

# Effects of Duct Geometry, Thickness and Types of Liners on Transmission Loss for Absorptive Silencers

M. Kashfi, K. Jahani

**Abstract**—Sound attenuation in absorptive silencers has been analyzed in this paper. The structure of such devices is as follows. When the rigid duct of an expansion chamber has been lined by a packed absorptive material under a perforated membrane, incident sound waves will be dissipated by the absorptive liners. This kind of silencer, usually are applicable for medium to high frequency ranges. Several conditions for different absorptive materials, variety in their thicknesses, and different shapes of the expansion chambers have been studied in this paper. Also, graphs of sound attenuation have been compared between empty expansion chamber and duct of silencer with applying liner. Plane waves have been assumed in inlet and outlet regions of the silencer. Presented results that have been achieved by applying finite element method (FEM), have shown the dependence of the sound attenuation spectrum to flow resistivity and the thicknesses of the absorptive materials, and geometries of the cross section (configuration of the silencer). As flow resistivity and thickness of absorptive materials increase, sound attenuation improves. In this paper, diagrams of the transmission loss (TL) for absorptive silencers in five different cross sections (rectangle, circle, ellipse, square, and rounded rectangle as the main geometry) have been presented. Also, TL graphs for silencers using different absorptive material (glass wool, wood fiber, and kind of spongy materials) as liner with three different thicknesses of 5mm, 15mm, and 30 mm for glass wool liner have been exhibited. At first, the effect of substances of the absorptive materials with the specific flow resistivity and densities on the TL spectrum, then the effect of the thicknesses of the glass wool, and at last the efficacy of the shape of the cross section of the silencer have been investigated.

**Keywords**—Transmission loss, absorptive material, flow resistivity, thickness, frequency

## I. INTRODUCTION

NOISE has become one of the main pollutants of modern the city life. Lots of methods exist for controlling the noise of duct systems like ventilation systems in automobiles. One of the common practical silencers for usage in duct systems is the absorptive silencer with the rounded rectangle duct cross section. Absorptive silencer is a kind of passive noise control methods. Controlling widest range of (medium to high) frequency is the main target in this paper. One of the advantages of absorptive silencers is producing low back pressure, and also it easily combines with other kind of silencers such as plate silencer in order to control wider range of frequency, from low to high [1]. Approximate optimizations of geometry, thickness, and flow resistivity of

the absorptive materials have been analyzed in this paper. TL happens because of the sound dissipating. Dissipating of sound will occur by porous absorptive materials, in which the acoustic energy of sound converts to the heat energy, and then it transfers to the environment. An analysis of the sound field inside a two-dimensional (2-D) circular lined duct was presented by Scott [2]. A completely arbitrary geometry, neglecting mean flow, has been addressed by Tarnow and Pommer [3]. TL spectrum condition depends on flow resistivity and the thickness of the absorptive materials, and also shape of the expansion chamber (cross section of the duct). In the present paper, in order to finding optimized condition of the TL spectrum, flow resistivity and thicknesses of the absorptive materials have been changed on the configuration of the main geometry of rounded rectangle duct cross section, shown in Fig. 1. Geometry and governing equations have been described in Section III. In Section IV, results have been investigated. Finally, in Section V, this work has been summarized and the conclusions have been written.

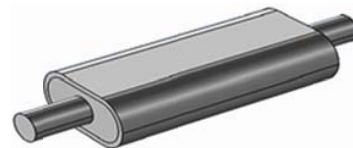


Fig. 1 Main geometry of the expansion chamber

## II. MATERIAL AND METHODS

At first the effect of the flow resistivity and thickness of the three kinds of absorptive materials include glass wool, wood fiber, and a kind of spongy material, on TL spectrum on an absorptive silencer with a cross section of rounded rectangle has been investigated, using FEM. Next, the volume of the materials has been kept in a constant value and the cross section of the main geometry has been changed to the shapes of rectangle (Fig. 2 (a)), ellipse (Fig. 2 (b)), circle (Fig. 2 (c)), square (Fig. 2 (d)), and then rounded rectangle (Fig. 2 (e)) as the main geometry.

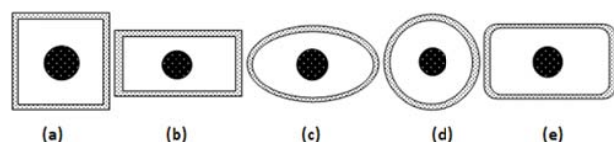


Fig. 2 Cross sections of absorptive silencer

Five different geometries for the silencer and other determinant parameters have been investigated in order to

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attain the best TL spectrum. So, TL spectrums have been presented separately for each condition. Sound assumed to be as plane wave condition at inlet and outlet regions of the silencer. Sound attenuation graphs that have been reached with applying FEM, have been compared. This model describes the pressure-wave propagation in an absorptive silencer for an automobile ventilation system. The approach is generally applicable to analyze the damping of propagation of harmonic pressure waves. The purpose of this model is showing how to analyze damping in acoustic pressure. The main output is the TL for the frequency range from 50 Hz to 1500 Hz.

### III. CALCULATION

The governing equations have been studied in this section are assumed to be for an absorptive silencer with circular cross section without considering mean flow. Fig. 3 shows a 2-D absorptive silencer with inlet and outlet regions and circular cross section.

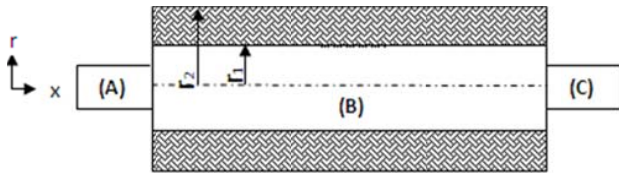


Fig. 3 2D absorptive silencer shows inlet, outlet, duct, and absorptive material regions

Equations that have been used for this specific cross section are extendable with few partial changes for other shapes of cross sections, which here have not been considered. Homogeneous and macroscopic porous materials by the density of  $\rho$ , sound speed of  $c$ , and flow resistivity of  $R_f$  have been employed in this paper. The material is between radius  $r_1$  and  $r_2$  in central part of the chamber. In the silencer chamber, a perforated membrane separates the mean flow in the duct and the absorptive materials. Inlet, duct, outlet, and absorptive material domains have been denoted by A, B, C and D, respectively. Presented theory is based on the prior study [4], [5]. Perforated membrane effect has been disregarded in this paper.

#### A. Formulation for a 2-D Absorptive Silencer

Acoustic wave equation in region B has given by (1) [4], [6]:

$$\frac{1}{c_0^2} \frac{\partial^2 P}{\partial t^2} - \nabla^2 P = 0 \quad (1)$$

where,  $c_0$  is the sound speed,  $P$  is the acoustic pressure, and  $t$  is the time. If time dependence assumed as  $e^{i\omega t}$  (where  $\omega$  is radiation frequency), (1) would be rewritten as (2):

$$k_0^2 P - 2iMk_0 \frac{\partial P}{\partial x} + (1 - M^2) \frac{\partial^2 P}{\partial x^2} + \frac{1}{r} \frac{\partial P}{\partial r} + \frac{\partial^2 P}{\partial r^2} = 0 \quad (2)$$

where,  $k_0 (= \omega/c_0)$  is the wave number and  $M$  is the Mach number of the duct mean flow (that is hereafter neglected).

Helmholtz equation solution for wave propagation in regions A and C can be expressed as (3) [7], [8]:

$$P(r, x) = \sum_{n=1}^{\infty} (A_n^+ \exp(-ik_{x,A,n} x) + A_n^- \exp(ik_{x,A,n} x)) \psi_{A,n}(r) \quad (3)$$

where  $i = (\sqrt{-1})$  is the imaginary unit,  $n$  is the mode number,  $(r, x)$  is the cylindrical coordinates,  $A_n^+$  and  $A_n^-$  are the wave propagation coefficients. Here  $\psi_{A,n}(r)$  is the eigenfunction for duct that is attained by zeroth order Bessel function of the first kind  $j_0(k_{r,A,n} r)$ , where radial wave number  $k_{r,A,n}$  satisfies the rigid wall boundary condition. Axial wave number  $k_{x,A,n}$  for mode  $n$  has been obtained by (4),

$$k_{x,A,n}^2 = k_0^2 - k_{r,A,n}^2 \quad (4)$$

Acoustic velocity in  $x$  direction in region A, using (3) and linearized momentum has been written as (5),

$$U(r, x) = \frac{1}{\rho_0 \omega} \sum_{n=1}^{\infty} k_{x,A,n} (A_n^+ \exp(-ik_{x,A,n} x) + A_n^- \exp(ik_{x,A,n} x)) \psi_{A,n}(r) \quad (5)$$

#### B. Model Equations

This model solves problem in the frequency domain. The model equation is a slightly modified version of the Helmholtz equation for the acoustic pressure,  $(P)$  [9], [10]:

$$\nabla \cdot \left( -\frac{\nabla P}{\rho} \right) - \frac{\omega^2 P}{c_c^2 \rho} = 0 \quad (6)$$

where  $\rho$  is the density,  $c_c$  equals the complex speed of sound, and  $\omega$  gives the angular frequency. In the absorbing glass wool (as the main absorptive material in this model), the damping enters the equation as a complex speed of sound,  $c_c = \omega/k_c$  and a complex density,  $\rho_c = k_c Z_c / \omega$ , where  $k_c$  is the complex wave number and  $Z_c$  equals the complex impedance.

For a highly porous material with a hard skeleton, these parameters have been given as (7) and (8),

$$k_c = k_a \cdot \left( 1 + 0.098 \left( \frac{\rho_a f}{R_f} \right)^{-0.7} - 0.189i \left( \frac{\rho_a f}{R_f} \right)^{-0.595} \right) \quad (7)$$

and

$$Z_c = Z_a \cdot \left( 1 + 0.057 \left( \frac{\rho_a f}{R_f} \right)^{0.734} - 0.087i \left( \frac{\rho_a f}{R_f} \right)^{-0.732} \right) \quad (8)$$

where,  $R_f$  is the flow resistivity, also  $k_a = \omega/c_a$  and  $Z_a = \rho_a c_a$  are the free-space wave number and impedance of air, respectively. For materials like glass wool, Bies and Hansen give an empirical correlation [10], [11]:

$$R_f = \frac{3.18 \times 10^{-9} \cdot \rho_{ap}^{1.53}}{d_{av}^2} \quad (9)$$

where,  $\rho_{ap}$  is the apparent density of material and  $d_{av}$  is the mean fiber diameter. Lightweight glass wool of  $\rho_{ap}=12 \text{ kg/m}^3$  and  $d_{av}=10 \text{ }\mu\text{m}$  has been used in this model as the main absorptive material.

### C. TL in an Absorptive Silencer

A significant parameter for every silencer is the TL or sound attenuation. It is defined as the ratio between the incoming and the outgoing acoustic energy. The TL (given in dB) of the acoustic energy is [10],

$$TL = 10 \log_{10} \left( \frac{E_{in}}{E_{out}} \right) \quad (10)$$

Here,  $E_{in}$  and  $E_{out}$  mark the incoming power at the inlet and the outgoing power at the outlet, respectively. Each of these quantities can be calculated as an integral over the corresponding surface,

$$E_{out} = \int \frac{|P|^2}{2\rho c_0} dA \quad (11)$$

Also,

$$E_{in} = \int \frac{P_0^2}{2\rho c_0} dA \quad (12)$$

Satisfactory range should be above a criterion value of 10 dB. It means that, if the amount of TL is more than 10 dB the performance would be acceptable [10].

### D. Boundary Conditions

There are three boundary conditions used in this model [10]. At the solid boundaries, which are the outer walls of the expansion chamber, the model uses sound hard-wall boundary condition. The condition imposes that the normal velocity at the boundary is zero, and is described mathematically by,

$$\left( -\frac{\nabla P}{\rho} \right) \cdot n = 0 \quad (13)$$

The boundary condition at the inlet includes a combination

of an incoming imposed plane wave and an outgoing radiating plane wave. Mathematically, it is formulated as,

$$n \cdot \frac{1}{\rho_0} \nabla P + ik \frac{P}{\rho_0} + \frac{i}{2k} \Delta_r P = \left( \frac{i}{2k} \Delta_r P_0 + (1 - (k \cdot n)) ik \frac{P_0}{\rho_0} \right) \exp(-ik(k, r)) \quad (14)$$

In (14),  $P_0$  represents the applied outer pressure,  $\Delta_r$  is the boundary tangential Laplace operator.

At the outlet boundary, the model specifies an outgoing radiating plane wave,

$$n \cdot \frac{1}{\rho_0} \nabla P + i \frac{k}{\rho_0} P + \frac{i}{2k} \Delta_r P = 0 \quad (15)$$

## IV. RESULTS AND DISCUSSION

Presented graphs, have compared TL spectrums in all mentioned situations. 3-D FEM calculation has been applied for gaining these graphs. All geometries have been analyzed in frequency range of interest (50 Hz-1500 Hz).

### A. Case Study 1: Consideration about Various Absorptive Materials with Constant Thickness for the Main Geometry (Rounded Rectangle)

The performances of absorptive materials have been compared in Fig. 4. In this case, the thicknesses of absorptive materials assumed to be constant at 15 mm. Absorptive materials in Fig. 4 are glass wool with density of  $\rho=12 \text{ kg/m}^3$  and flow resistivity of  $R_f=1425 \text{ Pa.s/m}^2$ , uncompressed wood fiber with density of  $\rho=50 \text{ kg/m}^3$  and flow resistivity of  $R_f=5000 \text{ Pa.s/m}^2$  and a kind of spongy material with density of  $\rho=8 \text{ kg/m}^3$  and flow resistivity of  $R_f=650 \text{ Pa.s/m}^2$ . The main geometry of the silencer has been illustrated in Figs. 1 and 2 (e).

As shown in Fig. 4, it is obvious that uncompressed wood fiber has the best performance between the proposed common absorptive materials. However, since the uncompressed wood fiber is rather dense, glass wool has been preferred in some cases, which being lightweight is considered as a determinant parameter. The frequency ranges above the criterion value ( $\geq 10 \text{ dB}$ ) are 657-1500 Hz, 939-1500 Hz, and 1234-1500 Hz for sponge, glass wool, and wood fiber, respectively. So, wood fiber has the widest frequency range between these three absorptive materials.

### B. Case Study 2: Consideration about Variation of Thicknesses of the Constant Absorptive Material for the Main Geometry

Different thicknesses of an absorptive material have been compared in Fig. 5. In all cases the absorptive material assumed to be the same (glass wool of mentioned density and flow resistivity). Thicknesses of 5 mm, 15 mm, and 30 mm have been analyzed in Fig. 5. The geometry is the same as illustrated in Figs. 1 and 2 (e).

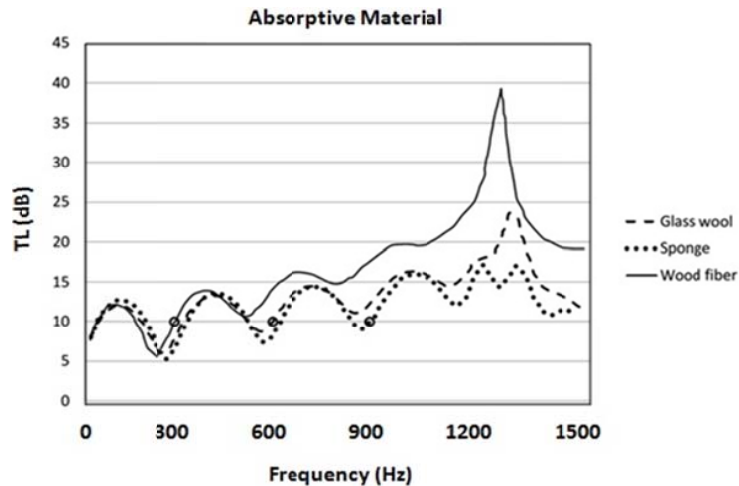


Fig. 4 Comparison between TL spectrums of absorptive silencer using different absorptive materials

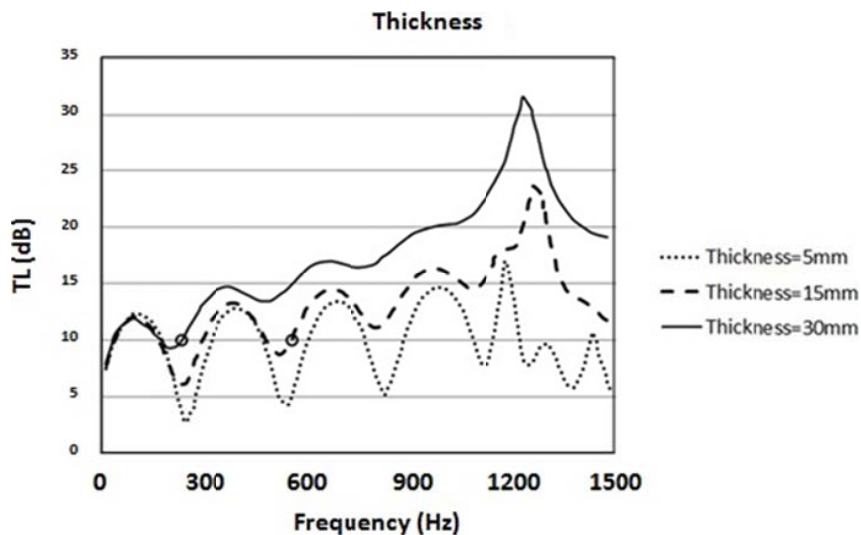


Fig. 5 Comparison between TL spectrums of absorptive silencer using different thicknesses of glass wool

As observed in Fig. 5, as thickness increases the performance of silencer has been improved. However, since in some situations the installation space is limited, with respect to the situation and the target, thinner thicknesses have been preferred. The frequency ranges above the criterion value ( $\geq 10$  dB) are 939-1500 Hz and 126-1500 Hz for glass wool liner with thicknesses of 15mm and 30mm, respectively. Note that the thickness of 5mm for the glass wool as the liner has some rupture in the frequency range above the criterion value, that it is unacceptable.

#### C. Case Study 3: Consideration about the Different Geometries of the Silencer with Constant Volume of an Absorptive Material

Various cross sections of expansion chambers of the absorptive silencers have been compared in Figs. 6 (a)–(e). In all cases the glass wool as the main absorptive material, has the constant volume (cross section area ( $10000 \text{ mm}^2$ )  $\times$  length of the silencer). Figs. 6 (a)–(e) show the TL spectrums for

square, rectangle, ellipse, circle, and rounded rectangle (main geometry) cross sections, respectively. In all figures, comparisons have been done between empty expansion chamber (without liner) and expansion chamber with existing of the glass wool as the main absorptive material.

As shown in Figs. 6 (a)–(e), absorptive silencer by rectangular cross section has the widest frequency range (total span 1284 Hz from whole 1500 Hz span) of TL above the criterion value. Rounded rectangle (the main geometry) (total span = 939 Hz or 5561-1500 Hz), ellipse (total span = 928 Hz or 572-1500 Hz) and circle (total span = 921 Hz or 579-1500 Hz) are seated in the next places, and after all, absorptive silencer of square cross section (total span = 811 Hz or 689-1500 Hz) has the worst performance between the mentioned geometries. Note that, the absorptive material volumes in all of the various geometry cases are equal, in order to a logical comparison. Also, using each of these shapes might be related to the installation space.

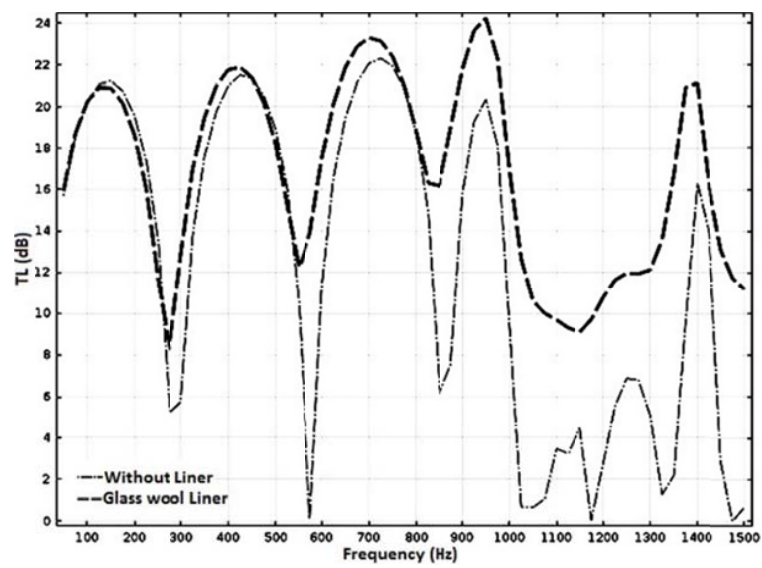


Fig. 6 (a) TL spectrum for an absorptive silencer with square cross section

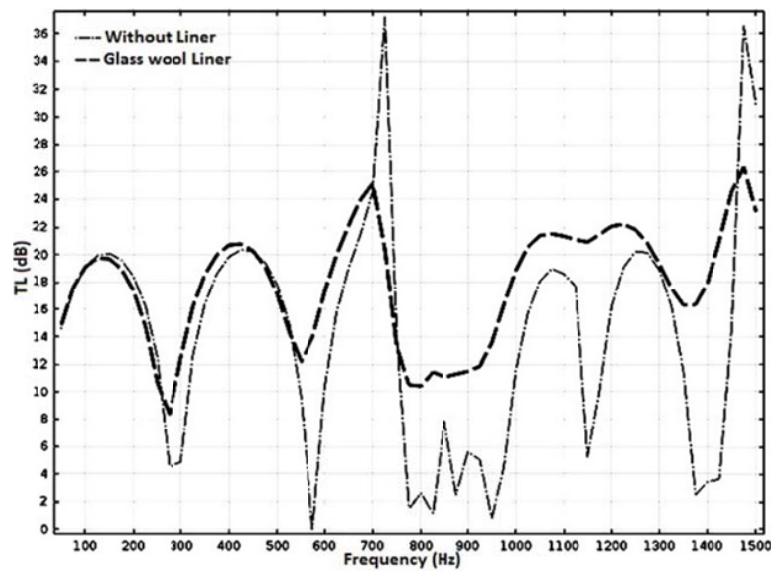


Fig. 6 (b) TL spectrum for an absorptive silencer with rectangle cross section

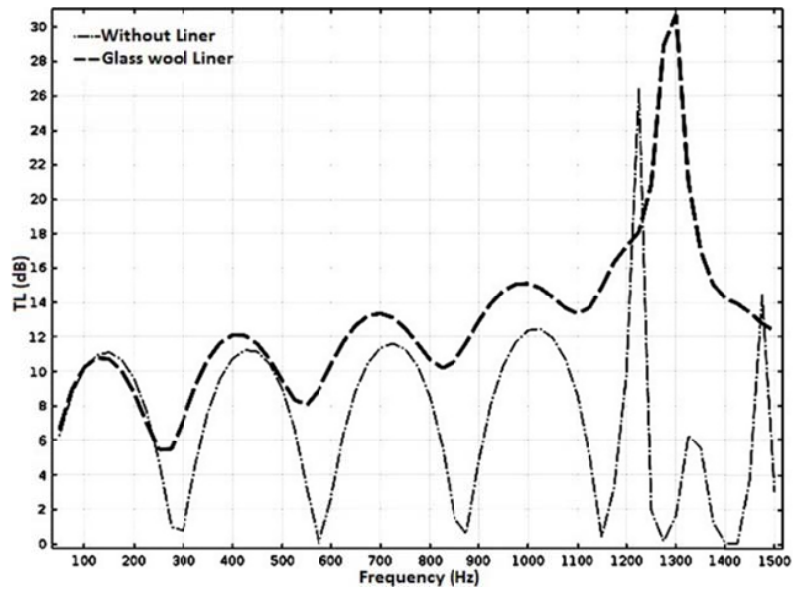


Fig. 6 (c) TL spectrum for an absorptive silencer with ellipse cross section

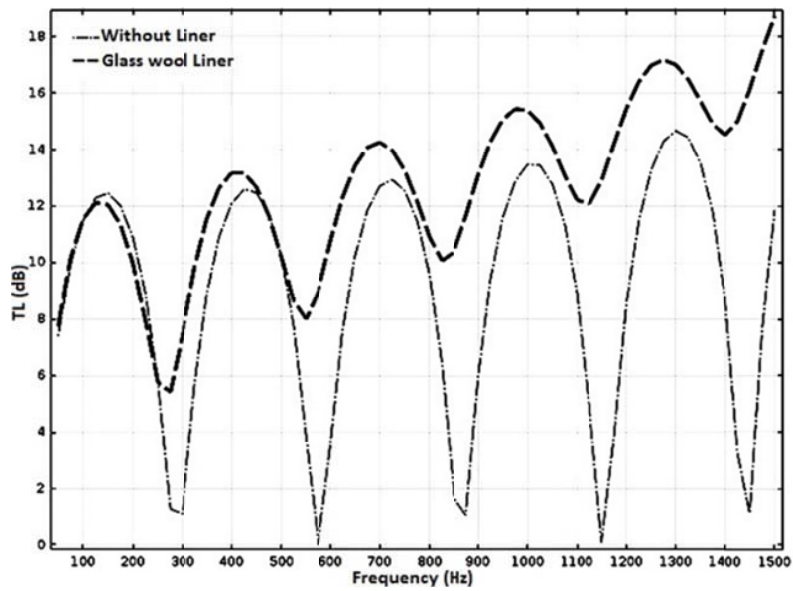


Fig. 6 (d) TL spectrum for an absorptive silencer with circle cross section



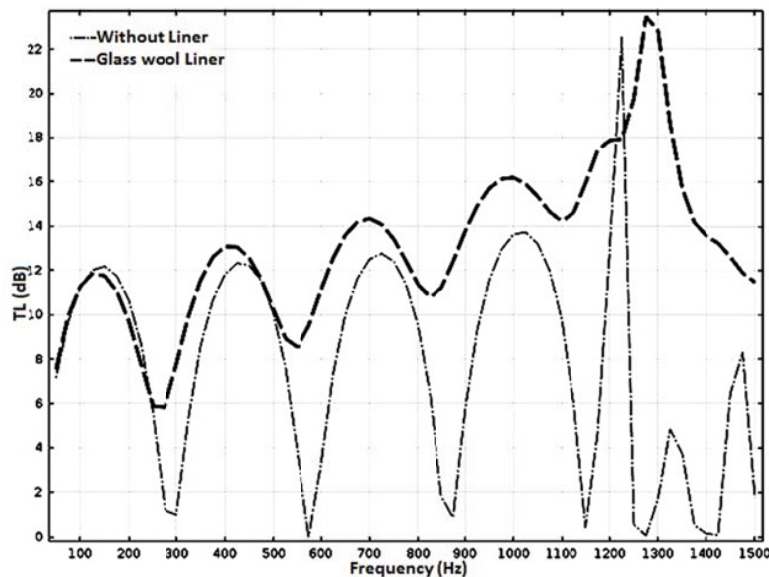


Fig. 6 (e) TL spectrum for an absorptive silencer with rounded rectangle cross section

#### V.CONCLUSION

Different geometries of the expansion chambers, types and thicknesses of absorptive materials for absorptive silencers, have been compared as the three important parameters in this paper. As deduced from the diagrams (Figs. 4-6), absorptive silencers show their best performance at medium to high frequencies, so the wider range of frequency means better performance. Here, frequency range has been identified as the frequencies that are above criterion value (10 dB) without any rupture. The higher frequencies have been considered as a constant value (=1500 Hz) in this paper, and the lower frequencies have been shown in Figs. 4 and 5 as open circles. Deducing lower frequencies from the higher frequency (=1500 Hz) gives the frequency range of TL above the criterion value. FEM results have been presented as the diagrams. As inducted from the graphs, the best performance between three considered absorptive materials as liner for the absorptive silencer refers to wood fiber, and the best geometry for the best TL refers to the absorptive silencer of rectangular cross section. Also, as the thickness of the liner increase the performance of the silencer increase too. With comparing Figs. 4 and 5 it can be recognized that using glass wool with thickness of 30 mm has better performance than wood fiber with thickness of 15 mm. So, if the installation space is not so important thicker liner of glass wool would be preferred. Also, absorptive silencer by rectangular cross section has the widest frequency range above the criterion value. So, comparing rounded rectangle, this geometry can be preferred if the installation space be large enough. Other limitations like installation space, weight of the device and so on, should be considered in designing and selecting silencers.

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