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Analysis and Simulation of TM Fields in Waveguides with Arbitrary Cross-Section Shapes by Means of Evolutionary Equations of Time-Domain Electromagnetic Theory

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***Abstract*—**The boundary value problem on non-canonical and arbitrary shaped contour is solved with a numerically effective method called Analytical Regularization Method (ARM) to calculate propagation parameters. As a result of regularization, the equation of first kind is reduced to the infinite system of the linear algebraic equations of the second kind in the space of L_2. This equation can be solved numerically for desired accuracy by using truncation method. The parameters as cut-off wavenumber and cut-off frequency are used in waveguide evolutionary equations of electromagnetic theory in time-domain to illustrate the real-valued TM fields with lossy and lossless media.

***Keywords*—**Arbitrary cross section waveguide, analytical regularization method, evolutionary equations of electromagnetic theory of time-domain, TM field.

I. Introduction

WAVEGUIDES are structures that direct electromagnetic energy along a desired path, such as transmission lines. Maxwell's equations predict that electromagnetic waves can also be guided through hollow metallic tubes.

In microwave engineering and microwave device manufacturing such as filters, couplers power dividers, polarizers and so, waveguides are a big fundamental partition. Analyses of propagation characteristics, such as cut-off wavenumber and propagation constant can be obtained easily, if the waveguide contour is rectangular, circular, or elliptical. For rectangular and circular case, it is easy to obtain with separation of variables (SVM) [1]. For elliptical case, it needs rigorous mathematical analysis and usage of Mathieu functions. Nevertheless, it is clear that for investigating problem of waveguides with arbitrary cross section, there is no paper which shows the usage of SVM.

Non-canonical, arbitrarily shaped waveguides are, in modern microwave systems, widely utilized, so accurate numerical methods are required to solve waveguide problems with rather arbitrary cross section.

Latest research shows that several techniques have been developed rapidly in this context, resulting in a very extensive literature [1].

In this paper, ARM is used to obtain the propagation parameters for arbitrary contours which are combined with the waveguide evolutionary equations of electromagnetic theory in time-domain. With the calculated parameters modal amplitudes and basis equations of TM field are derived. Finally, the real valued TM fields are illustrated for lossy and lossless media.

II. Previous Work

In literature, firstly an improved finite-difference technique is proposed for the electromagnetic eigenvalue problems. The improved implementation of finite-differences technique is presented based on well-known theories from [2]. Neumann and Dirichlet boundary conditions for homogenous Helmholtz's equation and second order Lagrange interpolation are used to determine the propagation constant, cut-off frequencies, modal field distributions (by eigenvectors) for finite differences technique. Nevertheless, error of calculating cut-off frequencies is not convergent to error of calculating eigenvector since there is no such parameterization for this calculation. Also error of calculating cut-off frequency is changing for each mode which does not show any convergence either [1]-[3].

Secondly, a new method for increasing the accuracy and decreasing the difficulty of the finite element method is presented in this paper [3]. Authors of the paper states that to improve the accuracy of such method, there are two ways. One way is to divide region into finer segments and the other one is to increase the order of the elements. However, increasing the order of elements is very difficult, and needs a lot of mathematical works. Previous papers focused on this method, show that difficulty comes from the related shape functions which are not properly described. In [3], rectangular waveguide is taken as an example. The geometry is divided into a triangle mesh. Some formulas for basic functions are presented to show the practicality of the method. But for more complicated shapes, more than a few dozen formulas are needed. Although with the

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usage of complex calculation the accuracy of the method could be increased, author avoided this hard mathematical work and decreased the accuracy of his own method.

Thirdly, a new method which is using boundary mode matching by dividing the boundary counter as planar part and circular arc is presented [4]. The transverse modes are presented in terms of circular waves whose cut-off wavenumber and amplitudes are the unknown values of the problem. Plane wave expression is used for the planar part of the boundary because Helmholtz's equation is automatically satisfied. As for the arcs, two different weighting functions are used since the center of the arc might be the same with the center of the circular wave or the different. Usage of fast Fourier transform does not require the necessity to use quadratic functions and accelerates the conversion between the different field representations [1].

III. Proposed Method

A. Analytical Regularization Method

ARM is used to solve the waveguides problems with arbitrary cross-section. The mentioned method has strong mathematical background and it is effective in numerical analysis [5]-[8].

Algorithm to solve the problem starts with the parameterization of the contour. Helmholtz' equation for the boundary value problem is solved respectively. By means of Green's functions, integral presentation of scattering field is obtained. Substitution of this integral representation into the boundary condition gives the relevant integral equation of first kind in space L_2. Then, the kernel is split into regular and singular parts, and expands every part into its Fourier series, obtaining finally an infinite algebraic system of the first kind. Then, by means of double-sided regularizator algebraic system of the first kind can be reduced to algebraic system of the second kind. The final equation system can be solved with any desired accuracy by truncation procedure [8].

Calculated results show that the presented method is mathematically strong, numerically accurate, and efficient not only for canonical shapes but also for non- canonical shapes of waveguides [9]-[11].

B. Waveguide Evolutionary Equations

Solving the Dirichlet boundary-value problem is essential to obtain the TM modal wave functions [12]. Wavenumbers are used to investigate the propagation of the electromagnetic wave along the waveguides. The details of evolutionary approach can be found in [12], [13]. We will introduce two variables which we scale by *z* and *t*.

$$\varsigma = k_m z \tag{1}$$

$$\iota = \omega_m t \tag{2}$$

where k_m and $\omega_m = k_m c$ are the propagation parameters which correspond to the cut-off wavenumber and cut-off frequency, respectively. TM waves can be written as,

$$\mathcal{H}_m = I_m(\varsigma,\iota)\mathbb{H}_m(\mathbf{r}) \tag{3}$$

$$\mathcal{E}_m = V_m(\varsigma,\iota)\mathbb{E}_m(\mathbf{r}) + f_m(\varsigma,\iota)\mathbb{Z}_m(\mathbf{r}) \tag{4}$$

where $\mathbb{H}_m(\mathbf{r})$, $\mathbb{E}_m(\mathbf{r})$ and $\mathbb{Z}_m(\mathbf{r})$ are the sets of modal basis obtained from solution to Dirichlet boundary-value problem [12]. The amplitudes $f_m(\varsigma,\iota)$, dependent on (ς,ι) are dimensionless quantities, and they should be found by solving equation,

$$\partial_\iota^2 f_m(\varsigma,\iota) + 2g\partial_\iota f_m(\varsigma,\iota) - \partial_\varsigma^2 f_m(\varsigma,\iota) + f_m(\varsigma,\iota) = 0 \tag{5}$$

where $g = \gamma/\omega_m$ is loss parameter and $\gamma = \sigma/2\epsilon_0$. The other modal amplitudes are,

$$V_m(\varsigma,\iota) = \partial_\varsigma f_m(\varsigma,\iota)$$

$$I_m(\varsigma,\iota) = -\partial_\iota f_m(\varsigma,\iota) - 2g\partial_\iota f_m(\varsigma,\iota) \tag{6}$$

Expected solution of $f_m(\varsigma,\iota)$ can be in the form of (7),

$$f_m(\varsigma,\iota) = e^{-g\iota}\tilde{f}_m(\varsigma,\iota) \tag{7}$$

where $\tilde{f}_m(\varsigma,\iota)$ is a new unknown function. Simple manipulations with (5) and (7) result in canonical Klein-Gordon Equation (KGE) as:

$$\partial_\iota^2 \tilde{f}_m(\varsigma,\iota) - \partial_\varsigma^2 \tilde{f}_m(\varsigma,\iota) + \eta^2 \tilde{f}_m(\varsigma,\iota) = 0 \tag{8}$$

$$V_m(\varsigma,\iota) = e^{-g\iota}\partial_\varsigma \tilde{f}_m(\varsigma,\iota) \tag{9}$$

$$I_m(\varsigma,\iota) = -e^{-g\iota}[\partial_\iota \tilde{f}_m(\varsigma,\iota) - g\partial_\iota \tilde{f}_m(\varsigma,\iota)] \tag{10}$$

where $\eta = \sqrt{1-g^2} \geq 0$, is the lossy parameter. The expected solution to (8) is in form of,

$$\tilde{f}_m(\varsigma,\iota) = A_m \sin[\Phi(\varsigma,\iota)] \tag{11}$$

$$\Phi(\varsigma,\iota) = (\varpi\iota - \varsigma\Gamma_m) + \varphi_m \tag{12}$$

$$\tilde{\Phi}(\varsigma,\iota) = \Phi(\varsigma,\iota) - \psi_m \tag{13}$$

where $\Gamma_m = \sqrt{\varpi^2 - \eta^2}$, $\varpi = \omega/\omega_m$, is dimensional frequency, $\psi_m = \arcsin(g/\sqrt{\varpi^2 + g^2})$ is phase shift of loss, ω is frequency parameter, $\omega_m = k_m c$, k_m is square root of an eigenvalue from Dirichlet problem, c is the speed of light. A_m and φ_m, are real-valued free numerical parameters. Exact explicit solutions lead us to have the energetic field properties in the time-domain [12]. The phase shift is ignored, and modal amplitudes are found as:

$$f_m^{\varpi}(\varsigma,\iota) = A_m e^{-g\iota} \sin[\Phi(\varsigma,\iota)] \tag{14}$$

$$V_m^{\varpi} = A_m\sqrt{\varpi^2 - \eta^2} e^{-g\iota} \cos[\Phi(\varsigma,\iota)] \tag{15}$$

$$I_m^{\varpi} = A_m\sqrt{\varpi^2 + \eta^2} e^{-g\iota} \cos[\tilde{\Phi}(\varsigma,\iota)] \tag{16}$$

Consequently, introducing a new energetic quantity yields:

$$P_m(\varsigma,\iota) = I_m^{\varpi}(\varsigma,\iota)\, V_m^{\varpi}(\varsigma,\iota) \tag{17}$$

$$W_m(\varsigma,\iota) = W_m^m(\varsigma,\iota) + W_m^e(\varsigma,\iota) \tag{18}$$

$$W_m^m(\varsigma,\iota) = [I_m^{\varpi}(\varsigma,\iota)]^2/2 \tag{19}$$

$$W_m^e(\varsigma,\iota) = [(V_m^{\varpi}(\varsigma,\iota)]^2 + [f_m^{\varpi}(\varsigma,\iota)]^2/2 \tag{20}$$

$$\dot{S}_M^{\varpi}(\varsigma,\iota) = 0.5(I_m^{\varpi 2} - V_m^{\varpi 2}) \tag{21}$$

$$w_M^{\varpi}(\varsigma,\iota) = 0.5 f_m^{\varpi 2} \tag{22}$$

P_m is power flow from waveguide contour, W_m is energy density, W_m^e and W_m^m are electric and magnetic energy density of the field. When the expected solutions (11) are applied to (21) and (22), we have:

$$S_M(\varsigma,\iota) = 0.5A_m^2\, e^{-2g\iota}\cos^2[\Phi_m(\varsigma,\iota)] \tag{23}$$

$$W_m(\varsigma,\iota) = 0.5A_m^2\, e^{-2g\iota}\sin^2[\Phi_m(\varsigma,\iota)] \tag{24}$$

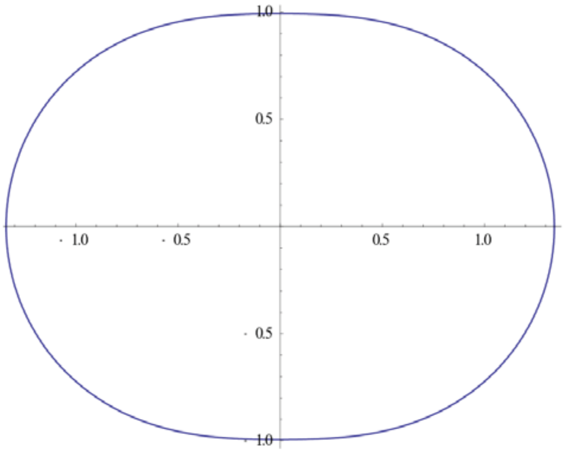


Fig. 1 Cassini Oval Contour (1) a=1.0 b=1.2

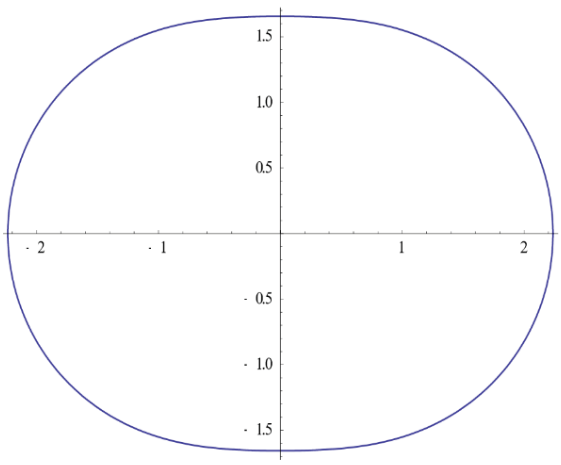


Fig. 2 Cassini Oval Contour (2) a=0.6 b=1.2

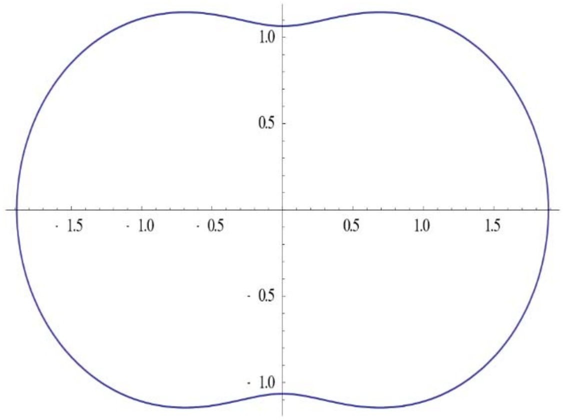


Fig. 3 Cassini Oval Contour (3) a=0.6 b=1.6

C. Arbitrary Shapes Used

The first object of our investigation has been the smooth-contoured waveguides of non-canonical cross section. We solved the ARM on shapes and formulae for them are taken from [14]. Equation of the Cassini Oval is,

$$r^4 - 2a^2r^2\cos 2\theta - b^4 + a^4 = 0. \tag{25}$$

IV. Results

TABLE I
CONVERGENCE OF K_M VALUES OF CONTOUR (1)

Contour (a)			
N	Km1	Km2	Km3
4	2,051107206	3,075268329	3,4696085839
8	2,051206388	3,052618821	3,4676661616
16	2,051206585	3,052589354	3,4677625896
32	2,051206585	3,052589354	3,4677625897
42	2,051206585	3,052589354	3,4677625897

TABLE II
CONVERGENCE OF K_M VALUES OF CONTOUR (2)

Contour (b)			
N	Km1	Km2	Km3
4	1,513912122	2,337541122	2,498500203
8	1,513898403	2,325088988	2,495522056
16	1,513898415	2,325079684	2,4955397
32	1,513898415	2,325079684	2,4955397
42	1,513898415	2,325079684	2,4955397

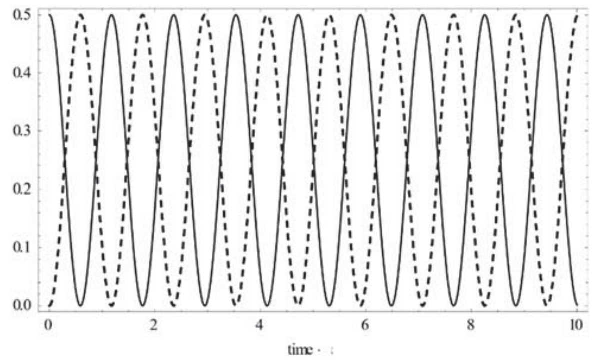
TABLE III
CONVERGENCE OF K_M VALUES OF CONTOUR (3)

Contour (c)			
N	Km1	Km2	Km3
4	2,54517904	3,204832497	4,467979862
8	2,520086608	3,107437562	3,951559388
16	2,519784987	3,105073113	3,941083358
32	2,51978494	3,105072588	3,941078545
42	2,51978494	3,105072588	3,941078545

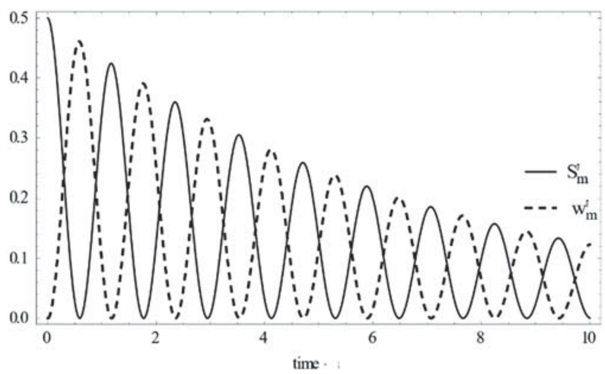
Calculation results of cut-off wavenumbers versus the dimension of system N for each contour are shown in the tables for fundamental mode respectively. The approximation of the results satisfies the reliability of analytical regularization method.

The energetic waves are obtained by means of evolutionary equations and illustrated graphically for lossy and lossless media where lossy parameter is $g = 0.07$ and dimensionless frequency $\varpi = 1.3$. Cut-off wavenumber of each contour is used for the propagation of the *TM*-modal wave with its amplitudes. Energy density $w_m(\varsigma, \iota)$ stored in the longitudinal component of the electric field and difference of the energy density $S_m(\varsigma, \iota)$ stored in the transverse components of the magnetic and electric fields are illustrated in Figs. 2, 4, and 6 for each cross section.

In Figs. 3, 5, and 7 the power and energy density of electric and magnetic field $P_m(\varsigma, \iota)$ and $W_m(\varsigma, \iota)$ stored in the waveguide cross section, which completely satisfies the theory of electromagnetic wave propagation, is simulated.

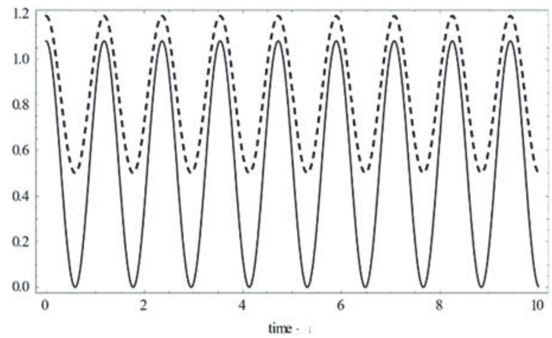


(a)

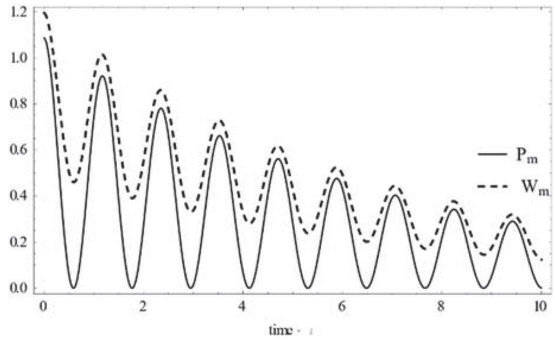


(b)

Fig. 4 Cassini Oval Contour (1) Field energy $S_M^{\varpi}(\varsigma, \iota)$ and energy density $w_M^{\varpi}(\varsigma, \iota)$with media, $k_m = 2.05$ (a) lossless (b) lossy

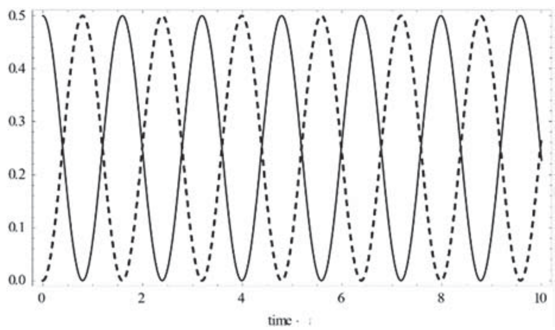


(a)

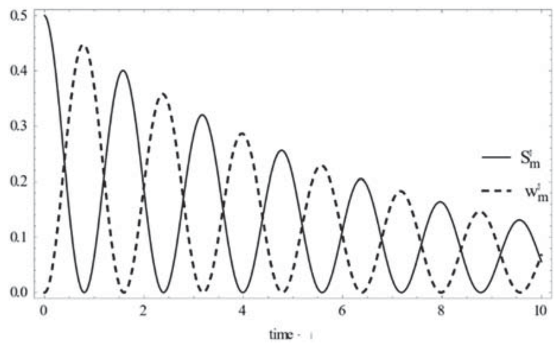


(b)

Fig. 5 Cassini Oval Contour (1) $P_m(\varsigma, \iota)$ and $W_m(\varsigma, \iota)$ for mediai $k_m = 2.05$ (a) lossless (b) lossy media.

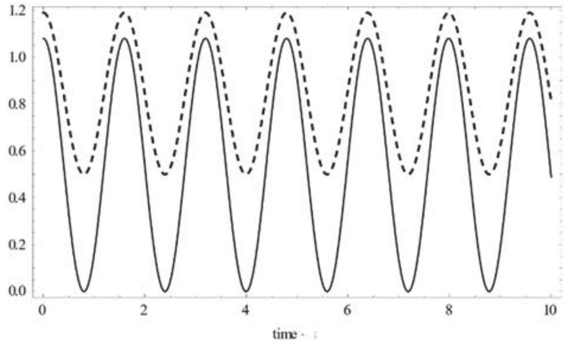


(a)

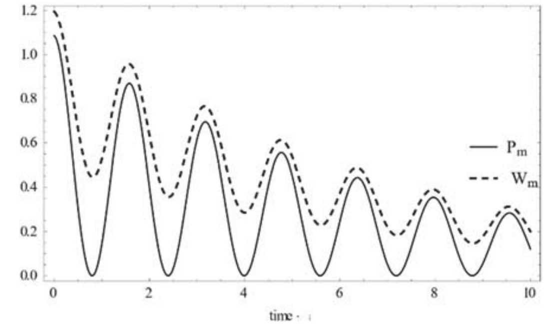


(b)

Fig. 6 Cassini Oval Contour (2) Field energy $S_M^{\varpi}(\varsigma, \iota)$ and energy density $w_M^{\varpi}(\varsigma, \iota)$with media, $k_m = 1.54$ (a) lossless (b) lossy

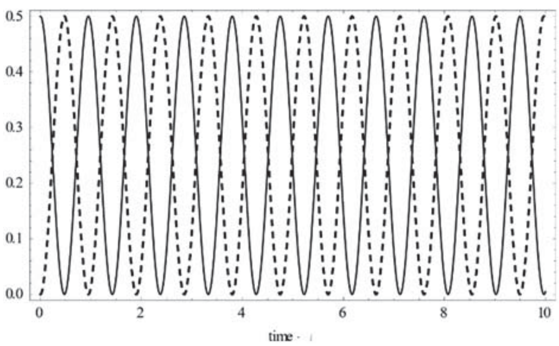


(a)

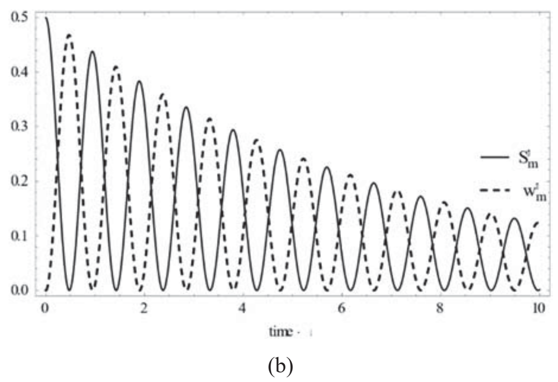


(b)

Fig. 7 Cassini Oval Contour (2) $P_m(\varsigma, \iota)$ and $W_m(\varsigma, \iota)$ for media, $k_m = 1.54$ (a) lossless (b) lossy media

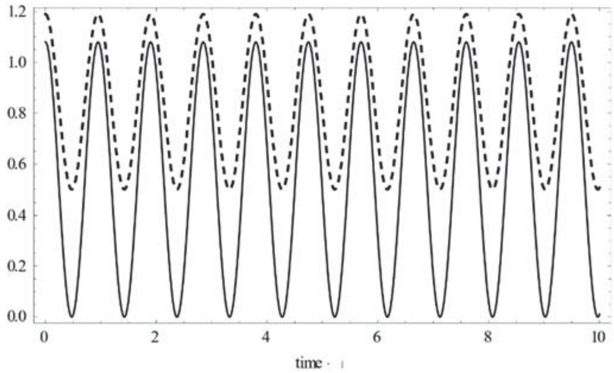


(a)

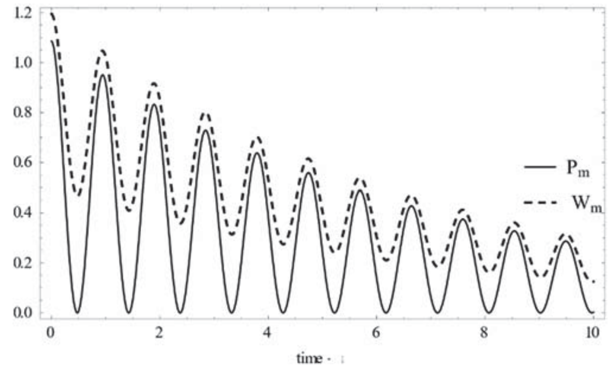


(b)

Fig. 8 Cassini Oval Contour (3) Field energy $S_M^{\varpi}(\varsigma, \iota)$ and energy density $w_M^{\varpi}(\varsigma, \iota)$with media, $k_m = 2.54$ (a) lossless (b) lossy



(a)



(b)

Fig. 9 Cassini Oval Contour (3) $P_m(\varsigma, \iota)$ and $W_m(\varsigma, \iota)$ for media, $k_m = 2.54$ (a) lossless (b) lossy media

V. Conclusion

Generally, difficulties arise due to the great complexity of many non-canonical resonant obstacles and cavities. In this paper, Cassini Oval is used as arbitrary shape. Each contour is solved by using ARM. Results (cut-off wavenumber and cut-off frequencies) are used to illustrate the energy densities of waveguide contour by means of the time domain evolutionary equations of waveguide electromagnetic theory.

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