# Modeling and Simulation of Flow Shop Scheduling Problem through Petri Net Tools

Joselito Medina Marin, Norberto Hernández Romero, Juan Carlos Seck Tuoh Mora, Erick S. Martinez Gomez

Abstract—The Flow Shop Scheduling Problem (FSSP) is a typical problem that is faced by production planning managers in Flexible Manufacturing Systems (FMS). This problem consists in finding the optimal scheduling to carry out a set of jobs, which are processed in a set of machines or shared resources. Moreover, all the jobs are processed in the same machine sequence. As in all the scheduling problems, the makespan can be obtained by drawing the Gantt chart according to the operations order, among other alternatives. On this way, an FMS presenting the FSSP can be modeled by Petri nets (PNs), which are a powerful tool that has been used to model and analyze discrete event systems. Then, the makespan can be obtained by simulating the PN through the token game animation and incidence matrix. In this work, we present an adaptive PN to obtain the makespan of FSSP by applying PN analytical tools.

**Keywords**—Flow-shop scheduling problem, makespan, Petri nets, state equation.

## I. INTRODUCTION

AFMS is a discrete event dynamic system that is composed of jobs and shared resources [1]. FMS engineers The typical problem that engineers faced when they are either designing a FMS or planning the master production plan for the FMS, is how they should make the best sequence of jobs in the FMS in order to carry all operations out in the minimum time [2], [3]. This problem is called FSSP, which is a combinatorial problem classified as NP-hard [4]. The makespan is the time that all the jobs are processed in the FMS, and it depends on the order that all the tasks are performed.

There have been published several research papers about finding the minimum value of makespan in the FSSP. For instance, a D.S. Palmer proposed a method to find an acceptable sequence in less time than exhaustive search [5]. Another algorithm based on heuristic strategies to find suitable solutions was proposed in [6]. Dannenbring performed a similar work, where he proposed eleven heuristics to solve the FSSP [7]. Nawas proposed an algorithm based on the assumption that jobs with higher processing time must be treated first; his algorithm is applied to static and dynamic sequencing environment [8]. In [9], Taillard applied taboo search to solve FSSP; moreover, he implemented a parallel version of taboo search to improve the algorithm execution time. Framinan and Leisten proposed a heuristic taking into account the optimization of partial schedules; instead of

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optimize the whole schedule [10]. Later, Framinan, Leisten and Ruiz-Usano proposed two multi-objective heuristics, whose objectives to solve are makespan and flowtime minimization [11].

Several metaheuristics have been used to find the minimum value for the makespan, such as Simulated Annealing [12], [13]; Taboo Search [14], [15]; Genetic Algorithms [16]-[18]; Ant Colony Optimization [19]-[20]; Iterated Local Search [21]; and Particle Swarm Optimization Algorithms [22], [23], [27]. These proposals can find reasonable results in less time than exact methods. The main outcome of these methods is that the global minimum could not be found; however, good approximations are obtained in a short time. Thus, all of them need a way to represent the FSSP in order to calculate the makespan. FSSP modeling should be understandable and able to calculate the makespan of a job operations sequence.

FMSs have been modeled via PNs in order to simulate and analyze them. PN theory is adequate to represent in a graphical and mathematical way Discrete Event Systems (DES) such as FMSs, because their dynamic behavior based on event occurrence can be modeled by PN elements (places and transitions) [24]. Moreover, PN theory offers analytical and graphical tools to study the modeled systems, based on the relationship among the FMS resources denoted as PN elements.

One important point in search methods is the calculus of the makespan, taking into account a certain processing order of the tasks. In this paper, we propose the use of an adaptive timed PN to calculate the makespan taking into account the PN state equation.

## II.FSSP

Scheduling tasks in a FMS is a typical combinatorial problem where it is needed to organize the processing of a set of jobs divided in operations, and each operation is carried out in a shared resource [25], [26].

In the FSSP, given the processing times  $p_{jk}$  for each job j on every machine k, and a job sequence  $S = (s_1, s_2, ..., s_n)$  where n jobs (j = 1, 2, ..., n) will be processed by m machines (k = 1, 2, ..., m), so the aim of FSSP is to find a sequence order for operation processing with the minimum value for the makespan. For instance, Table I shows a FMS with three machines, four jobs, and each job has three serial operations.

# III. PNs Concepts

A PN is a graphical and mathematical tool that has been used to model concurrent, asynchronous, distributed, parallel, non-deterministic, and/or stochastic systems.

TABLE I FMS CONFIGURATION WITH OPERATION TIMES

Items	Jobs -	Operation Serial Number		
		$M_1$	$M_2$	$M_3$
Operation time	$J_1$	96	90	35
	$J_2$	74	57	91
	$J_3$	13	5	7
	$J_4$	71	23	38

The graph of a PN is directed, with weights in their arcs, and bipartite, whose nodes are of two types: places and transitions. Graphically, places are depicted as circles and transition as boxes or bars. PN arcs connect places to transitions or transition to places; it is not permissible to connect nodes of the same type. The state of the system is denoted in PN by the use of tokens, which are assigned to place nodes.

A formal definition of a PN is presented as follows [24]: A PN is a 5-tuple,  $PN = (P, T, F, W, M_0)$  where:

- $P = \{p_1, p_2, ..., p_m\}$  is a finite set of places,
- $T = \{t_1, t_2, ..., t_n\}$  is a finite set of transitions,
- $F \subseteq \{P \times T\} \cup \{T \times P\}$  is a set of arcs,
- $W = F \rightarrow \{1, 2, 3, ...\}$  is a weight function,
- $M_0 = P \rightarrow \{0, 1, 2, 3, ...\}$  is the initial marking,
- $P \cap T = \emptyset$  and  $P \cup T \neq \emptyset$ .

The token movement through the PN represents the dynamical behaviour of the system. In order to change the token position, the following transition firing rule is used [24]:

- A transition t ∈ T is enabled if every input place p ∈ P of t has w(p,t) tokens or more. w(p,t) is the weight of the arc from p to t.
- An enabled transition t will fire if the event represented by t takes place.
- 3. When an enabled transition t fires, w(p,t) tokens are removed from every input place p of t and w(t,p) tokens are added to every output place p of t. w(t,p) is the weight of the arc from t to p.

A Timed Place Petri Nets (TPPN) is an extended PN, where a new element is added. It is a six-tuple  $TPPN = \{P, T, F, W, M_0, D\}$ , where the first fifth elements are similar to PN definition presented above, and  $D = \{d_1, d_2, ..., d_m\}$  denotes the time-delay for each place  $p_j \in P$  [28]. Output transitions  $t_i$  for each  $p_j$  will be enabled once the time indicated in  $p_j$  is reached.

## A. Analysis Methods

In this paper, we are applying the matrix equation approach as the analytical method of PN theory in order to calculate de makespan of the FMS modelled.

#### B. Incidence Matrix and State Equation

A *PN* with *n* transitions and *m* places can be expressed mathematically as an  $n \times m$  matrix of integers  $A = [a_{ij}]$ . The values for each element of the matrix are given by:  $a_{ij} = a_{ij}^+ - a_{ij}^-$ , where  $a_{ij}^+$  is the weight of the arc from  $t_i$  to  $p_j$ , and  $a_{ij}^-$  is the weight of the arc from  $p_j$  to  $t_i$ .

The state equation is used to determine the marking of a *PN* after a transition firing, and it can be written as:

$$M_k = M_{k-1} \times A^T U_k, k=1,2,...$$
 (1)

where  $u_k$  is a  $n \times 1$  column vector of n - 1 zeros and one nonzero entries, which represents the transition  $t_j$  that will fire. The nonzero entry is located in the position j of  $u_k$ .  $A^T$  is the transpose of incidence matrix.  $M_{k-1}$  is the marking before the firing of  $t_j$ . And Mk is the reached marking after the firing of  $t_j$  denoted in  $u_k$ .

#### IV. ADAPTIVE TIMED PLACE PN

In this paper we apply an adaptive TPPN (ATPPN), which adds some arcs according to tasks sequence of the FMS.

The formal definition of an ATPNN is as follows: An ATPNN is a seven-tuple  $(P, T, F, W, M_0, D, F_d)$ , where the first six elements are similar to TPNN elements, and the last one,  $F_d$ , is the set of dynamic arcs that change depending on the job operations order.  $F_d \subseteq \{P \times T\} \cup \{T \times P\}$ .  $F \cap F_d = \emptyset$ .

## A. One Operation Modeling

The ATPNN to model one operation  $O_{ijk}$  of a job  $J_i$  processed by machine  $M_k$  is depicted in Fig. 1.

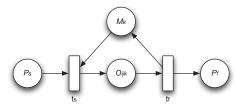


Fig. 1 Operation  $O_{ijk}$  processed by machine  $M_k$  denoted as a PN model

#### B. One Job Modeling

As we mentioned above, one job  $J_i$  is composed of operations  $O_{ijk}$ , so the PN model for each  $J_i$  is a concatenation of its operations  $O_{ijk}$  (Fig. 2).

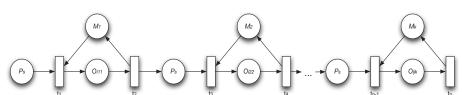


Fig. 2 Operations  $O_{ijk}$  of job  $J_i$  processed by machines  $M_k$  denoted as a PN model

## C. FMS Modeling

In order to model the whole FMS, we add the PN structure for each job  $J_i$  and connect every  $M_k$  place with its corresponding input (output) transition from (to) operation  $O_{ijk}$ . Fig. 3 shows the PN model for the FMS described in Table I.

In Fig. 3, each column corresponds to each job  $J_i$ , and some places have a label d, which denotes the time delay for processing an operation  $O_{ijk}$  in the connected machine  $M_k$ .

### D.Algorithm to Convert a TPPN into an ATPPN

At this time, PN model of Fig. 3 only represents the FMS, but it is also necessary to set the priority in the operations processing by means of arcs connection in the PN model. So, we need to define the elements of  $F_d$  to denote this priority.

First of all, the operations sequence is defined in a row vector  $OS = [os_1 \ os_2 \ ... \ os_{i \not\sim j}]$ , where each OS value corresponds to one operation  $O_{ijk}$ . The following algorithm receives as inputs the row vector OS and the TPPN model, as output of the algorithm we obtain the ATPPN.

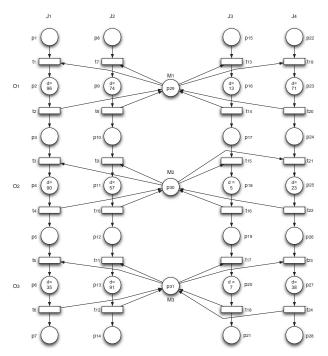


Fig. 3 PN model for the FMS described in Table I

Algorithm TTPN\_into\_ATPPN Input: TPPN, OS
Output: ATPPN

```
1. For q=1 to i \times j
k = \text{machineOf}(OS(q))
add(M_0(k),OS(q))
End For
2. For k = 1 to NumberOfMachines
For i = 1 to NumberOfJobs - 1
p_1 = \text{placeOf}(M_0(k,i))
p_2 = \text{placeOf}(M_0(k,i+1))
t_1 = p_1^{\bullet}
```

In Step 1, a  $k \times i$  matrix called  $M_O$  is created. Operations  $os \in OS(O_{ijk})$  carried out by the same machine  $M_k$  are added in the row k of  $M_O$ . The sequence order for the same machine is taking into account. In Step 2, new arcs  $(t,p) \in F_d$  are created, which connect output transitions of places representing operations  $O_{ijk}$  with the input place of next operation  $O_{ijk}$  in the sequence order defined in  $M_O$ . Moreover, a value 2 is assigned to weight  $W(p_3,t_2)$ , to assure the order in the operations processing.

To illustrate the algorithm result, Fig. 4 shows the ATPPN obtained, based on the operations denoted in Fig. 3 and following the order:  $OS = [O_1J_2, O_1J_4, O_2J_2, O_1J_3, O_2J_4, O_1J_1, O_3J_2, O_3J_3, O_2J_1, O_3J_1, O_2J_3, O_3J_3].$ 

The ATPPN model presented in Fig. 4 is used to calculate the makespan for the sequence defined in vector *OS*.

## V.ALGORITHM TO OBTAIN THE MAKESPAN

The proposed algorithm takes into account the mathematical representation of the ATPPN. In particular, the incidence matrix and the state equation are utilised to obtain the time delay for each  $O_{ijk}$ .

As input data the algorithm needs the ATPPN which includes its input arcs matrix  $(a_{ij}^-)$ , the output arcs matrix  $(a_{ij}^+)$ , the time delays column vector D, and the initial marking  $M_0$ , the total number of jobs (nj), the total number of operations per job (no), and the total number of shared machines (nm).

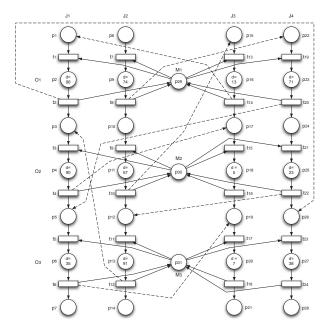


Fig. 4 ATPPN model obtained applying the algorithm TTPN\_into\_ATPPN

Algorithm Calculate\_Makespan Input: ATPPN,  $a_{ij}^{-}$ ,  $a_{ij}^{+}$ , D,  $M_0$ , nj, no, nm

Output: makespan

1. Initialise variables:

```
txj = |T| / nj

pxj = (|P| - nm) / nj
     AT = [0 \ 0 \ ... \ 0]'_{P/ \times 1}
     TV = [0 \ 0 \ ... \ 0]_{1 \times (nj)}
2. ET = enabledTransitions(Mo, aii)
3. \forall et \in ET, U_k (et) = 1
4. While |ET| > 0
     For each t \in ET
        indexT = ceil(t/txj)
        Uk_{tmp}^{T} = [0 \ 0 \ \dots \ 0]'_{/T/\times 1}
        Uk_{tmp}(t) = 1
         \tau = D' \times (a_{ij})' \times Uk_{tmp}
        \tau_{accum} = AT' \times (a_{ij}^-)' \times Uk_{tmp}
        max\_\tau_{accum} = max(TV(indexT), \tau_{accum}/2) + \tau
        For each p ∈ t*
            indexP = ceil(p/pxj)
           If indexT == indexP
              TV(indexP) = \tau_{accum}
           Else
              AT(p) = \tau_{accum}
           End if
        End For
     End For
   M_i = M_{i\_1} + (a_{ij}^-)' * Uk
   \texttt{ET} = \texttt{enabledTransitions}(\texttt{M}_{\texttt{i}}, \texttt{a}_{\texttt{i}\,\texttt{j}}^{\texttt{-}})
  Uk^{T} = [0 \ 0 \ \dots \ 0]'/_{T/\times 1}
  \forall \text{et} \in \text{ET}, \text{Uk(et)} = 1
End While
5. makespan = max(TV)
```

In step 1, four variables are initialised: the number of transitions per job (txi), the number of places per job (pxi), a column vector AT to assign the accumulative time for each place, and a row vector TV utilised to save the time used for each job. Step 2 obtains the enabled transitions (ET) for an initial marking  $M_0$ . Step 3 creates the Uk vector from ET transitions. Step 4 makes an iterative process while the ATTPN is alive, i.e., while there exists at least one enabled transition in the current marking. So, for every enabled transition t, we identify the job  $J_i$  where the transition belongs (indexT), initialise a temporal Uk ( $Uk_{tmp}$ ) to fire transition t. The time delay  $\tau$  corresponding to current operation  $O_{ijk}$  is calculated multiplying the transpose of the time delay vector D' by the transpose of the input arcs matrix  $(a_{ij})$ , and the result is multiplied by the firing vector  $Uk_{tmp}$  taking into account only transition t.

The time accumulated, denoted as  $\tau_{\rm accum}$ , represents the time that the needed machine  $M_k$  has been busy previous to the current operation  $O_{ijk}$ . And it is calculated in a similar way that  $\tau$ , but in this case we use an auxiliary vector AT where the accumulative time for each place is stored, instead of the time delay vector D. Then, we compare both times, the time when the machine Mk is ready and the time when the operation  $O_{ijk}$  is also ready to be processed. The maximum time plus the time delay for operation  $O_{ijk}$  is assigned to  $\max_{\tau_{accum}}$ . For every  $p \in t^{\bullet}$ , if p and t are in the same job line then  $\max_{\tau_{accum}}$  is assigned to the time vector variable TV. On the other hand, if p and t belong to different job lines,  $\max_{\tau_{accum}}$  is assigned to the vector AT.

Finally, the ATPPN marking  $M_i$  changes according to the result of the equation state. From this new marking  $M_i$ , the new enabled transitions are assigned to vector ET and vector Uk is generated from them.

#### VI. CONCLUSIONS AND FUTURE WORK

FSSP is a NP-hard problem that has been analysed applying different kinds of techniques, such as exact models and heuristics strategies. One important calculus in the FSSP is the makespan value, which depends on the sequence of operations for each job and the order of machine utilisation.

In this paper, we use a different way to calculate the makespan by means of mathematical tools of PNs, such as the equation state and the incidence matrix. Firstly, we describe an algorithm to create an ATPPN from a FMS description. The ATPPN arcs indicate the order in which operations  $O_{ijk}$  must be done in each job. Moreover, arcs are connected adequately to set the operations order for each machine. And secondly, the marking evolution by using the state equation is taken into advantage to calculate the makespan. We added a time delay vector in order to consider the processing time for every operation involved in the FMS, and it is included in the matrix operations to be able to obtain the makespan for each job.

As future work, we are including these algorithms as part of a study based on evolutionary computing. Moreover, we are interested in analyse the feasibility of PN tools as part of a heuristic to obtain de minimum makespan in the FSSP.

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#### International Journal of Information, Control and Computer Sciences

ISSN: 2517-9942 Vol:10, No:5, 2016

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