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Model Updating-Based Approach for Damage Prognosis in Frames via Modal Residual Force

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***Abstract*—**This paper presents an effective model updating strategy for damage localization and quantification in frames by defining damage detection problem as an optimization issue. A generalized version of the Modal Residual Force (MRF) is employed for presenting a new damage-sensitive cost function. Then, Grey Wolf Optimization (GWO) algorithm is utilized for solving suggested inverse problem and the global extremums are reported as damage detection results. The applicability of the presented method is investigated by studying different damage patterns on the benchmark problem of the IASC-ASCE, as well as a planar shear frame structure. The obtained results emphasize good performance of the method not only in free-noise cases, but also when the input data are contaminated with different levels of noises.

***Keywords*—**Frame, grey wolf optimization algorithm, modal residual force, structural damage detection.

I. Introduction

In the past few years, the problems on damage detection and health monitoring of structures have received considerable attention. Once the structural damage has been accurately detected, the rehabilitation procedures should be conducted in order to prevent catastrophic events. Therefore, in the field of maintenance of structures, structural damage identification consists of basic steps which not only reasonably decreases the costs of structural repairing, but also can accelerate the rehabilitation procedures by concentrating only on the damaged sections of the structure. Although different kinds of input structural feedbacks may be used for damage prognosis, vibration-based approaches are more preferred. The fundamental purpose of the vibration-based damage detection is that the damage-induced changes in physical properties (mass, damping, and stiffness) will result in detectable changes in modal properties (natural frequencies, and mode shapes). Therefore, it seems that by employing an inverse strategy, damage features can be identified from changes in the modal properties. A complete review of vibrational-based damage identification methods can be found in [1], [2]. Some of these methods are based on natural frequencies [3], [4], while the other methods employ mode shapes or their derivatives [5]–[10]. Subsequently, optimization-based methods have been proposed in order to find a proper solution for the damage identification problem [11]–[18]. In fact, optimization-based damage detection methods are aimed at detecting damage location as well as damage severity by updating numerical finite element model (FEM) of the intact structure in a way that the numerical model can behave close to the monitored structure. Evaluating amount of correlation between monitored structure and updated numerical model, and deciding about approaching of the two models is done via optimization algorithms. For instance, Ghodrati Amiri et al. [13] employed the free vibration equilibrium of the plate-like structures for introducing a damage-sensitive cost function. They utilized pattern search and genetic algorithms for finding optimal solution of the problem. Vincenzi et al. [15] proposed an optimization-based damage detection method based on coupled local minimizers and differential evolution algorithm. Mohan et al. [16] employed particle swarm optimization and genetic algorithms for solving damage identification problem by considering a modal data-based cost function. Recently, Zare Hosseinzadeh et al. [18] formulated the damage identification problem by presenting a cost function based on calculated static displacements and reached to global extremums of the problem by means of cuckoo optimization algorithm.

In this paper, an effective optimization-based model updating method is presented for damage localization and quantification in structural frames. First, a damage-sensitive cost function is defined based on a generalized version of MRF. Then, GWO algorithm is utilized for solving the optimization problem to find global extremums as the damage detection results. The efficiency of the presented method is demonstrated by applying it for detecting simulated damages on the first phase of the benchmark problem provided by the International Association for Structural Control (IASC)-American Society of Civil Engineers (ASCE) Task Group on Structural Health Monitoring. Moreover, some other studies are carried out on a planar four-story shear frame. The acquired results clearly indicate that the proposed method can precisely locate and quantify damage of frames not only in free noise state, but also when the noisy input data are fed.

II. GWO Algorithm

GWO is an evolutionary optimization algorithm that is inspired by the particular lifestyle of a wolf family, called grey wolf community [19]. The grey wolves have a social dominant hierarchy, and they mostly prefer to live in a pack. The top of hierarchy pyramid are leaders namely alphas. The alphas (α)

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are a male and a female, and their decision has the most priority in the pack. The betas (β) are in the second level of social hierarchy, and they help the alpha in making decisions and other fundamental activities. If one of alpha wolves passes away or becomes very old, the best candidates for substitution are beta wolves. Omega (ω) wolves are the lowest level of hierarchy, and they are supposed to submit to all the other dominant wolves. Therefore, omegas are the last wolves that are allowed to eat in the pack. If a wolf is not alpha, beta or omega, he/she is called subordinate or delta (δ). Delta wolves have to submit to alphas and betas, but they dominate the omegas [19]. The social hierarchy of grey wolves is shown in Fig. 1.

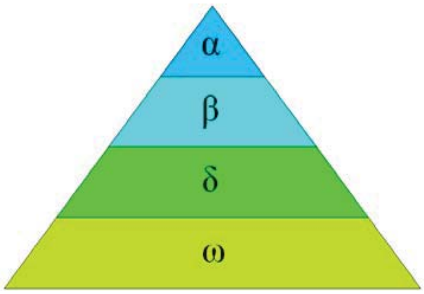

Fig. 1 Social dominant hierarchy of grey wolves is shown as a pyramid

Another social behavior of grey wolves is *group hunting*. Their hunt involves three phases. The first phase is related to tracking, chasing, and approaching the prey. The wolves pursue, encircle, and harass the prey until it stops running away in the second phase, and in the last phase they attack towards the prey [19].

Their behavior and lifestyle can be suggested through a mathematical model. In this model the first, second, and third best solutions are assumed to be alpha, beta, and delta wolves, respectively. The rest of solutions are supposed to be omega wolves. In GWO framework, the hunting is leaded by be alpha, beta, and delta wolves, and the omega wolves follow these three wolves. The following framework is presented to model encircling behavior of grey wolves [19]:

$$\mathbf{D}_j = \left|\mathbf{C}_j.\mathbf{Y}_p^t - \mathbf{Y}_j^t\right| \tag{1}$$

$$\mathbf{Y}_j^{t+1} = \mathbf{Y}_p^t - \mathbf{A}_j.\mathbf{D}_j \tag{2}$$

$$\mathbf{A}_j = 2\mathbf{a}_j.\mathbf{R}_{1j}\mathbf{a}_j - \mathbf{a}_j \tag{3}$$

$$\mathbf{C}_j = 2\mathbf{R}_{2j} \tag{4}$$

where $\mathbf{A}_j$ and $\mathbf{C}_j$ are coefficient vectors, $\mathbf{Y}_p^t$ is the prey in iteration t, $\mathbf{Y}_j^t$ is the jth grey wolf in iteration t, $\mathbf{R}_{1j}$ and $\mathbf{R}_{2j}$ are random vectors between [0,1] which is generated by MATLAB software, and $\mathbf{a}_j$ is a vector that its components are linearly decreased from 2 to 0 during incremental iterations.

For simulation of grey wolves' hunting, it is presumed that the alpha, beta, delta have better knowledge of the potential location of prey. As a consequence, the three best solutions should be saved. Afterwards, omegas will update their positions based on the best solution of the wolves located around the prey. The following equations represents this position updating:

$$\mathbf{D}_\alpha = \left|\mathbf{C}_1.\mathbf{Y}_p^t - \mathbf{Y}_1^t\right| \tag{5}$$

$$\mathbf{D}_\beta = \left|\mathbf{C}_2.\mathbf{Y}_\beta^t - \mathbf{Y}_2^t\right| \tag{6}$$

$$\mathbf{D}_\delta = \left|\mathbf{C}_3.\mathbf{Y}_\delta^t - \mathbf{Y}_3^t\right| \tag{7}$$

$$\mathbf{Y}_1^{t+1} = \mathbf{Y}_\alpha^t - \mathbf{A}_1.\mathbf{D}_\alpha \tag{8}$$

$$\mathbf{Y}_2^{t+1} = \mathbf{Y}_\beta^t - \mathbf{A}_2.\mathbf{D}_\beta \tag{9}$$

$$\mathbf{Y}_3^{t+1} = \mathbf{Y}_\delta^t - \mathbf{A}_3.\mathbf{D}_\delta \tag{10}$$

$$\mathbf{Y}^{t+1} = \frac{1}{3}(\mathbf{Y}_1^t + \mathbf{Y}_2^t + \mathbf{Y}_3^t) \tag{11}$$

In Fig. 2, the searching procedure is depicted which is leaded through updating the position based on alpha, beta, and delta in a planar search space. It is obvious that the final location would be a random position within a circle which is determined by the position of alpha, beta, and delta wolves. Hence, the position will be estimated based on alpha, beta, and delta wolves' orders, and omega wolves will update their positions randomly around the prey.

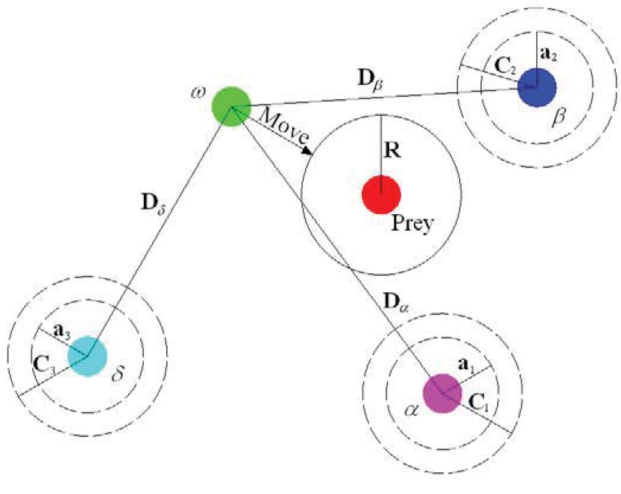

Fig. 2 Position updating of wolves based on GWO algorithm

As mentioned before, the final phase in the grey wolves' hunt is attacking towards the prey as soon as it stops running away. In the mathematical model, approaching the prey is modelled by decreasing the values of entries of $\mathbf{a}_j$ from 2 to 0 during incremental iterations. Since, $\mathbf{A}_j$ is a value between [-$\mathbf{a}_j$

, $\mathbf{a}_j$], the fluctuation range of $\mathbf{A}_j$ is decreased, and it provides the exploitation ability of the algorithm. Also, when $|\mathbf{A}_j|$ is greater than 1, it provides the exploration ability of the GWO.

III. Damage Detection Method

The free vibration problem for a structure with N degrees of freedoms (DOFs) can be expressed as:

$$\left(\mathbf{K}-\mathbf{M}\,\omega_i^2\right)\boldsymbol{\varphi}_i=\mathbf{0}\ ,\ i=1,2,\ldots,N \quad (12)$$

where $\mathbf{K}$ and $\mathbf{M}$ are the stiffness and mass matrices, respectively. ω_i and $\boldsymbol{\varphi}_i$ are the natural frequency and the mode shape vector of the ith mode, respectively. If superscripts u and d are utilized for undamaged and damaged states, (12) can be rewritten for undamaged and damaged structures:

$$\left(\mathbf{K}^u-\mathbf{M}^u\left(\omega_i^u\right)^2\right)\boldsymbol{\varphi}_i^u=\mathbf{0}\ ,\ i=1,2,\ldots,N \quad (13a)$$

$$\left(\mathbf{K}^d-\mathbf{M}^d\left(\omega_i^d\right)^2\right)\boldsymbol{\varphi}_i^d=\mathbf{0}\ ,\ i=1,2,\ldots,N \quad (13b)$$

As mentioned at Section I, damage is defined as some changes in the physical properties of the structures. So, $\mathbf{K}^d$ and $\mathbf{M}^d$ can be defined as below:

$$\mathbf{K}^d=\mathbf{K}^u+\Delta\mathbf{K}^d \quad (14)$$

$$\mathbf{M}^d=\mathbf{M}^u+\Delta\mathbf{M}^d \quad (15)$$

where $\Delta\mathbf{K}^d$ and $\Delta\mathbf{M}^d$ are the changes in stiffness and mass matrices of the structure as a consequence of damage. Since the damage has negligible effect on the system's mass, the change of mass is ignored ($\Delta\mathbf{M}^d$=$\mathbf{0}$). By substituting (14) and (15) into (13b) and doing some mathematical simplifications, (16) can be yielded:

$$\Delta\mathbf{K}^d\boldsymbol{\varphi}_i^d+\left(\mathbf{K}^u-\mathbf{M}^u\left(\omega_i^d\right)^2\right)\boldsymbol{\varphi}_i^d=\mathbf{0} \quad (16)$$

or:

$$\left(\mathbf{K}^u-\mathbf{M}^u\left(\omega_i^d\right)^2\right)\boldsymbol{\varphi}_i^d=-\Delta\mathbf{K}^d\boldsymbol{\varphi}_i^d \quad (17)$$

From the left side of (17), it can be concluded that because of damage occurrence, the free vibration equilibrium results in some non-zero values if this equation is formed via mass and stiffness matrices of the intact structure, and those modal data related to the ith mode of the monitored (or damaged) structure. These non-zero values, which are interpreted as non-absorbed forces, can be physically justified by considering presented strategy via (14) and (15) for damage simulation. The non-absorbed forces are defined as MRF:

$$\mathbf{R}_i=\left(\mathbf{K}^u-\mathbf{M}^u\left(\omega_i^d\right)^2\right)\boldsymbol{\varphi}_i^d \quad (18a)$$

or:

$$\mathbf{R}_i=-\Delta\mathbf{K}^d\boldsymbol{\varphi}_i^d \quad (18b)$$

in which $\mathbf{R}_i$ is the MRF for the ith mode. Without considering noise effect, the kth entry of $\mathbf{R}_i$ will be zero if none of the elements related to the kth DOF is damaged. On the other hand, a non-zero value will be assumed for the kth entry of $\mathbf{R}_i$ if any elements that are related to this DOF have been damaged. As a result, by detecting non-zero entries in $\mathbf{R}_i$, the damage locations can be identified. It is worth noting that this method will work properly with one mode and free-noise state. However, in practice, more than one mode is usually needed considering noise effects. It is possible that some entries in the modal residual vector will be regarded as non-zero elements because of measurement or numerical errors. Hence, a generalized version of the residual force vector proposed as [20]:

$$\mathbf{R}^g=\left\{r_1\quad r_2\quad\ldots\quad r_N\right\}^T \quad (19)$$

where r_k is obtained from (20):

$$r_k=\left(\left(\left|r_k\right|_1\right)\times\left(\left|r_k\right|_2\right)\times\ldots\times\left(\left|r_k\right|_p\right)\right)^{(1/p)} \quad (20)$$

in which $|r_k|_i$ is the absolute value of the kth entry of $\mathbf{R}_i$ from (18), and p is the number of measured frequencies and mode shapes of the damaged structure. This paper uses generalized version of the MRF for formulating damage detection problem as an optimization problem. The objective is minimizing the cost function which is suggested by means of data-fitting strategy as:

$$Cost=\left\|\mathbf{R}_D^g-\mathbf{R}_X^g\right\| \quad (21)$$

where $\mathbf{R}^g{}_D$ is the generalized MRF for the monitored structure which can be calculated via the measured first p modes' data using (18a). Moreover, $\mathbf{R}^g{}_X$ is the generalized MRF for the numerical model of the damaged structure which is calculated by the first p modes' data via (18b). It should be mentioned that in the numerical model of the damaged structure, unknown damage severities are considered for all elements as:

$$\mathbf{K}_n^d=(1-x_n)\mathbf{K}_n^u\ ,\ 0\le x_n\le 1 \quad (22)$$

where $\mathbf{K}^d{}_n$ and $\mathbf{K}^u{}_n$ are stiffness matrices of the nth element in damaged and undamaged states, respectively; and x_n is unknown damage severity for the nth element. Also, related modal data for the numerical model may be extracted via classic modal analysis. In the next step, the GWO algorithm is employed to find unknown damage severities (i.e. x_ns). The flowchart of GWO algorithm can be depicted as Fig. 3.

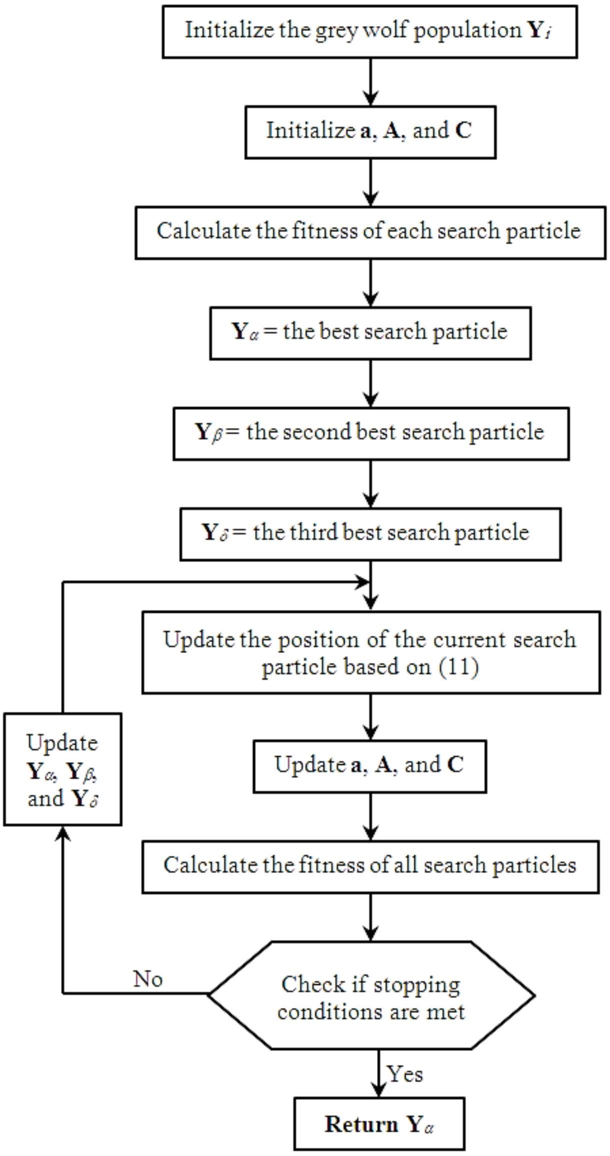

Fig. 3 Flowchart of GWO algorithm

IV. Numerical Studies

In order to validate the effectiveness of the proposed method, it is applied to the first phase of IASC-ASCE benchmark structure. This is followed by studying a planar four-story plane frame.

A. IASC-ASCE Benchmark Structure

The benchmark structure is a four-story steel frame, two-bay by two-bay and quarter-scale model structure, constructed in the Earthquake Research Laboratory at the University of British Columbia. The geometry of the benchmark structure is shown in Fig. 4. Details of the benchmark problem are described by Johnson et al. [21].

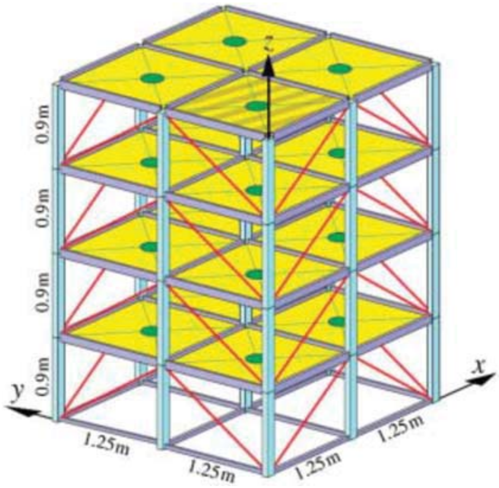

Fig. 4 Geometry of the IASC-ASCE benchmark structure constructed in the Earthquake Research Laboratory at the University of British Columbia

In this study, a FEM of the benchmark structure with 12 DOFs (as a three dimensional shear frame structure with three DOFs at each floor) is considered. The following two damage patterns were simulated:

- Damage Pattern 1: All the braces of the first and third stories are broken,
- Damage Pattern 2: One brace at the first and third stories is broken.

The mass and horizontal story stiffnesses for healthy and damaged structure based on each damage pattern are listed in Table I.

TABLE I
MASS AND HORIZONTAL STIFFNESS OF UNDAMAGED AND DAMAGED 12 DOFS MODEL OF THE BENCHMARK STRUCTURE

			Stiffness (MN/m)		
				Damage pattern	
Story	DOF	Mass (kg)	Undamaged	1	2
1	x	3452.4	106.60	58.37	106.60
2	x	2652.4	106.60	106.60	106.60
3	x	2652.4	106.60	58.37	94.54
4	x	1809.9	106.60	106.60	106.60
1	y	3452.4	67.90	19.67	55.84
2	y	2652.4	67.90	67.90	67.90
3	y	2652.4	67.90	19.67	67.90
4	y	1809.9	67.90	67.90	67.90
1	θ_z	3819.4	232.00	81.32	213.12
2	θ_z	2986.1	232.00	232.00	232.00
3	θ_z	2986.1	232.00	81.32	213.12
4	θ_z	2056.9	232.00	232.00	232.00

TABLE II
INPUT PARAMETERS FOR GWO ALGORITHM

Number of initial population of wolves	30
Upper bound	0
Lower bound	1
Maximum number of iterations	500

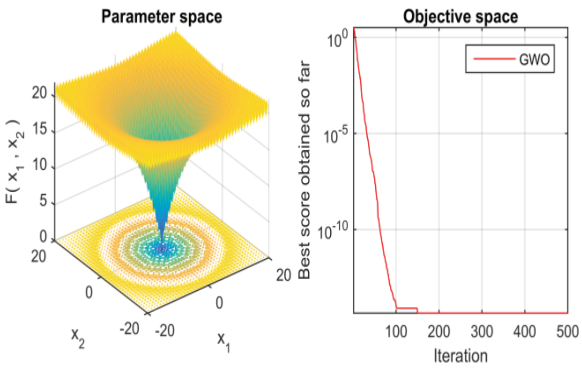

Fig. 5 Convergence curve of GWO for Damage Pattern 2 of benchmark structure

The damage severity is determined for each damage case by applying the proposed method. The values of parameters of the GWO algorithm are listed in Table II. These parameters were selected based on trial and error, which depend on the number of variables and the complexity of the cost function.

In Fig. 5, the convergence curve of the GWO algorithm is shown for Damage Pattern 2. In addition, this figure shows the search space for the first two unknown parameters in terms of cost function value.

The results of the damage identification method are summarized in Table III and IV, for Damage Pattern 1 and Damage Pattern 2, respectively. Parameter q in these tables indicates the number of modal data which were utilized for solving the damage identification problem. Moreover, e represents the noise levels used in order to contaminate the natural frequencies. The effects of noise on the natural frequencies are simulated using random noise as follows [17]:

$$\omega_i^n = \omega_i (1 + e\,\varepsilon_i) \tag{23}$$

where ω_i is the ith natural frequency without noise, ω_i^n is the ith natural frequency that is polluted by noise. As mentioned before, e is the noise level, and ε_i is a random value between (-1) and (1).

TABLE III
OBTAINED RESULTS (%) FOR THE FIRST DAMAGE PATTERN OF THE BENCHMARK PROBLEM

			Estimated					
			$q = 3$			$q = 6$		
Story	DOF	Actual	$e = 0\%$	$e = 3\%$	$e = 5\%$	$e = 0\%$	$e = 3\%$	$e = 5\%$
1	x	45.24	45.243	43.322	45.699	45.243	46.250	44.195
2	x	0	0.000	0.037	0.030	0	0.013	0.047
3	x	45.24	45.243	43.460	45.658	45.243	46.260	43.767
4	x	0	0.000	0.023	0.023	0	0.061	0.039
1	y	71.03	71.022	71.037	71.071	71.022	71.011	71.064
2	y	0	0.000	0.018	0.050	0	0.024	0.001
3	y	71.03	71.022	71.037	71.045	71.022	70.993	71.025
4	y	0	0.000	0.038	0.018	0	0.036	0.037
1	θ_z	64.95	64.946	65.098	65.265	64.946	64.990	64.951
2	θ_z	0	0.000	0.038	0.010	0	0.031	0.025
3	θ_z	64.95	64.946	65.098	65.109	64.946	64.986	64.978
4	θ_z	0	0.000	0.013	0.002	0	0.014	0.004

DOF = degrees of freedom; q = number of utilized modal data for damage detection procedure; e = noise level.

TABLE IV
OBTAINED RESULTS (%) FOR THE SECOND DAMAGE PATTERN OF THE BENCHMARK PROBLEM

			Estimated					
			$q = 3$			$q = 6$		
Story	DOF	Actual	$e = 0\%$	$e = 3\%$	$e = 5\%$	$e = 0\%$	$e = 3\%$	$e = 5\%$
1	x	0	0.000	0.003	0.278	0.000	0.233	0.303
2	x	0	0.000	0.011	0.011	0.000	0.144	0.343
3	x	11.31	11.319	11.492	11.508	11.319	11.443	11.722
4	x	0	0.000	0.010	0.492	0.000	0.445	0.491
1	y	17.76	17.767	17.931	18.143	17.767	17.877	18.059
2	y	0	0.000	0.032	0.010	0.000	0.010	0.024
3	y	0	0.000	0.050	0.035	0.000	0.006	0.014
4	y	0	0.000	0.020	0.007	0.000	0.049	0.044
1	θ_z	8.14	8.145	8.154	8.186	8.145	8.353	8.437
2	θ_z	0	0.000	0.005	0.032	0.000	0.017	0.018
3	θ_z	8.14	8.145	8.160	8.178	8.145	0.017	8.380
4	θ_z	0	0.000	0.047	0.036	0.000	0.017	0.010

DOF = degrees of freedom; q = number of utilized modal data for damage detection procedure; e = noise level.

It is obvious that localizing of damage in noisy state may be faced with some uncertainty; however, the results indicate that the proposed method has outstanding capability of identifying and quantifying simulated damages in noisy states. For this example, the structure was considered as a three dimensional model. In section *B*, a planar four-story shear frame structure is utilized for evaluating the performance of the suggested method.

B. Planar Four-Story Shear Frame

In the second example, the proposed method is employed for damage localization and quantification in a planar four-story shear frame. Table V presents the physical properties of this structure. In this Section, two damage patterns are considered as listed in Table VI. It is assumed that two different sets of modal data are available for damage identification (q=1 and 3). In addition, the optimization parameters are selected similar to those which utilized in the previous example (Table II).

The obtained damage detection results are shown in Figs. 6 and 7 for damage patterns I and II, respectively. It is clear that the presented method has robust capability to detect and quantify damage extent. The effect of noise existence in the input data has been considered in the simulated damage patterns, by means of presented strategy via (23). Based on the obtained results, it is obvious that the noise effects are negligible. Therefore, the proposed method can be considered as a viable method for damage detection.

TABLE V
PHYSICAL PROPERTIES OF PLANAR FOUR-STORY SHEAR FRAME

Story No.	Mass (ton)	Stiffness (MN/m)
1, 2	80	7.5
3, 4	65	7.5

TABLE VI
SIMULATED DAMAGE PATTERNS IN THE PLANAR FOUR-STORY SHEAR FRAME

Damage pattern I		Damage pattern II	
Story	Damage (%)	Story	Damage (%)
3	10	1	5
		3	10

V. CONCLUSION REMARKS

This paper has presented a novel method for damage identification in frames by defining a cost function based on the generalized version of MRF and an optimization framework. The proposed cost function was solved by GWO algorithm. The GWO is a novel algorithm inspired by the particular lifestyle of a pack of grey wolves. The applicability of the suggested method was investigated by studying different damage patterns on two numerical examples. Furthermore, the effects of noise appearance in the input data and the number of utilized modal data were considered. The outcomes indicate good performance of the presented procedure for damage localization and quantification in frames.

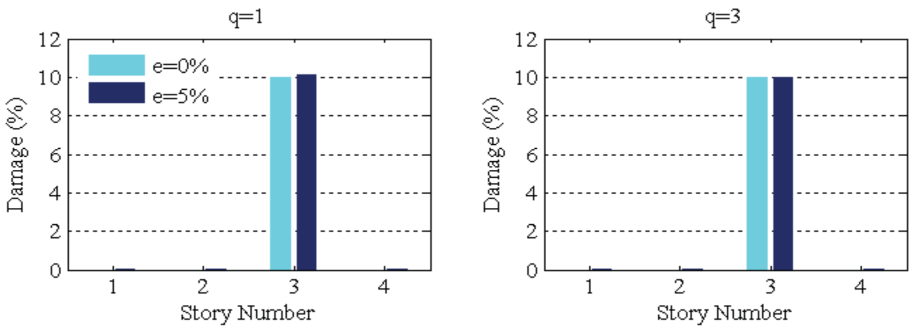

Fig. 6 Damage detection results for the first damage pattern of the planar four-story shear frame

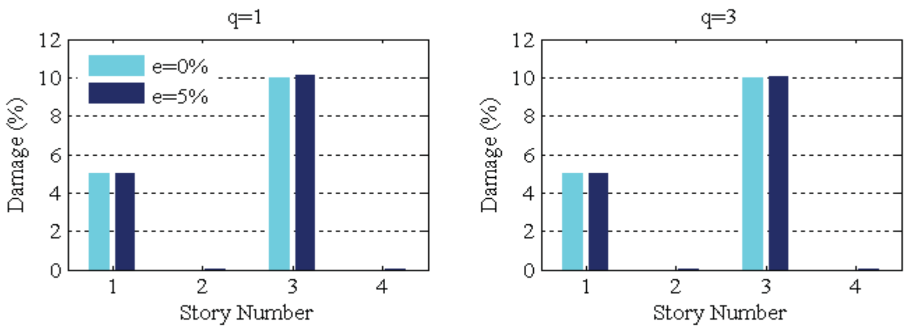

Fig. 7 Damage detection results for the second damage pattern of the planar four-story shear frame

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