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Combined Analysis of Sudoku Square Designs with Same Treatments

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***Abstract*—**Several experiments are conducted at different environments such as locations or periods (seasons) with identical treatments to each experiment purposely to study the interaction between the treatments and environments or between the treatments and periods (seasons). The commonly used designs of experiments for this purpose are randomized block design, Latin square design, balanced incomplete block design, Youden design, and one or more factor designs. The interest is to carry out a combined analysis of the data from these multi-environment experiments, instead of analyzing each experiment separately. This paper proposed combined analysis of experiments conducted via Sudoku square design of odd order with same experimental treatments.

Keywords**—**Sudoku designs, combined analysis, multi-environment experiments, common treatments

I. Introduction

MANY researchers such as [1], [3] and [6] conduct field experiments by randomized block design (or Latin square design or balanced incomplete block design or Youden design) over different environments such as locations or periods (seasons). The purpose is to investigate effects of treatments over each of the environments, or interaction between treatments and location/periods. Analysis of data from these experiments is usually carried out through joint analysis of all the experiments, instead of individual experiment as discussed in [1]-[6]. Analysis of multi-environment experiments is often referred to as the combined analysis of multiple experiments. As stated in [5] that field experiments are frequently replicated over multiple environments, the most common environments being locations and years. There are two main reasons for multiple-environment trials (METs) other than simply increasing sample sizes through increased replication. The reasons are neither mutually exclusive nor all inclusive. One reason is to estimate the effects and comparative effects of treatments for specific environments or to estimate the effects and comparative effects of treatments over broad populations of environments [5]. The second reason was to estimate the consistency of treatment effects for specific environments or over broad populations of environments, which are discussed in [5], [7], [8].

A method of joint intrablock analysis of a series of randomized block designs having some treatments common to all the experiments was discussed in [1]. Subsequently [2] following the same approach and obtained the joint intrablock analysis of a series of balanced incomplete block designs with some common treatments. The combined analysis of Youden squares and Latin square designs with some common treatments was discussed in [3]. Combined analysis of experiments for one or more factor designs are also reported in [4], [9]. In [9] the procedure for one factor experiments is discussed, but do not describe the test of the average response to treatments over years or locations as indicated in [4]. For two or more factors [4] described the appropriate analyses of combined experiments based on the constraint that the effects of random interactions. This assumption reflects a tradition in the analysis of fixed effects that, in most cases, is not appropriate for a random effect see [6], [12]. It was also presented in [4] that some completes analysis of variance tables for combining balanced experiments that can be used by researchers to quickly and correctly identify source of variation and the appropriate F-ratios in a two or more factors experiments. Analyses for experiments combined across years and locations are presented in [6]. This is similar to [4], although the *F* tests are specified based on the alternative assumption about mixed interactions, i.e., the fixed effects do not sum to zero in a mixed interaction. Analyses are presented for experiments where year is the only random factor, year and location are random, and all factors are random [6]. Although, several field experiments are conducted over two or more locations or years, yet the is no standard reference which provides all of the details necessary for combining analyses of experiments with Sudoku square designs. In addition, the combined analysis of multiple experiments is not common for some designs especially Sudoku square designs discussed by [10], [11]. Two of the references [10], [11] most commonly used for Sudoku square designs do not contain combined analysis of Sudoku designs over Multi-environment experiments.

Sudoku, or Su Doku, is a Japanese word (or phrase) meaning something like Number Place, see [20], [21]. It is widespread puzzle nowadays. It is can be found in most of the newspapers, magazines and in web sites. Many mathematicians such as [17], [18], [22] considered Sudoku as a matrix of which every element appears only once in each of the sub-blocks, rows and columns of the matrix. Such matrices are called Sudoku matrices. Sudoku matrices have an interesting combinatorial application in the design theory as discussed in [10], [11], [14], [15], [19]. For instance, [14] described Sudoku, as experimental designs, build upon the Latin square and accommodate one more component of variation through the concept and formation of 'internal blocks'.

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		1		2	
		1	2	1	2
1	1	A	B	C	D
	2	C	D	A	B
2	1	B	A	D	C
	2	D	C	B	A

Fig. 1 p =2 and q = 2 Sudoku Square

		1			2			3		
		1	2	3	1	2	3	1	2	3
1	1	G	I	B	A	F	H	E	D	C
	2	F	D	C	G	E	B	I	A	H
	3	E	A	H	C	D	I	B	F	G
2	1	D	E	A	F	H	G	C	B	I
	2	I	C	G	D	B	E	A	H	F
	3	B	H	F	I	A	C	G	E	D
3	1	A	F	I	E	G	D	H	C	B
	2	H	G	D	B	C	A	F	I	E
	3	C	B	E	H	I	F	D	G	A

Fig. 2 p = 3 and q = 3 Sudoku Square

TABLE I
ANOVA TABLE FOR THE COMBINED ANALYSIS OF THE SEQUENCE OF SUDOKU SQUARES FOR MODEL 1

Source	df	ss
Experiments	g – 1	SSEE
Treatments	k – 1	SSt
Rows	g(k – 1)	SSr
Columns	g(k – 1)	SSc
Sub-squares	g(k – 1)	SSs
Experiments × treatments	(g – 1)(k – 1)	SSI
Error	g(k – 1)(k – 3)	SSe
Total	gk^2- 1	SST

A Sudoku square design is an experimental design with *k* experimental units that are divided into *k* rows, *k* columns, and *k* sub-blocks (i.e., each box contains *k* experimental units with 1 through *k* treatments) see [16]. In this design each treatment has *k* replications [10]. It was reported by [10] that when compared with the Latin square design, the Sudoku square design adds a term of box effect in the source of variation, and that the soil-environment variation can be better controlled when the Sudoku square is used in a field experiment. It should show a smaller error and more precise tests for treatment means. Particularly, when a field experiment is conducted on the area with lumpy soil variation, the Sudoku square design should be recommended [10]. The Sudoku design of order $k = m^2$ was discussed by [11], [16]. Reference [11] gives a detailed analysis of data for $k = 3^2$. Detail description on how to design and randomized a Sudoku Square are presented in [10]. However, the Sudoku design presented by [10] does not contain the analysis of row-blocks or column-blocks effects. However, [11] extended the design to include analysis of row-blocks and column-blocks effects with four types of data analyses procedures. Recently, [13] discussed a simple method of constructing Quasi-Sudoku-based sliced space-filling designs.

This paper proposed combined analysis of multi-environment experiments conducted via Sudoku square designs of odd order where the treatments are common to the whole set of experiments.

II. ANALYSES

Assume that we have *g* multi-environment experiments each conducted in Sudoku square design, each with k treatments with each treatment occurs only once in each row (column or sub-block) such that $k = m^2$ where both k and m (number of row-block or column-block) are odd see [11]. For example, the square in Fig. 1 is a 4 × 4 Latin square design or 2×2 Sudoku square design, while the square in Fig. 2 is a 9 × 9 Latin square design or 3×3 Sudoku square design. In the figures, p and q denote the number row boxes and the number of column boxes, respectively.

In this study, five models of Sudoku designs were modified and combined analysis from each were discussed. The first model is by [10] and the other four are by [11]. For all the models considered in this study, it is assumed that all effects are fixed. Analyses of random and mixed effects models are not considered in this paper. In addition, only $k = m^2$ number of treatments is discussed and the same treatments are applied to all the environments.

Model 1: The [10] linear model for Sudoku square design is modified to include the multi-environment experiments as follows:

$$y_{ijlmx} = \mu + \theta_x + \alpha_i + \beta_{jx} + \delta_{lx} + \gamma_{mx} + (\alpha\theta)_{ix} + \varepsilon_{(ij)lmx} \begin{cases} i = 1,2,...,k \\ j = 1,2,...,k \\ l = 1,2,...,k \\ m = 1,2,...,k \\ x = 1,2,\ldots,g \end{cases}$$

where $y_{(ij)lm}$ is an observed value of the plot in the *l*th row and *m*th column, subjected to the *i*th treatment, *j*th box of the xth experimental environment (location); μ is the grand mean, α_i, β_j, δ_l, $\gamma_m, \theta_x, (\alpha\theta)_{ix}$, are the main effects of the *i*th treatment, *j*th box, *l*th row, *m*th column, x environment and interaction between treatments and environment, respectively, ε_{ijm} is the random error.

Where SSEE is the sum of squares of the experimental environments, SSt is total treatments sum of squares for all the multi-environment experiments, SSr is total rows sum of squares for all the multi-environment experiments, SSc is total columns sum of squares for all the multi-environment experiments, SSs is total sub-blocks sum of squares for all the multi-environment experiments, SSI is Interaction sum of squares between treatments and the multi-environment experiments. SSe is total sum of squares for all the multi-environment experiments.

Let S_x^r, S_x^c , S_x^s, S_x^e be the xth environment row, column, sub-square and error sum of squares respectively. Then;

$$SSr = \sum_{x=1}^{g} S_x^r$$

$$SSc = \sum_{x=1}^{g} S_x^c$$

$$SSs = \sum_{x=1}^{g} S_x^s$$

$$SS_e = \sum_{x=1}^{g} S_x^e$$

Alternatively, the above sum of square may be obtained using the following:

$$T_1 = \frac{y^2_{\bullet\bullet\bullet\bullet\bullet}}{gk^2},\quad T_2 = \frac{\sum_x y^2_{\bullet\bullet\bullet\bullet x}}{k^2},\quad T_3 = \frac{\sum_l \sum_x y^2_{\bullet\bullet l\bullet x}}{k},$$

$$T_4 = \frac{\sum_m \sum_x y^2_{\bullet\bullet\bullet mx}}{k},\quad T_5 = \frac{\sum_i y^2_{i\bullet\bullet\bullet\bullet}}{gk},\quad T_6 = \frac{\sum_x \sum_i y^2_{i\bullet\bullet\bullet x}}{k},$$

$$T_7 = \frac{\sum_j \sum_x y^2_{\bullet j\bullet\bullet x}}{k} \quad \text{and} \quad T_8 = \sum_i \sum_j \sum_g \sum_m y^2_{ijlmx}.$$

Thus,

$$SSr = T_3 - T_2,\ SSc = T_4 - T_2,\ SSs = T_7 - T_2,$$

$$SSt = T_5 - T_2,\ SSEE = T_2 - T_1,\ SST = T_8 - T_1,$$

$$SSI = T_6 - T_2 - T_5 + T_1, \text{ and}$$

$$SSe = T_8 - T_1 - T_3 - T_4 - T_6 - T_7 + 4T_2$$

The computations of sum of squares for models 2 to 5 are similar to Model 1 and therefore not reported in this paper. In addition, for individual analysis using Sudoku designs with models 2 to 5, see [11].

Model 2: *The modification of Type I* of [11] linear model is as:

$$y_{ij(klpq)x} = \mu + \alpha_{ix} + \beta_{jx} + \tau_k + r_{lx} + c_{px} + s_{qx} + \theta_x + (\tau\theta)_{kx} + \varepsilon_{ij(klpq)x}$$

$i, j = 1, 2, ..., m$ and $k, l, p, q = 1, 2, ..., m^2$, $x = 1, 2, …, g$

where μ is general mean effect, α_{ix} is i^{th} Row block effect in xth environment experiment, β_{jx} is j^{th} Column block effect in xth environment experiment, τ_k is k^{th} Treatment effect, r_{lx} is l^{th} Row effect in xth environment experiment, c_{px} is p^{th} Column effect in xth environment experiment, S_{qx} is q^{th} *sub*-Square effect in xth environment experiment , θ_x is x^{th} environment-experiment, $(\tau\theta)_{kx}$ is interaction between k^{th} treatment and x^{th} environment, and ε_{ij} is the error component with mean zero and variance σ^2.

TABLE II
ANOV TABLE FOR THE COMBINED ANALYSIS OF THE SEQUENCE OF SUDOKU SQUARES FOR MODEL 2

Source	df	SS
Experiments	g – 1	SSEE
Treatments	k – 1	SSt
Rows	g(k – 1)	SSr
Columns	g(k – 1)	SSc
Sub-blocks	g(k – 1)	SSs
Row- blocks	g(m – 1)	SSrb
Column-block	g(m-1)	SScb
Experiments × treatments	(g – 1)(k – 1)	SSI
Error	By subtraction	SSe
Total	$gk^2 – 1$	

Model 3: *The modification of Type II* of [11] linear model is as:

$$Y_{ij(klpq)x} = \mu + \alpha_{ix} + \beta_{jx} + \tau_k + \phi(\alpha)_{l(i)x} + c(\beta)_{p(j)x} + s_{qx} + \theta_x + (\tau\theta)_{kx} + \varepsilon_{ij(klpq)x}$$

$i, j, l, p = 1, 2, ..., m$ and $k, q = 1, 2, ..., m^2$, $x = 1, 2, …, g$

where μ is General mean effect, α_i is i^{th} block (Row) effect, β_j is j^{th} block (Column) effect, τ_k is k^{th} treatment effect, $\phi(\alpha)_{l(i)x}$ is l^{th} row effect nested in i^{th} block (row) of x^{th} environment experiment effect, $c(\beta)_{p(j)x}$ is p^{th} column effect nested in j^{th} block (column) of x^{th} environment experiment effect, Sqx is q^{th} square effect in the x^{th} environment experiment effect, θ_x is x^{th} environment- experiment, $(\tau\theta)_{kx}$ is interaction between k^{th} treatment and x^{th} environment $\varepsilon_{ij}$is Error component with mean zero and variance σ^2.

TABLE III
ANOV TABLE FOR THE COMBINED ANALYSIS OF THE SEQUENCE OF SUDOKU SQUARES FOR MODEL 3

Source	df
Experiments	g – 1
Treatments	k – 1
Rows within row-blocks	gm(m – 1)
Columns within column-blocks	gm(m – 1)
Sub-blocks	g(k – 1)
Row blocks	g(m – 1)
Column blocks	g(m-1)
Experiments × treatments	(g – 1)(gk – 1)
Error	(g – 1)(k – 1)
	By subtraction
Total	$gk^2 – 1$

Model 4: *The modification of Type III* of [11] linear model is as:

$$Y_{ij(klpq)x} = \mu + \alpha_{ix} + \beta_{jx} + \tau_k + \phi_{lx} + c_{px} + s(\alpha)_{q(i)x} + \delta(\beta)_{r(j)x} + \theta_x + (\tau\theta)_{kx} + \varepsilon_{ij(klpqr)x}$$

i, j, q, r = 1, 2,. . .,*m andk, l, p* = 1, 2,. . .,m^2, x = 1, 2, …, g

where μ is the General mean effect, α_{ix} is the i^{th} Row block effect, β_{jx} is the j^{th} Column block effect, τ_k is the k^{th} Treatment effect, ϕ_{lx} is the l^{th} Row effect, c_{px} is the p^{th} Column effect, $s(\alpha)_{q(i)x}$ is the *qth* Horizontal Square effect nested in i^{th} row block effect, $\delta(\beta)_{r(j)}$x is the r^{th} Vertical Square effect nested in j^{th} column block effect, θ_x is the x^{th} environment- experiment, $(\tau\theta)_{kx}$ is the interaction between k^{th} treatment and x^{th} environment, ε_{ij} is the Error component with mean zero and variance σ^2

Model 5: *The modification of Type IV* of [11] linear model is as follows:

$$Y_{ij(klpq)x} = \mu + \alpha_{ix} + \beta_{jx} + \tau_k + r(\alpha)_{l(i)x} + c(\beta)_{p(j)x} + s(\alpha)_{q(i)x} + \delta(\beta)_{r(j)x} + \theta_x + (\tau\theta)_{kx} + \varepsilon_{ij(klpqr)x}$$

i, j, l, p, q, r = 1, 2,. . .,*m and k* = 1, 2,. . .,m^2, x = 1, 2, …, g

where μ is General mean effect, α_{ix} is i^{th} Row block effect, β_{jx} is j^{th} Column block effect, τ_k is k^{th} Treatment effect, $r(\alpha)_{l(i)x}$ is l^{th} Row effect nested in i^{th} row block effect, $c(\beta)_{p(j)x}$ is p^{th} Column effect nested in j^{th} column block effect, θ_x is x^{th} environment- experiment, $(\tau\theta)_{kx}$ is interaction between k^{th} treatment and x^{th} environment.

TABLE IV
ANOV TABLE FOR THE COMBINED ANALYSIS OF THE SEQUENCE OF SUDOKU SQUARES FOR MODEL 4

Source	df
Experiments	g – 1
Treatments	k – 1
Rows	g(k – 1)
column	g(k – 1)
Rows within row-blocks	gm(m – 1)
Columns within column-blocks	gm(m – 1)
Sub-blocks	g(k – 1)
Row- blocks	g(m – 1)
Column- blocks	g(m-1)
Experiments × treatments	(g – 1)(k – 1)
Error	By subtraction
Total	$gk^2 – 1$

TABLE V
ANOV TABLE FOR THE COMBINED ANALYSIS OF THE SEQUENCE OF SUDOKU SQUARES FOR MODEL 5

Source	df
Experiments	g – 1
Treatments	k – 1
Rows within row-blocks	gm(m – 1)
Columns within column-blocks	gm(m – 1)
Sub-blocks within row-blocks	gm(m – 1)
Sub-blocks within column-blocks	gm(m – 1)
Row- blocks	g(m-1)
Column- blocks	g(m-1)
Experiments × treatments	(g – 1)(k – 1)
Error	By subtraction
Total	$gk^2 – 1$

III. CONCLUSION

This paper presents method of combining analyses of Sudoku square designs of odd order when experiments are conducted in g environments with the same treatments. The study modified five linear models that are usually applied to study data from Sudoku design by adding environmental effect term and environments by treatments interaction term to the models. The Sum of squares for rows, columns, and sub-squares of the g environments are obtained by adding all the individual g sum of squares for rows, columns and sub-squares, respectively. Sum of squares for row-blocks, column-blocks, rows within row-blocks, columns within column-blocks, sub-block within row-blocks and sub-blocks within column-blocks are obtained similarly.

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