

Network-Constrained AC Unit Commitment under Uncertainty Using a Bender's Decomposition Approach

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Abstract—In this work, the system evaluates the impact of considering a stochastic approach on the day ahead basis Unit Commitment. Comparisons between stochastic and deterministic Unit Commitment solutions are provided. The Unit Commitment model consists in the minimization of the total operation costs considering unit's technical constraints like ramping rates, minimum up and down time. Load shedding and wind power spilling is acceptable, but at inflated operational costs. The evaluation process consists in the calculation of the optimal unit commitment and in verifying the fulfillment of the considered constraints. For the calculation of the optimal unit commitment, an algorithm based on the Benders Decomposition, namely on the Dual Dynamic Programming, was developed. Two approaches were considered on the construction of stochastic solutions. Data related to wind power outputs from two different operational days are considered on the analysis. Stochastic and deterministic solutions are compared based on the actual measured wind power output at the operational day. Through a technique capability of finding representative wind power scenarios and its probabilities, the system can analyze a more detailed process about the expected final operational cost.

Keywords—Benders' decomposition, network constrained AC unit commitment, stochastic programming, wind power uncertainty.

I. INTRODUCTION

UNIT Commitment (UC) is a crucial short-term decision-making problem in power system operations, whose objective is to determine the least-cost commitment and dispatch of generating units to serve the load. The deterministic form of the UC problem and its solution strategies are extensively documented in the literature, e.g., [1]–[3]. However, the recent increase of stochastic production units, especially wind power, in generation portfolios calls for a stochastic form of the UC problem, instead of a deterministic one. Moreover, a precise modeling of the physical laws characterizing this problem is needed as increasing wind power production generally results in stressed operating conditions. Hence, the need for an AC modeling arises.

Large-scale integration of wind power increases significantly the level of uncertainty [4], hence the need of a stochastic UC approach. The stochastic UC problem was first studied in mid 1990s [5], [6]. More recent works include [7]–

[9]. These approaches embed a dc representation of transmission system, rendering a mixed-integer linear UC problem (network-constrained dc-UC problem), which is generally tractable [10]–[13]. It is worth mentioning that due to the simplifications considered in the dc-UC problems, i.e., the exclusion of voltage magnitude and reactive power constraints, an ex-post verification is required to check that the results obtained are implementable.

A UC problem including an AC network representation (network-constrained ac-UC problem) provides a comparatively more precise description of power system operations, particularly as operating conditions become increasingly stressed due to increasing wind production. However, the ac unit commitment (ac-UC) problem is mixed-integer nonlinear, and thus hard to solve. In the technical literature, there are few works addressing the ac-UC problem. Reference [14] proposes an approach based on Benders' decomposition to solve an AC network-constrained hydrothermal scheduling problem. A security-constrained ac-UC problem is proposed in [15], whose objective is to minimize the system's operating cost while maintaining appropriate security. The solution strategy proposed in [15] is to decompose the original ac-UC problem into a master problem without enforcing the network constraints, and a subproblem to check the feasibility of the master solution from the network constraint point of view. Note that wind power uncertainty is not modeled either in [14] or [15]. References [16], [17] formulate a security-constrained stochastic ac-UC problem under wind power uncertainty, and discusses potential solution techniques, but numerical results are not reported.

This paper presents a network-constrained ac-UC problem in which the wind power uncertainty is characterized by a set of suitable scenarios. To cope with wind power uncertainty, a two-stage stochastic programming model is considered, whose first-stage represents the day-ahead market, and whose second-stage represents the real-time operating conditions involving wind power realizations.

II. AC-UC MODEL

A. Stochastic Programming

Linear programming is a mathematical method to optimize a function given a set of constraints which are linear in nature. But not always are the constraints linear. That let us to non-linear programming where the constraints become non-linear

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& further improving on it comes Stochastic Programming where the constraints are based on random variables. Stochastic programming is a framework for modeling optimization problems involving uncertainty. There are more or less always some unknown parameters involved in real world optimization problems, which are assumed to follow some probability distributions. There are 3 main types of stochastic programming models:

- **Anticipative model:** Every decision is taken without knowing anything about the future.
- **Adaptive model:** Some information is available about the uncertain future before the decisions are made.
- **Recourse model:** Combines the above 2 models. Here some of the decisions are anticipative & some are adaptive.

A Stochastic 2-Stage Fixed Recourse problem looks like:

$$\begin{aligned} \text{Min } & c^T x + Q(x) \\ \text{s.t., } & A_x = b, x \geq 0 \end{aligned} \quad (1)$$

where;

$$Q(x) = \sum_j p_j Q(x, \xi_j)$$

Here, it is assumed that the random variable ξ has a discrete distribution where

$$\text{Prob}(\xi = \xi_j) = p_j \text{ for every } j \quad (2)$$

B. Bender's Decomposition

Bender's Decomposition in simple words is a way to divide complicated mathematical programming problems into 2 parts thereby simplifying the solution by solving one master problem & one sub-problem. In Stochastic Programming it is more common to refer to bender's decomposition as the L-shaped decomposition method.

Bender's Decomposition can be divided into 2 stages:

- Feasibility Cut
- Optimality Cut

1. Feasibility Cut

Feasibility cut is actually a constraint on the 1st stage variables. For a particular solution $x = 0$ of the 1st stage problem we need to check if it will yield feasible 2nd stage problems for all possible scenarios. If for any value of ξ the 2nd stage problem becomes infeasible then immediately it generates a feasibility cut so that it removes that particular solution from the solution set. Thus feasibility cuts are generated to make the 2nd stage problems feasible.

2. Optimality Cut

Optimality cuts are constraints based on the 1st stage variables x as well as θ . Optimality cuts are generated only after all the feasible cuts are found. The idea of the optimality cut is to gradually take us towards the optimal solution. The system see that our original problem is a minimization problem. So, it will be helpful if it could find a lower bound of

θ . Optimality cuts can therefore be looked upon as lower bounds of θ .

B. Modeling Assumptions

For the sake of clarity, the modeling assumptions considered in this work are listed as follows:

- The first-stage of the proposed UC problem (that represents the day-ahead market) embodies a dc network representation, while the second-stage (that represents the real-time operation) embeds an ac one. This assumption is consistent with the functioning of most real-world electricity markets.
- For the sake of simplicity, only wind power uncertainty is taken into account. However, other uncertainties can be incorporated into the model. The uncertainty of wind power production is modeled through a set of plausible scenarios based on the available forecasted data.
- The minimum up-time and minimum down-time constraints of thermal units are not considered in this paper. To consider them, additional binary variables are required [18].
- A number of units are available to provide reserve.
- The wind power production cost is assumed to be nil.
- All loads are assumed to be inelastic.
- Wind farms of Type 3 DFIG and Type 4 full converter are able to provide voltage support in steady-state and dynamically [19]. However, for the sake of simplicity, we assume unit power factor for all wind farms.
- The security constraints are not modeled in this work. However, such constraints can be easily incorporated in the proposed framework.

C. Formulation

The considered two-stage ac-UC problem is formulated as (3)–(23). Objective function (1) represents the system's expected cost, and is subject to first-stage constraints (2) and second stage constraints (3). The optimization variables of the ac-UC problem (1)–(3) are the elements of the set:

$$\begin{aligned} E^{UC} = & \sum_{(i \in G)t} [C_{it}^{SU} + C_i P_{it}^{DA}] + \\ & \sum_s \rho_s [\sum_{(i \in G)t} C_{it}^{r_{its}} + \sum_{(d \in D)t} V_d^{SH} L_{dts}^{SH}] \end{aligned} \quad (3)$$

The first two terms of (3) correspond to the system's cost at scheduling time (first-stage), while the other two terms refer to the expected cost in real-time operation (second-stage). The first term represents the start-up cost of the units, the second one refers to their production cost, and the third term represents the reserve deployment cost. Note that the reserve deployment cost (third term) refers to the production cost of the additional energy produced in real-time operation to offset the energy imbalance occurred due to wind power variability. This term is in fact the product of the generating unit's marginal cost and the production increment from day-ahead to real-time operation. Finally, the last term of (3) is the load curtailment cost.

The first-stage constraints are:

$$\sum_{(i \in G)t} P_{it}^{DA} + \sum_{(k \in k_n)t} W_{kt}^{DA} - \sum_{d \in d_n} L_{dt}^p = \sum_{m \in \Omega_n} P_{nmt}^{DA}(\theta), \quad \forall_n, \forall_t \quad (4)$$

$$P_i u_{it} \leq P_{it}^{DA} \leq \bar{P}_i, \quad \forall_i \in G, \forall_t \quad (5)$$

$$0 \leq W_{kt}^{DA} \leq \bar{W}_{kt}^{DA}, \quad \forall_k \in k, \forall_t \quad (6)$$

$$-R_i^- \leq [P_{i(t=1)}^{DA} - P_i^{ini}] \leq R_i^+, \quad \forall_i \in G \quad (7)$$

$$-R_i^- \leq [P_{i(t=1)}^{DA} - P_{i(t=1)}^{DA}] \leq R_i^+, \quad \forall_i \in G, \forall_t > 1 \quad (8)$$

$$-\pi \leq \theta_{nt}^{DA} \leq \pi, \quad \forall_n, \forall_t \quad (9)$$

$$\theta_{(n=1)t}^{DA} = 0, \quad \forall_t \quad (10)$$

$$P_{nmt}^{DA}(\theta) \leq \bar{S}_{nm}, \quad \forall_n, \forall_m \in \Omega_n, \forall_t \quad (11)$$

$$C_{it(t=1)}^{SU} \geq \lambda_i^{SU} [u_{i(t=1)} - u_i^{ini}], \quad \forall_i \in G \quad (12)$$

$$C_{it(t=1)}^{SU} \geq \lambda_i^{SU} [u_{i(t=1)} - u_{i(t=1)}], \quad \forall_i \in G, \forall_t > 1 \quad (13)$$

$$C_{it}^{SU} \geq 0, \quad \forall_i \in G, \forall_t \quad (14)$$

Constraints (4) represent the active power balance at scheduling time at each node and for each time period. Constraints (5) and (6) enforce the lower and upper bounds for active power production of generating units and wind farms, respectively. Constraints (7) and (8) ensure that the hourly changes of scheduled power do not violate the ramp-rate limits. Constraints (9) enforce lower and upper bounds of voltage angles. Constraints (10) set as the reference node. The capacity of each transmission line is enforced through (11). Constraints (12)–(14) allow calculating the start-up cost of the units.

The second-stage constraints are:

$$\sum_{(i \in G_n)} (P_{it}^{DA} + r_{its}) + \sum_{k \in k_n} (W_{kts} - W_{kts}^{SP}), \quad \forall_n, \forall_t, \forall_s \quad (15)$$

$$\sum_{(i \in G)t} [Q_{its} - \sum_{d \in d_n} L_{dt}^Q] = \sum_{m \in \Omega_n} Q_{nmt}^{DA}(v, \theta), \quad \forall_n, \forall_t, \forall_s \quad (16)$$

$$0 \leq L_{dts}^{SH} \leq L_{dt}^p, \quad \forall_k \in D, \forall_t, \forall_s \quad (17)$$

$$0 \leq W_{kts}^{SP} \leq W_{kts}, \quad \forall_k \in K, \forall_t, \forall_s \quad (18)$$

$$-R_i^D \leq r_{its} \leq R_i^U, \quad \forall_i \in G, \forall_t, \forall_s \quad (19)$$

$$P_i u_{it} \leq [r_{its} P_{it}^{DA}] \leq \bar{P}_i u_{it}, \quad \forall_i \in G, \forall_t, \forall_s \quad (20)$$

$$Q_i u_{it} \leq Q_{its} \leq \bar{Q}_i u_{it}, \quad \forall_i \in G, \forall_t, \forall_s \quad (21)$$

$$-R_i^- \leq [(P_{i(t=1)}^{DA} - r_{i(t=1)s}) - (P_i^{ini} + r_i^{ini})] \leq R_i^+, \quad \forall_i \in G, \forall_s \quad (22)$$

$$-R_i^- \leq [(P_{it}^{DA} + r_{its}) - (P_{i(t=1)}^{DA} + r_{i(t=1)s})] \leq R_i^+, \quad \forall_i \in G, \forall_t > 1 \quad (23)$$

Constraints (15) and (16) represent the active and reactive power balance in real-time operation at each node and for each time period and scenario. Active power balance constraints (15) enforce that the deviations of wind production are met with reserve deployment of generating units, and/or wind power spillage of farms, and/or curtailment of loads. Constraints (17)–(21) bound the value of unserved load, wind power spillage, active power reserve deployed, total active power production and reactive power production of generating units, respectively. Constraints (22) and (23) enforce ramp-rate limits. but for real-time operation. Finally, note that the proposed ac-UC problem (3)–(23) is mixed-integer, nonlinear and generally intractable. To make it solvable, Benders' decomposition is applied as described in the next section.

III. BENDER'S SOLUTION

This section proposes a solution strategy based on Benders' decomposition to solve (3)–(23).

A. Complicating Variables and Convexification

If first-stage variables and are fixed to given values in problem (3)–(23), this problem decomposes into 1) a scenario-independent mixed-integer linear problem (representing the first-stage), and 2) a set of nonlinear continuous problems, one per scenario (representing the second-stage). Therefore, and are complicating variables, and Benders' decomposition can be potentially applied [20].

Although the original ac-UC problem (3)–(23) is non-convex and Benders' decomposition is not generally applicable, if the number of wind power scenarios is large enough, the objective function (1) as a function of the complicating variables convexifies as shown in [21]. In other words, the objective function of an expected value stochastic programming problem convexifies as the number of scenarios increases. The reason of this is that the objective function represents the expectation over a number of scenarios. Thus, as the number of scenarios increases, the diversity of objective functions increases, while the weight of each single-scenario decreases. This results in a smoothing effect leading to the convexification of the expected value objective function. This convexification allows a successful implementation of Benders' decomposition. Benders' convergence is guaranteed if the objective function of the original problem projected on the subspace of the complicating variables has a convex envelope. The proposed ac-UC problem (1)–(3) is "sufficiently" convexified by considering a large enough number of scenarios, and our numerical analysis confirms the well-functioning of the proposed decomposition algorithm. Nevertheless, convergence cannot be generally guaranteed for the considered problem.

Finally, note that the asymptotic convexification yielded by increasing the number of scenarios is not a heuristic, provided.

B. Decomposition by Scenario and Time Period

Fixing the complicating variables and given values decomposes the ac-UC problem (3)–(23) by scenario. However, the ramping constraints (22) and (23), which links time periods, impede the ac-UC problem to decompose by time period. In general, an appropriate balance is needed between model accuracy and computational burden. To this end, a heuristic technique is used in this paper to decompose the proposed ac-UC problem by time period. This technique allows reducing the computational burden, but at the potential cost of introducing imprecision in the final solution. According to this heuristic technique, the inter-temporal ramping constraints (22) and (23) are relaxed and enforced just locally. That is, at time, ramping limits are enforced with respect to time, and time periods are processed successively from the first to the last one. However, note that if needed, a reduced number of hours (e.g., 3 or 4) may be processed at the same time, which may be helpful for periods with high increase/decrease in demand and/or renewable production levels. Our extensive numerical simulations show that the results obtained with and without such a heuristic technique are close enough. Note that the temporal decomposition used is myopic, and it cannot be applied in power systems with inter-temporal constraints (e.g. a power system with a significant number of hydro units or large-scale energy storage facilities) since these inter-temporal constraints cannot be locally relaxed. Hydroelectric systems are not prevalent in most parts of the world [22], but for such systems the proposed ac-UC problem is still applicable and computationally efficient by decomposing the problem only by scenario (and not by time period). Note that the number of

scenarios is generally much larger than the number of hours within the time horizon considered. The formulation of Benders' master problem and subproblems are provided in the next subsections.

C. Subproblem

The subproblem for scenario and time period is formulated as (24) below. All variables pertain to Benders' iteration:

$$E_{ts}^{SP} = \sum_{(i \in G)} C_i^{f_{its}^{(v)}} + \sum_{d \in D} V_d^{SH} L_{dts}^{SH(v)} \quad (24)$$

Subject to (3) and the constraints below,

$$P_{it}^{DA(V)} = P_{it}^{DA, fixed}, \quad \forall_i \in G \quad (25)$$

$$u_{it}^{(v)} = u_{it}^{fixed}, \quad \forall_i \in G \quad (26)$$

Objective function (24) represents total operation costs in real time operation. Constraint (25) comprises the second-stage constraints. Constraints (26) obtained from the solution of the master problem. The formulation of the master problem is provided in the next subsection. To prevent infeasibility, a number of non-negative slack variables are included in the reactive power and voltage magnitude constraints, along with penalties in the objective function (24) [20], [23].

D. Master Problem

The master problem corresponding to the original problem (3)–(23) is formulated as (27). All variables refer to Benders iteration. The objective function (27) corresponds to (3), where represents the expected cost in real-time operation:

$$Z_{down}^{(v)} = \sum_{(i \in G)_t} [C_{it}^{SU(v)} + C_i P_{it}^{DA(v)}] + \alpha^{(v)} \quad (27)$$

Subject to:

$$Z^{(j)} + \sum_{(i \in G)_t} \mu_{it}^{u(j)} (P_{it}^{DA(v)} - P_{it}^{DA(j)}) + \mu_{it}^{u(j)} (u_{it}^{(v)} - u_{it}^{(j)}) \leq \alpha^{(v)} \quad j = 1, \dots, v-1 \quad (28)$$

$$\alpha^{(v)} \geq \alpha^{down} \quad (29)$$

$$(2a), (2c), -(2k) \quad (30)$$

$$P_{it}^{DA(v)} + [\max_s r_{its}^{(v-1)}] u_{it}^{(v)} \leq \bar{P}_{it} u_{it}^{(v)}, \quad \forall_i \in G, \forall_t \quad (31)$$

$$P_{it}^{DA(v)} + [\max_s r_{its}^{(v-1)}] u_{it}^{(v)} \geq \bar{P}_{it} u_{it}^{(v)}, \quad \forall_i \in G, \forall_t \quad (32)$$

Constraints (28) are Benders' cuts, which are generated one per iteration. Note that the feasibility cuts are not required in the master problem because always-feasible subproblems are used within the proposed Benders' algorithm. This "trick" has proven to be computationally efficient. Constraint (29) imposes a lower bound on to accelerate convergence. Constraint (30) enforce all first-stage constraints, except (5). Instead of (5), constraints (31) and (32) are included in the master problem to improve convergence. Note that and are parameters obtained from the solution of the subproblems in the previous iteration. In fact, in addition to the Benders' cuts (28), constraints (31) and (32) further link the master problem and the subproblems. The value of objective function (27), that is a lower bound for the optimal value of the objective function of problem (2)-(23). The solution of the master problem (27) updates the values of complicating variables.

E. Bender's Algorithm

- 1) Input: a small tolerance ϵ to control convergence, and initial guesses of the complicating variables, $P_{it}^{DA(v)}$ and

$$u_{it}^{(v)} \quad \forall_i \in G, \forall_t$$

- 2) Initialization: Set $v=1, z_{down}^{(v)} = -\infty$

F. Flow Chart

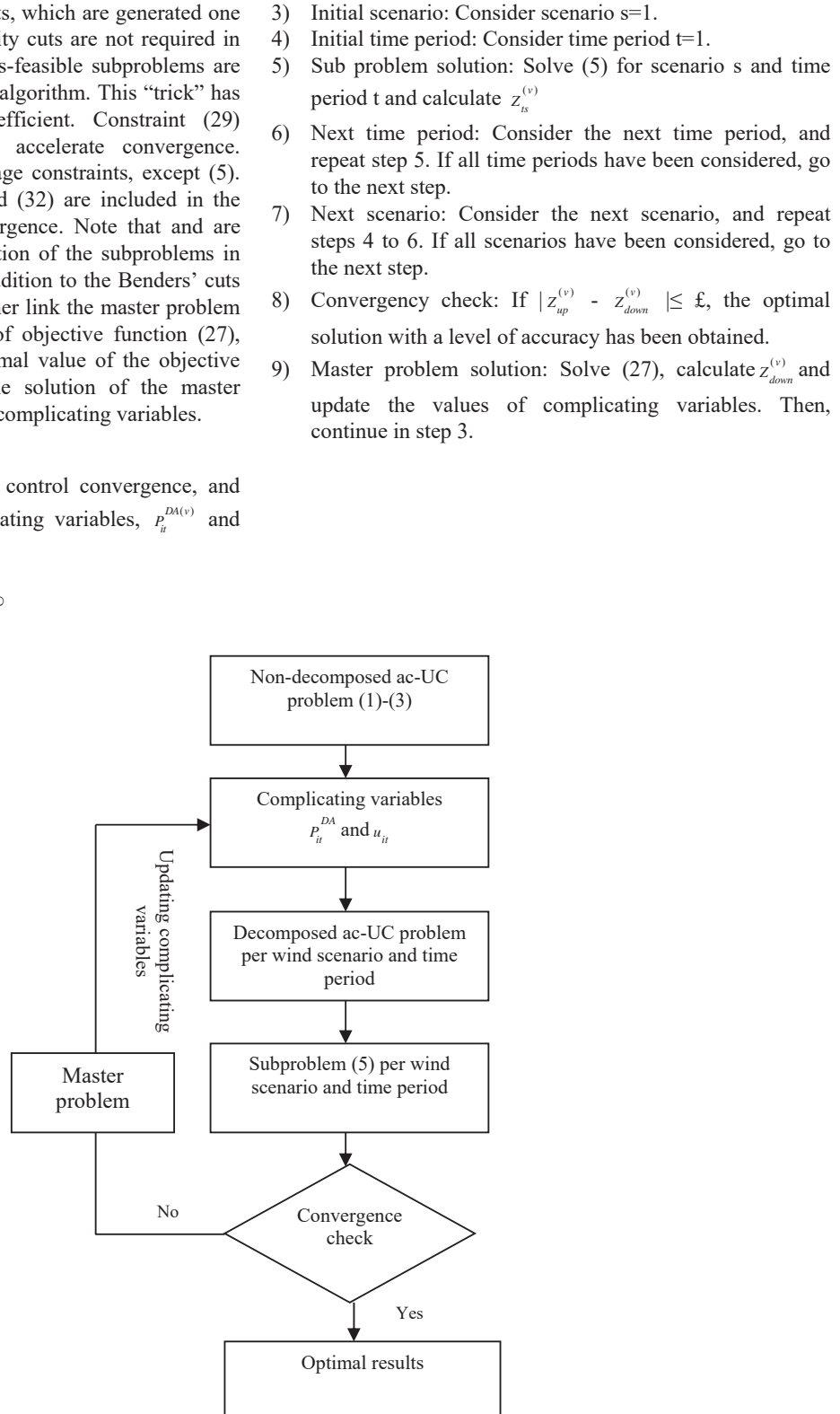


Fig. 1 Flowchart for Bender's Algorithm

TABLE I
NETWORK DATA

Transmission line (n,m)	1-3	3-9	6-10	10-11	11-14	14-16
$\overline{S}_{n,m}$	2.3	1.6	2.5	3.0	2.3	2.1

TABLE II
LOAD FACTOR CORRESPONDING TO EACH TIME PERIOD

Time period	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12
Load factor	0.75	0.70	0.65	0.60	0.62	0.63	0.65	0.68	0.70	0.72	0.75	0.78
Time period	t13	t14	t15	t16	t17	t18	t19	t20	t21	t22	t23	t24
Load factor	0.80	0.85	0.85	0.90	0.92	0.95	0.98	1.00	0.97	0.93	0.91	0.92

TABLE III
DATA FOR GENERATING UNITS

Unit (i)	Node	p_i	\overline{p}_i	Q_i	\overline{Q}_i	R_i^U, R_i^D	R_s^+, R_s^-	$C_i [\frac{\$}{p.u.}]$	$\lambda_i^{SU} [\$]$	u_i^{ini}	p_i^{ini}	r_i^{ini}
1,2	1	0.100	0.200	0.00	0.10	0.000	0.100	1109	300	1	0.20	0
3,4	1	0.152	0.760	-0.25	0.30	0.110	0.500	1246	400	1	0.76	0
5,6	2	0.100	0.200	0.500	0.10	0.000	0.100	1109	300	1	0.20	0
7,8	2	0.152	0.760	-0.25	0.30	0.100	0.500	1246	400	1	0.76	0
9	7	0.800	3.500	0.00	1.50	1.000	2.500	1720	100	0	0.00	0
10,11	7	0.150	1.000	0.00	0.60	0.550	0.850	1660	275	1	0.55	0.45
12-14	13	0.620	1.970	0.00	0.80	0.450	1.150	1408	300	1	1.97	0
15-19	15	0.024	0.120	-0.50	0.06	0.096	0.096	2141	400	0	0.00	0
20	15	0.500	1.550	-0.50	0.80	0.450	1.000	1592	200	1	1.10	0.45
21	16	0.500	1.550	-0.50	0.80	0.450	1.000	1592	200	1	1.10	0.45
22	18	1.000	4.000	-0.50	2.00	1.500	2.800	1917	250	0	0.00	0
23	21	1.000	4.000	-0.50	2.00	1.500	2.800	1917	250	0	0.00	0
24-29	22	0.000	0.500	-0.10	0.16	0.150	0.500	0	100	1	0.50	0
30-31	23	0.500	1.550	-0.50	0.80	0.450	1.000	1592	200	1	1.10	0.45
32	23	0.800	3.500	-0.25	1.50	1.000	2.500	1720	100	0	0.00	0

TABLE IV
PROBABILITY OF EACH SCENARIO

s	ρ_s	s	ρ_s	s	ρ_s	s	ρ_s	s	ρ_s	s	ρ_s	s	ρ_s	s	ρ_s
s1	0.01	s6	0.03	s11	0.05	s16	0.01	s21	0.03	s26	0.01	s31	0.02	s36	0.03
s2	0.01	s7	0.03	s12	0.05	s17	0.01	s22	0.04	s27	0.01	s32	0.02	s37	0.03
s3	0.01	s8	0.03	s13	0.01	s18	0.02	s23	0.04	s28	0.01	s33	0.02	s38	0.04
s4	0.02	s9	0.04	s14	0.01	s19	0.02	s24	0.05	s29	0.01	s34	0.02	s39	0.04
s5	0.02	s10	0.04	s15	0.03	s20	0.03	s25	0.05	s30	0.01	s35	0.02	s40	0.04

IV. SIMULATION RESULTS

Fig. 2 represents the simulation output for power consumption with respect to day. The power consumption varies with respect to demand. The power demand is calculated during scheduling time for real time operation. The power consumption varies from 100 to 900MW.

Fig. 3 shows the graph representing the cost for the power consumed per day. The cost of power for a particular day depends upon the unit of power consumption per day. Thus, this system helps in committing the required generators at low cost.

V. CONCLUSION

The increasing share of renewable energy, namely wind power generation, brings new challenges and concerns to power systems operators. They have to guarantee the power demand and generation balance. For systems with high

presence of wind power generation, the maintenance of this balance can be problematic. Periods with high levels of wind power generation combined with low demand create over-generation that causes difficulties to the system operation. As consequence, wind curtailments are expected, representing a waste of natural resources. Studies refer that good wind forecasting has an important influence on the reduction of the Unit Commitment costs. Another solution suggested is the aggregation of wind plants over wider geographical areas providing a mechanism capable of reducing wind plant variability. Recent studies indicate that taking into account the stochastic nature of the wind in the Unit Commitment procedure, more robust schedules could be produced. The ramping capacity of the power systems is pointed as being a crucial factor on the accommodation of wind power generation. The impact that the wind power forecasting has on the UC problem was evaluated by introducing wind power output scenarios. Performances of the deterministic and

stochastic solutions are compared based on the actual evaluation that considers all possible wind power scenarios. measured values of wind power generation and based on a risk

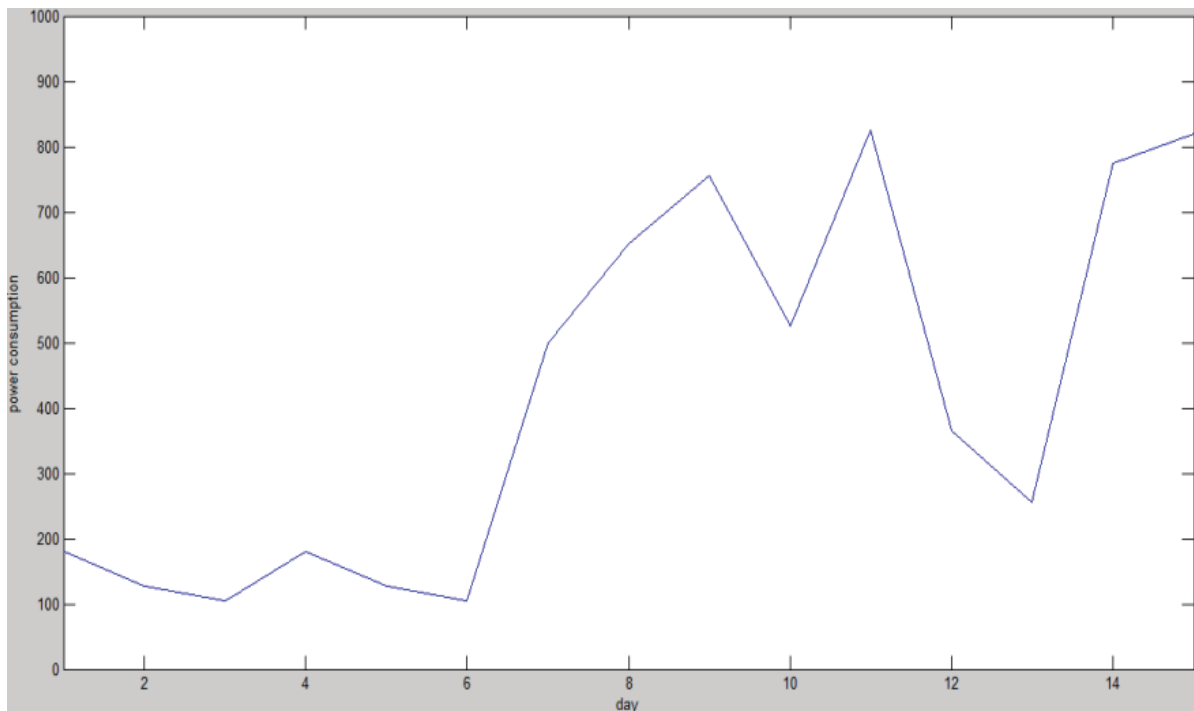


Fig. 2 Simulation Waveform for Power Consumption

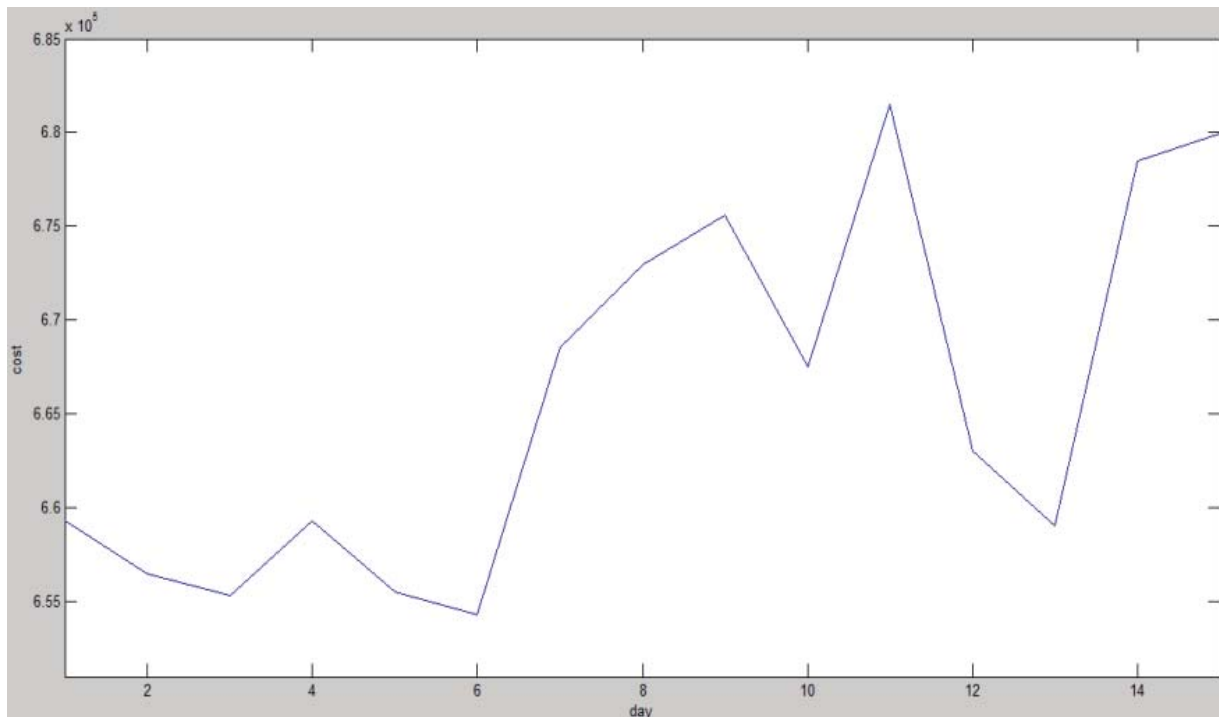


Fig. 3 Simulation Waveform for cost versus day

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