could analyze the relativity of time and its relation to the relativity of space.

Delivering the knowledge which is already developed with meaning and sense questions that students have never done does not allow the development of analytical skills and the logic they need.

The teaching of physics from a strictly utilitarian perspective is insufficient for future graduates of scientific and technological careers and does not give rise to deepening from research to different modes of scientific production and to the study of more abstract and complex phenomena. The joy of seeing and understanding is the most beautiful gift of nature [8].

## II. CONCLUSIONS

Even though the established idea has not been carried out, it is possible to analyze certain counterfactual matters such as asking what would have happened with students' academic performance if the axiomatic method would be less marginalized and primary and secondary teachers would be better trained in this matter?

Naturally, we do not have answers for this question. However, when methodologic, didactic and disciplinary proposals join together for a common goal we will, undoubtedly, be able to answer it. So far, and at least for the next 10 months, we will be devoted to establish possible actions to carry out the physical axiomatic plan (at least in some areas).

While lower animals are alone in the world, man tries to understand; and on the basis of their imperfect but perfectible, intelligence world, man tries to make it more comfortable. In this process, build an artificial world: the growing body of ideas called "science" can be characterized as rational, systematic, accurate, verifiable and therefore fallible knowledge. By through scientific research, man has reached a reconstruction conceptual world that is increasingly broad, deep, and accurate [9].

This work aims to raise awareness on issues related to the real reason for higher education based on the idea of a psychologist dedicated to education that "look slow" requires some attention. "Understanding" takes time, patience, and waiting. Both actions become incompatible with the speed and success with anxiety [10].

## REFERENCES

- [1] P. Cullen, *El Renacimiento Educativo. La salida de la Crisis educativa Argentina como oportunidad*. Ed. edUTecNe.
- [2] Bosch, *Teoría especial de la Relatividad. Enfoque Axiomático y Epistemológico* Ed. Ediciones Universidad CAECE. Serie Ciencias Exactas.
- [3] P. Cullen, *Universidades para el Siglo XXI*. Ed. edUTecNe Personal reflection on his thinking.
- [4] S.I.Grossman, *Algebra Lineal*. 5ta. Ed. Mc Graw Hill.
- [5] A. Badiou, *El Concepto de Modelo*. Ed. La Bestia 2009.

- [6] G. Klimovsky, M.J. C. de Asúa, El Método Axiomático Formal. Elementos de Matemática. Publicación Didáctica Científica de la Universidad Caece. 1988.
- [7] P.A. Tipler, G. Mosca, Física para la ciencia y la tecnología. 6ta Edición. Edit. Reverté.
- [8] A. Einstein, Mis ideas y opiniones. Aforismos para Leo Baeck. Pg 38
- [9] M. Bunge, "La Ciencia, su método y su filosofía" Ed. Siglo XXI.
- [10] G. Rivelis, Construcción Vocacional. ¿Carrera o camino? Noveduc.

**Liliana M. Marinelli** is a Physics Professor at the Facultad Regional Delta and I.S.F.T 195. She was born in 1975 and earned her degree in Math and Physic in 1996 at Colegio de Nivel Medio y Superior Dr. Julio Ossola – Rosario del Tala- Entre Ríos. In 2003 she got her Licenciatura in Educational Technologies-Universidad Tecnológica Nacional Facultad Regional Delta-Campana Buenos Aires. In 2007 got the Licenciatura in Teaching Physics-Universidad CAECE- Buenos Aires.

**Cristina T. Varanese** is a Physics Professor at the Facultad Regional Delta. She earned her degree in Chemistry.