Modeling, Analysis and Control of a Smart Composite Structure

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Abstract—In modern engineering, weight optimization has a priority during the design of structures. However, optimizing the weight can result in lower stiffness and less internal damping, causing the structure to become excessively prone to vibration. To overcome this problem, active or smart materials are implemented. The coupled electromechanical properties of smart materials, used in the form of piezoelectric ceramics in this work, make these materials well-suited for being implemented as distributed sensors and actuators to control the structural response. The smart structure proposed in this paper is composed of a cantilevered steel beam, an adhesive or bonding layer, and a piezoelectric actuator. The static deflection of the structure is derived as function of the piezoelectric voltage, and the outcome is compared to theoretical and experimental results from literature. The relation between the voltage and the piezoelectric moment at both ends of the actuator is also investigated and a reduced finite element model of the smart structure is created and verified. Finally, a linear controller is implemented and its ability to attenuate the vibration due to the first natural frequency is demonstrated.

Keywords—Active linear control, Lyapunov stability theorem, piezoelectricity, smart structure, static deflection.

I. INTRODUCTION

A smart structure is a structure that can sense external disturbances or oscillations and respond to that with active control in real time to maintain the requested requirements. Smart structures consist of highly distributed active devices which are primarily sensors and actuators either embedded or attached to an existing passive structure. As for sensors, mechanically induced deformations can be determined by measuring the induced electrical potential, whereas deformation or strains can be controlled through the introduction of an appropriate electric potential in actuator applications [1]. Recent innovations in piezoelectric materials and developments in control theory have made it possible to control the dynamics of these structures, and have led to an extensive amount of research activity in this field [2].

The work presented in this paper can be classified into three parts. In the first part, the electromechanical behavior of the smart structure is statically investigated, and the relation between deflection and applied voltage is derived. In the second part, the piezoactuator is modeled and the relationship

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between the applied voltage and moment at its ends is investigated. The structural analytical model is then produced by using the finite element (FE) method, and the number of elements and nodes of the FE model is reduced by using the super element (SE) technique. Finally, a linear active controller based on the Lyapunov stability theorem is implemented, and the ability of the controller to attenuate the vibration of the smart structure once excited with its first eigenfrequency is demonstrated.

II. THEORETICAL ANALYSIS

A. Modeling of Static Deflection of the Smart Structure

As previously mentioned, the smart structure used in this work is composed of a cantilevered steel beam, a bonding layer (adhesive) and a piezoelectric actuator. A similar work with only two layers of the smart beam is presented in [3]. A schematic of the cross section with the piezoelectric patch is shown in Fig. 1. The curvature δ'' can be expressed as [4]:

$$\delta'' = \frac{M}{EI} = \frac{\varepsilon}{Z_{na} - t_b} \tag{1}$$

where, E is the Young's modulus, I is the moment of inertia, M is the bending moment, t_b is the thickness of the steel beam, Z_{na} is the distance to the neutral axis, and ε is the strain.

Using the constitutive piezoelectric equation for strain without the stress component gives [5]:

$$\varepsilon = d_{31}\xi = d_{31}\frac{V}{2t_p} \tag{2}$$

where d_{31} is the piezoelectric strain coupling coefficient, ξ is the electric field component, V is the voltage, and t_p is the thickness of the piezoelectric layer.
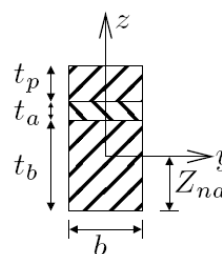

Fig. 1 A schematic of the cross section of the smart structure

Substituting (2) in (1) and double integrating along the longitudinal x-axis yields the static deflection of the smart structure. It is in the form of:

$$\delta(x,V) = d_{31} \frac{V}{4t_p(Z_{na} - t_b)} x^2 \qquad (3)$$

The position of the neutral axis can be determined by deriving the equation of the total bending stress in the composite structure and setting it equal to zero [6], i.e.:

$$E_b b \int_{-Z_{na}}^{t_b - Z_{na}} z\,dz + E_a b \int_{t_b - Z_{na}}^{t_a + t_b - Z_{na}} z\,dz + E_p b \int_{t_a + t_b - Z_{na}}^{t_p + t_a + t_b - Z_{na}} z\,dz = 0 \qquad (4)$$

where b is the width of the smart structure at the location of the piezoelectric patches, and t_a and t_p are the thicknesses of the adhesive and piezolayers, respectively. This gives:

$$Z_{na} = \frac{E_p t_p^2 + 2E_p t_p t_a + 2E_p t_p t_b + E_a t_a^2 + 2E_a t_a t_b + E_b t_b^2}{2E_p t_p + 2E_a t_a + 2E_b t_b} \qquad (5)$$

Substituting (5) into (3) represents the equation of the deflection as function of the voltage and the material and geometric properties of the smart structure. It has the form:

$$\delta(x,V) = d_{31} \frac{V}{2t_p} \frac{(E_b t_b + E_a t_a + E_p t_p)}{[E_a t_a^2 + E_p(t_p^2 + 2t_p t_a) - E_b t_b^2]} x^2 \qquad (6)$$

If the adhesive layer is neglected, (6) becomes:

$$\delta(x,V) = d_{31} \frac{V}{2t_p} \frac{(E_b t_b + E_p t_p)}{(E_p t_p^2 - E_b t_b^2)} x^2 \qquad (7)$$

Townley [7] derived the equation for tip displacement without considering the Young's modulus of the various materials of the smart structure:

$$\delta(x,V) = d_{31} \frac{V}{2t_p(t_b + t_p)} x^2 \qquad (8)$$

B. Deriving the Relation between Piezoelectric Actuator Voltage and Moment

Researchers who studied the stress-strain-voltage behavior of piezoelectric elements bonded to beams observed that the actuator voltage can be represented by two equal moments with opposite directions concentrated at both actuator ends [8]. This will mainly help in the finite element modeling of the smart structure since only the mechanical degrees of freedom will then be considered. Another reason behind investigating the relation between actuator moments and voltage is to use the equivalent moments as input to the controller. A linear

behavior of the piezoelectric material and a one dimensional deformation are assumed.

The longitudinal stress σ_p, defined by using Hooke's law, generates a bending moment M_p around the neutral axis of the smart structure. It is calculated by using:

$$M_p = \int_{t_a + t_b - Z_{na}}^{t_p + t_a + t_b - Z_{na}} \sigma_p b z\,dz = 0, \text{ where } \sigma_p = E_p \frac{d_{31}}{t_p} V \qquad (9)$$

Integrating (9) and substituting for Z_{na} delivers the relation between the actuator moments and voltage:

$$M_p = \frac{E_p E_a(t_p t_a + t_a^2) + E_p E_b(t_b^2 + t_p t_b + 2t_a t_b)}{E_p t_p + E_a t_a + E_b t_b} \frac{d_{31} b}{2} V \qquad (10)$$

C. Creating the FE Model and the SE Model of the Smart Structure

The FE package SAMCEF® is used to create the FE and SE models of the smart structure. In order to find the best representative for the FE model, the optimal type and size of the finite elements must be selected. Thus, a modal analysis of the real system is indispensable. Since the number of nodes in the FE model might induce a problem in designing and implementing the controller later, the SE technique, also known as Craig-Bampton method, is used to reduce the number of nodes in the FE model. It implements that the basic structure can be split into a certain number of substructures and the nodes are classified into retained nodes and condensed nodes [9]. The retained nodes are usually those where boundary conditions are applied, or where stresses, displacements, etc. are imposed or measured. The remaining nodes will be condensed. For the SE model created in this work, there are only four retained nodes; the first node corresponds to the clamp constraint, the second and the third nodes correspond to the location of the piezoactuator moments and the fourth node corresponds to the sensor that measures the tip deflection or vibration.

D. Design and Implementation of the Lyapunov Function Linear Controller

Although there is no general procedure for constructing a Lyapunov function, any function can be considered a Lyapunov function if its positive definite is equal to zero at the equilibrium state and its derivative is less than or equal to zero. For that reason, the energy equation of a thin Bernoulli-Euler beam [10], modeled as a single FE, with actuator length h and left point coordinate of actuator x_i in a one-dimensional system is considered as a Lyapunov function candidate. The derivative of this function has the form:

$$\dot{U} = \int_{x_i}^{x_i + h} \left[\rho \dot{y} \ddot{y} + EI \frac{\partial^2 y}{\partial x^2} \frac{\partial}{\partial t}\left(\frac{\partial^2 y}{\partial x^2}\right) \right] dx \qquad (11)$$

For (11) to become smaller than zero and after applying the equations for bending moment and shear force for a uniform Bernoulli-Euler beam [11], the following form should apply:

$$M_p = -k \left(\dot{y}'_{x_i + h} - \dot{y}'_{x_i} \right) \qquad (12)$$

where k is a positive number whose value is determined by trial and error. The terms inside the parenthesis indicate the difference in angular velocity between the second and the third node of the SE model. This difference is multiplied by a constant and then fed back as a bending piezoelectric moment at each side of the actuator. Both the design and implementation of this type of linear active controllers are discussed in detail elsewhere [7].

III. Results and Discussions

A. Validation of the Equation for Static Deflection

The relation between static deflection and applied voltage is validated by comparing the outcome of (6) to similar results from literature. As a reference, the work done by Townley [7] has been utilized.

The cantilevered unimorph beam used for analysis is approximately 400 μm long and it has a width of 100 μm. The thickness of the piezoelectric layer is $t_p = 1$ μm and that of the beam is $t_b = 0.2$ μm. The piezoelectric layer is made of Aluminum Nitride (AIN), and the beam layer is made of Platinum. The relevant material properties used are shown in Table I. The adhesive layer is ignored.

TABLE I
MATERIAL PROPERTIES OF THE CANTILEVERED UNIMORPH BEAM

Layer	E (GPa)	ρ (kg/m^3)	d_{31} (pC/N)
AIN	292	3200	-1.98
Platinum	168	21450	-

The tip deflection at the end of the beam is depicted in Fig. 2, together with the calculated and the measured results from Townley [7]. A very good agreement between the results is noticed.
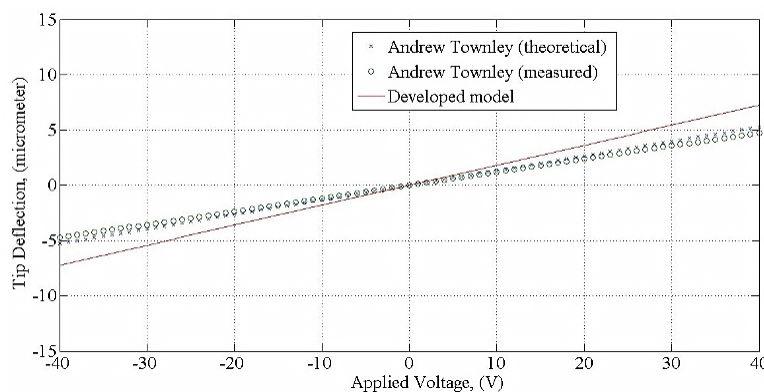

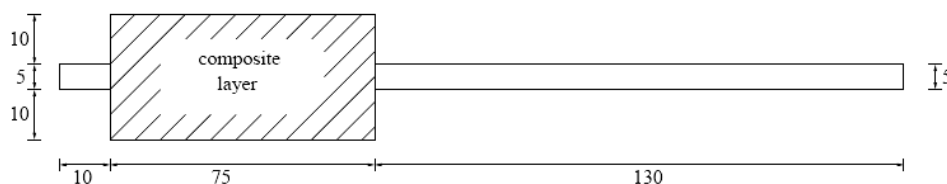Fig. 2 Applied voltage vs. deflection of developed and other models

Fig. 3 A detailed geometry of the smart structure (in mm)

B. Validation of the FE and the SE Models

To check if the FE and the SE models represent the real physical model of the smart structure, an example of a smart structure is shown for comparison in Fig. 3. Only the composite layer consists of the steel beam layer, the adhesive layer, and the piezoelectric layer, respectively, while the rest is made of steel. The thickness and material properties for each layer are depicted in Table II.

From the modal analysis and by comparing the results of the first and the second eigenfrequency to the experiment, it has been concluded that the optimal element size is 1 mm. The results of the modal analysis are shown in Table II. Regarding the FE type, shell elements based on the first order shear deformation theory have been used. Performing the modal analysis for the FE model, as well as for the SE model and comparing the results with the experimental results have shown that the difference in eigenfrequencies is less than 3% as shown in Table III. This proves that the SE model, which is composed of only four retained nodes, is a good representative of the real physical model of the smart structure.

TABLE II
MATERIAL COMPONENTS OF EACH LAYER OF THE SMART STRUCTURE

Material	Beam Steel	Adhesive epoxy resin	Piezoelectric Actuator PIC 151
Thickness (mm)	0.5	0.036	0.25
Density (kg/m^3)	7900	1180	7800
Elastic Modulus (MPa)	210000	3546	66667

TABLE III
VALIDATION OF FE TYPE AND SIZE BASED ON MODAL ANALYSIS

No. of eigenfrequency	FE Model	SE Model	Experimental
1	13.81 Hz	14.25 Hz	13.26 Hz
2	42.67 Hz	43.41 Hz	41.14 Hz

C. Demonstrating the Effectiveness of the Lyapunov Active Controller

The smart structure is initially excited with its first eigenfrequency for about 20 seconds, and after that it is left to vibrate freely. Exactly at that moment, the controller, which is based on the Lyapunov stability theorem, is activated. The results are represented in Fig. 4.

It is obvious that the controller reduced the time of free body vibration from 20 seconds to about 2 seconds only, and this demonstrates its effectiveness.
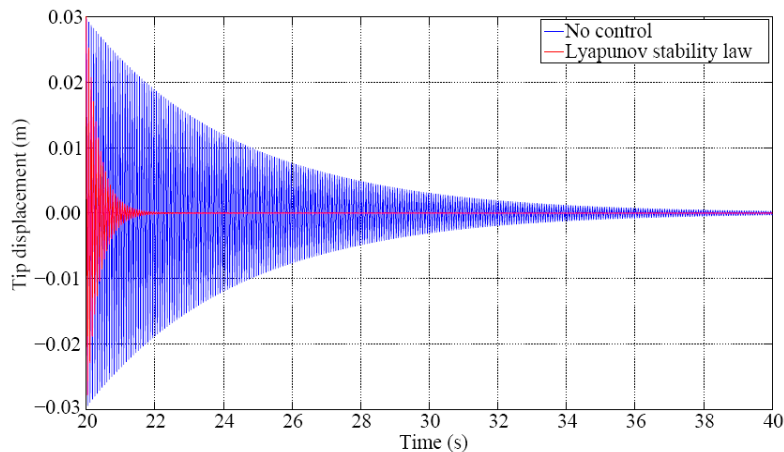

Fig. 4 Tip displacement vs. time with and without controller

IV. CONCLUSION

In this paper, a smart structure was proposed, and the relation between static deflection and actuator voltage was derived and verified. The relation between the piezoelectric moment and voltage was also investigated, and a finite element model of the structure was created with a reduced number of nodes. An active linear controller based on the Lyapunov stability theorem was designed and implemented successively to attenuate the vibration of the smart structure once it is excited with its first eigenfrequency.

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